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UNRAVELING THE SPREADING PATTERN OF COLLUSIVELY EFFECTIVE COMPETITION CLAUSES

Michael Trost
University of Hohenheim

Institute of Economics

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Michael Markus Trost†

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Abstract

Meanwhile, the Industrial Organization literature gives several reasons why retailers adopt competition clauses (CCs) such as price matching or price beating guarantees. The motivations underlying the CCs might affect their forms and spread. In this paper, we unravel the spreading pattern of CCs in markets where they are used as a device to facilitate tacit collusion. It turns out that in homogeneous markets with capacity-constrained retailers, the retailers with the largest capacities are most inclined to adopt CCs. Our finding is in line with results of earlier studies on the formation of price leadership, which suggest that the retailers with the largest capacities take on the leadership position. On the other side, we find that in some market instances, retailers have to resort to CCs of non-conventional forms (i.e., of forms uncommon in real commercial life) to induce the most robust collusion. However, it turns out that this peculiar finding can be resolved for markets with additional characteristics. For example, if there exist market dominant retailers or the residual market demand is specified by efficient rationing, the most resilient collusion can also be enforced by CCs of conventional forms.

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†Institute of Economics, University of Hohenheim, trost.michael@uni-hohenheim.de.

1 Introduction

Competition clauses (CCs) are a widespread practice in retail markets, in particular in market segments such as consumer electronics, office supplies, DIY products, and car components. Such a clause is a promise of a retailer to refund its customers whenever they are able to prove that its merchandise is offered at a lower price by some of its competitors.

In real commercial life, different variants of CCs are applied. If the customers are eligible for a refund so that the price they actually pay at the retailer corresponds to the lowest announced price in the market, the competition clause is referred to as a *price-matching guarantee* or a *meeting competition clause* (MCC). If they are entitled for a refund so that the price they actually pay is less than the lowest announced price in the market, the clause is called a *price-beating guarantee* or a *beating competition clause* (BCC).

Since the seminal papers of Hay (1982) and Salop (1986), many competition economists consider such clauses with certain unease. Despite of their pro-competitive appeal, competition clauses are regarded as a cunning device facilitating tacit collusion. The reasoning backing this view seems plausible: If a retailer offers the MCC or a BCC, then any price undercutting announced by one of its competitors is immediately matched or even beaten by the retailer as its customers would then exercise the clause. This in turn entails that the retailer offering such form of a CC is able to announce an excessive price without worrying about being undersold.

The fundamental issue underlying this paper is whether there exists a pattern in the adoption of CCs if they are used by the retailers to facilitate tacit collusion. Being more precise, we ask given that the retailers have different market shares, which of them are most incentivized to implement CCs to enforce collusive behavior in the market.

Earlier theoretical studies such as the ones of Doyle (1988) and Corts (1995) provide a definite answer to this question. These studies follow the above logic throughout to its end and conclude that a market-wide adoption of CCs is imperative for inducing collusion. They argue that even if only one of the retailers abstained from offering a CC, but announced a supracompetitive price, its competitors would profit by slightly underselling this retailer and, thus, unleashing the standard price-cutting competition.

However, the requirement of market-wide adoption of CCs is in stark contrast with empirical findings. It has been observed that if any, then usually only a few of the retailers in the market adopt CCs, see e.g. the expositions in Arbatskaya et al. (2006), Table 2 in Moorthy and Zhang (2006) or Table I in Jiang et al. (2017). If one embraced the claim of the classical studies we mentioned above, one would have to acknowledge that CCs are adopted for other reasons than facilitating collusion.

Meanwhile, there are numerous theoretical studies suggesting other motives behind the implementation of CCs than the collusive one as well as predicting (or at least being consistent with) partial adoption of CCs. A common feature of those studies is that they take for granted that some form of incomplete knowledge prevails in the market so that CCs are introduced with the purpose to exploit or remove these information gaps.

Several of those studies such as the pioneering work of Png and Hirshleifer (1987) as well as the papers of Corts (1997), Chen et al. (2001), and Hviid and Shaffer (2012) segment the demand side of the market in differently informed consumer groups. For example, one group of consumers, the so-called non-shoppers or tourists, are aware of only one retailer while others, the so-called shoppers, are well-informed and know the clause policy as well as the price chosen by any retailer. In such setting of asymmetric information, CCs are used by the retailers as a device to price differentiate between the less- and well-informed consumers.

More recent studies such as the ones of Janssen and Parakhonyak (2013), Yankelevich and Vaughan

(2016), and Jiang et al. (2017) propagate this motive on the basis of more sophisticated economic models. They endogenize the asymmetric information distribution by assuming that consumers are heterogeneous regarding their price search costs.

Other scholars such as Jain and Srivastava (2000), Moorthy and Winter (2006), and Moorthy and Zhang (2006) take for granted that all consumers know the clause policies of all retailers, but not the prices the retailers actually charge for the merchandise. In such setting of symmetric, but imperfect information, a CC might become an effective device for signaling low prices. In opposite to the high cost retailer, the low cost retailer is incentivized to adopt a CC in order to signal to its customers that it charges the lowest price in the market.

Recently, Mehra et al. (2018), Chen and Chen (2019), and Xu et al. (2021) give a further reason for implementing CCs. They claim that such clauses prevent "showroomming", i.e., the well-known practice of examining the suitability or quality of the merchandise in a traditional brick-and-mortar store, but then buying it online. By promising to match any price charged by the online competitors, brick-and-mortar retailers might attract customers to buy the merchandise from them instead of acquiring it online.

This brief literature review suggests that partial adoption of CCs might occur if retailers have other purposes in mind than inducing collusion. However, there have been also attempts to reconcile the theory of CCs as a facilitating practice with the general observation of partial adoption of CCs.

Logan and Lutter (1989) as well as Hviid and Shaffer (1999) show that in asymmetric duopolies (i.e., retailers are asymmetric regarding costs or demands), supracompetitive prices results even if only one retailer adopts the MCC. However, these prices are only modest above the competitive ones. Hviid and Shaffer (2010) study the collusive efficacy of clauses combining the MCC with the most favored customer clause (MFC) in spatially differential duopolies.[1] They conclude that if at least one of the retailers offers such composite clause, then supracompetitive prices become enforceable. Unlike the former papers, Belton (1987), Trost (2016), and Pollak (2017) assume sequential price setting. In such setting, the monopolistic price can be reached by the price leader adopting a CC.

Additional characteristics of the market environment are taken into account in the competition model of Trost (2021). It is assumed that capacity-constrained retailers interact repeatedly. Moreover, the CCs are regarded as binding commitments and, thus, are chosen by the retailers before they compete in prices. A crucial result of this model is that collusion is enforceable even by partial adoption of CCs, at least for a wide range of common discount factors.

An implicit assumption of the above mentioned studies on CCs is that the commodity is provided only by the indirect supply channel (i.e., all retailers source the commodity from an independent manufacturer). This assumption is dropped in Nalca (2017) and Corts (2018). They consider retail markets with dual supply channels. The manufacturer of the commodity is part of a vertically integrated firm. This firm not only delivers the commodity to an independent retailer, but also offers it directly to the consumers. Both studies conclude that if the vertically integrated firm is dominant, then none of the retailers adopts the MCC, and if the independent retailer is dominant, then both do it. However, as argued in Nalca (2017), in the case that none of the two is too dominant, it suffices that the independent retailer offers the MCC to induce the monopolistic price.[2]

[1] A MFC guarantees the customers to pay not more than the lowest price the retailer ever charges in the future. More precisely, whenever the commodity they bought is offered by the retailer at a lower price at some later time, they are entitled to receive a refund in the amount of the difference between the price they paid and the lower price.

[2] This result might be at odds with observations of several market segments such as air travel, accommodation services, and computer hardware; see for example the exposition in Jiang and He (2021). In those markets, the vertically integrated manufacturers are often the ones offering CCs. Jiang and He (2021) provide a market model with CC-insensitive and channel-favoring consumers to explain these observations. However, unlike Nalca (2017), they take for the granted that only the vertically integrated manufacturer is able to offer CCs.

Obviously, the welfare effects of CCs are unambiguously negative if they are used as a device inducing collusion. However, such unambiguity cannot be upheld *a priori* if they are adopted for other purposes like the ones we mentioned above. Indeed, in some of these instances, the implementation of CCs might increase the consumer welfare; see e.g. the results of Corts (1997) in case of price differentiating, the results of Moorthy and Winter (2006) in case of signaling, and the results of Chen and Chen (2019) in case of showrooming.

From an antitrust perspective, it might be interesting to know whether there are observable criteria delineating cases in which CCs might induce collusion from other case in which CCs are adopted for less consumer harming purposes. The partial adoption of CCs might not be such a criterion. As suggested by our above literature review, such an observation could be consistent with tacit collusion as well as other less concerning purposes.

This finding is where our paper steps in. We aim at figuring out whether there exists a specific pattern in the (partial) adoption of CCs indicating tacit collusion. To accomplish this task, we resort to the competition model proposed by Trost (2021). This model might have several advantages over other models used so far to study the competitive effects of CCs.

First, the number of retailers is arbitrary and not restricted to two as in most of the other studies. Moreover, the retailers are assumed to be heterogeneous regarding their sales capacities, entailing different market shares. A further striking feature is that the types of the CCs between which the retailers can choose are not restricted to the MCC or specific BCCs. Apart from that, the CCs are viewed in our model as binding commitments the retailers enter towards their customers and, thus, are chosen before they set the prices of the merchandise. This assumption reproduces the reasonable conjecture that CCs might be more difficult to change than prices in real commercial life. The competition between the retailers is modeled as a multi-stage game so that the retailers interact repeatedly and can also resort to punishment strategies to enforce collusion.

The main finding of the paper is that the largest retailers (i.e., the retailers with largest market shares) are the ones most incentivized to adopt CCs in order to facilitate collusion. This insight could be used as a guidance for antitrust authorities. If such pattern in the adoption of CCs is observed, antitrust authorities might be induced to take a closer look at the market and examine whether further indications of tacit collusion prevail.

Remarkably, there is a analogy between the finding of our paper and earlier results concerning the facilitating practice of price leadership. Deneckere and Kovenock (1992) as well as Ishibashi (2008) establish for market environments very similar to the one we consider that the retailer with largest capacity becomes the price leader. Indeed, as will be pointed out later, our paper can be seen complementary to those two studies.

The paper is structured as follows. In Section 2, the competition model underlying our analysis is presented. As mentioned above, we adopt the framework proposed by Trost (2021) and consider an infinitely repeated Bertrand competition with clause policies and capacity constraints. The game is solved by the concept of subgame perfectness in Section 3. In particular, we characterize the set of clause policies inducing collusion in a compact way.

As there is a plethora of such clause policies, plausible refinement criteria are introduced in Section 4. Based on cost-efficiency and robustness considerations, we single out the most collusive clause policies which we henceforth term “robustly collusive”. The core result of our analysis is derived in this section: It turns out that the retailers with the largest capacities are the ones which are most incentivized to adopt CCs. However, we detect market environments in which none of the retailers adopt a conventional CC (i.e., a CC of a form widely used in real commercial life) in any robustly collusive outcome. The latter finding is provoking and induces us to analyze specific market regimes.

In Section 5, we consider markets with dominant retailers, i.e., retailers with capacities large enough to serve the market demand at the competitive price. It turns out that if at least two dominant retailers exist in the market, robust collusion can always be reached by conventional CCs. Remarkably, it suffices in this case that the two largest retailers adopt CCs.

In Section 6, we consider markets with specific rationing rules. Rationing rules specify the residual demand faced by the retailer undersold by some of its (capacity-constrained) competitors. We will show that under regular rationing rules the most collusive spreading pattern of conventional CCs is the one in which the largest retailers adopt them. Nevertheless, the assumption of regular rationing is too weak to ensure that robust collusion can be enforced by conventional CCs. However, if efficient rationing is assumed, this becomes possible.

Section 7 summarizes our findings and points to some of their limitations. The proofs of the results are relegated to the Appendix.

2 Bertrand Competition with Competition Clauses

The competition model on which our analysis is based corresponds to the one proposed by Trost (2021). Nevertheless, to provide a self-contained paper, we entirely describe this model in the following subsections. In addition, we point to some specific features to which we will resort later.

The markets we analyze are oligopolistic retail markets. Their supply side consists of a set $I := \{1, \dots, n\}$ of retailers where $n \geq 2$. All retailers offer the same commodity and the provision of the commodity causes constant and identical marginal costs $c \geq 0$ for any of them. Moreover, each retailer faces a (positive and real-valued) capacity constraint, the upper limit of the commodities the retailer is able to supply. Without loss of generality, we assume that their capacities differ from each other so that we can arrange them in an increasing order $k_1 < k_2 < \dots < k_n$.[3] The capacity of the non-empty set $J \subseteq I$ of retailers is summarized by $K_J := \sum_{j \in J} k_j$. For the sake of convenience, we define $K_\emptyset := 0$ and denote the total (market-wide) capacity by $K := K_I$. The retailer i's share of the total capacity is denoted by $\kappa_i := \frac{k_i}{K}$ and J's share of the total capacity by $\kappa_J := \frac{K_J}{K}$.

The competition between the retailers is modeled as a multi-stage game with perfect information and infinite horizon. In the first stage (period $t = -1$), the retailers simultaneously announce their competition clause policies. Afterwards, the retailers participate in an infinitely repeated Bertrand competition, i.e., they simultaneously announce prices for the commodity in any of the succeeding and infinitely countable stages (periods $t = 0, 1, \dots$). We call the first stage the *clause implementation phase* and the succeeding stages the *price competition phase*. The timing of our competition game is depicted by the below figure in which the arrow represents the time axis.

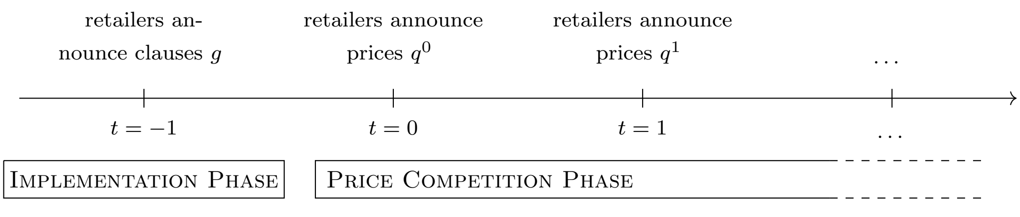

Figure I: The timing of the competition game

[3] The analysis could be extended to include cases in which retailers have identical capacities. However, such generalization would expand the set of solutions without any added value. It then would hold: If outcome o satisfies property P, then outcome $\tilde{o}$ which differs from o only by permuting the retailers so that the capacities are still ordered in a non-decreasing way would also satisfy this property. To circumvent such multiplicity, we have refrained from such generalization.

Our game-theoretical setup resembles that adopted in the literature on (partial) cartel formation, e.g. that of Selten (1973), D'Aspremont et al. (1983), Escrihuela-Villar (2008), and Bos and Harrington (2010). Those studies endogenize the formation of cartels by multi-stage games where a cartel participation stage in which the firms decide about whether to enter the cartel precedes the competition stage. The competition between the firms is modeled differently in those studies; either as a finite game like in the two former articles or as an infinitely repeated game like in the later two articles. Besides this time horizon, our setup has in common with Bos and Harrington (2010) that the firms are heterogeneous with regard to their capacities.

A review of the game-theoretical studies on clause policies reveals that different timing structures regarding the retailers' decisions have been considered. Some authors like Doyle (1988), Corts (1995), and Kaplan (2000) suppose that each retailer decides simultaneously about the adoption of CCs and the advertised prices. A sequential timing structure in which the competition clauses take the form of binding commitments and are announced before the prices are fixed is adopted e.g. in Logan and Lutter (1989), Zhang (1995), Chen (1995), and Liu (2013). However, different time horizons of the price competition phase are assumed in these articles. The former three regard price competition as a one-stage game, whereas the latter models it as an infinitely repeated game like we do.

The only other paper we know which studies the collusive efficacy of CCs in an infinitely repeated competition game is that of Cabral et al. (2021). Unlike Liu (2013), they assume that the retailers decide about the prices alternately. Nevertheless, both papers are more restrictive in several aspects than ours. First, they only consider duopolies. Moreover, the retailers are assumed to have no capacity constraints and can choose only among specific forms of CCs. A further crucial difference is that the adoption of the CCs is not endogenized in those studies.

To the best of our knowledge, the only paper which has so far examined the collusive efficacy of CCs in markets with capacity-constrained retailers is that of Tumennasan (2013). It rests on the two-stage duopoly framework proposed by Kreps and Scheinkman (1983); the retailers choose their capacities in the first stage and the commodity prices in the second stage. The novel feature in the model of Tumennasan (2013) is that in the second stage, each duopolist has the additional option to implement the MCC.[4]

Our approach turns the approach of Tumennasan (2013) upside down. While Tumennasan (2013) examines the effect of the MCC on the sales capacities of the retailers, we study the other direction. The retailers' clause policies rather than their capacities are endogenized in our paper. In doing so, we suppose that the capacities of the retails are invariable for an indefinite period of time. We do not deny that clause policies might have a feedback effect on the retailers' capacities. However, we think that these effects become into force only in the long run.

In the following three subsections, we will detail the peculiarities of our market model. The exposition follows the chronological order of the game. First, we specify the options available for the retailers in the clause implementation phase. After that, we turn to the price competition phase and describe the market environment the retailers face. A comprehensive game-theoretical description of our model is provided in the last subsection.

To abridge the succeeding presentation, we introduce additional notation. Let A be some set. The indicator mapping of set A is denoted by $\mathbf{1}_A : X \to \mathbb{R}$, i.e., $\mathbf{1}_A(x) = 1$ if $x \in A$ and $\mathbf{1}_A(x) = 0$ otherwise. The cardinality of A is expressed by $|A|$. By definition, $|I| = n$. As is standard, we denote the sets of integers and real numbers by $\mathbb{Z}$ and $\mathbb{R}$, respectively. The set of the non-negative integers and non-negative real numbers are represented by $\mathbb{Z}_+$ and $\mathbb{R}_+$, respectively. If the number zero is excluded from $\mathbb{R}_+$, we write $\mathbb{R}_{++}$. The set of the non-negative real n-tuples is denoted by $\mathbb{R}^I_+$.

[4] The results of this paper are mixed. The MCC might not necessarily increase the prices in this framework. If the capacity costs are sufficiently large, then MCC either has no effect on the prices or even leads to a price decrease.

Subsets of I are called *coalitions* of retailers. If number 0 is added to I, we write $I_0 := I \cup \{0\}$. For any $k \in I_0$, we define $J_k := \{i \in I : i \leq k\}$, i.e., J_k is the coalition of the k smallest retailers. Obviously, $J_0 = \emptyset$ and $J_n = I$. The complement of coalition J is denoted by $-J := I \setminus J$. According to this notation, $-J_k$ represents the coalition of the $n-k$ largest retailers. As usual, we simply denote the coalition of all retailers except for retailer i by $-i$.

The set of all permutations on I is denoted by Σ_n. Pick some $i, j \in I$ where $i \neq j$. The permutation $\tau_{i,j}$ specified by $\tau(i) := j$, $\tau(j) := i$ and $\tau(k) := k$ for any $k \neq i, j$ is called the *transposition swapping i and j*. A permutation σ is said to be *increasing on* J whenever $\sigma(j) \geq \sigma(i)$ for any $i, j \in J$ satisfying $j > i$. If $\sigma(j) > \sigma(i)$ for any $i, j \in J$, then it is *strictly increasing* on J. A permutation σ is called an *upshifting on* J whenever $\sigma(j) \geq j$ for any $j \in J$ and a *downshifting on* J whenever $\sigma(j) \leq j$ for any $j \in J$.[5] We say it is *non-constant on* J whenever $\sigma(j) \neq j$ for some $j \in J$.

The *extreme upshift permutation on* J is the permutation υ defined by

$$\upsilon(i) := \begin{cases} n + 1 - |\{j \in J : j \geq i\}| & \text{if } i \in J \\ n + 1 - |J| - |\{j \in I \setminus J : j \geq i\}| & \text{otherwise.} \end{cases}$$

In words, the extreme upshift permutation on J changes the indices of the retailers in the way so that the retailer with the k largest index in coalition J will be tagged with index $n + 1 - k$ and the retailer with the k largest index in coalition $-J$ will be tagged with index $n + 1 - |J| - k$. As can be easily checked, υ proves to be the unique permutation which is increasing on both J and $-J$, upshifting on J, and satisfying $\upsilon(J) = I \setminus J_{n-|J|}$. Obviously, if $J = \emptyset$ or $J = I$, then the extreme upshift permutation on J corresponds to the identity mapping.

An n-tuple $x := (x_i)_{i \in I}$ is referred to as a *profile of realizations*. We sometimes express profile x by (x_J, x_{-J}) and, if $J = \{i\}$, simply by (x_i, x_{-i}). In the case that all components of profile x take on the same value α (i.e., $x_i = \alpha$ for any $i \in I$), we simply write α instead of $(\alpha)_{i \in I}$. This simplification might not cause confusion as it should become clear from the context whether α represents a value or a profile. If all values of profile x are real numbers, then x is termed numerical. The lowest value of a numerical profile x is denoted by $x_{\min} := \min\{x_i : i \in I\}$ and its greatest value by $x_{\max} := \max\{x_i : i \in I\}$. Consider some numerical profile $x := (x_i)_{i \in I}$ and some $\alpha \in \mathbb{R}$. We denote the set of all retailers which have realized a value equal to α by $[x = \alpha]$. In general, if R is binary relation on $\mathbb{R}$, we define $[xR\alpha] := \{i \in I : x_i \, R \, \alpha\}$.

As is standard, the composition of permutation $\sigma \in \Sigma_n$ with n-tuple $x := (x_i)_{i \in N}$ is denoted by $x \circ \sigma$. Suppose x is numerical and σ be a permutation on I so that composition $y := x \circ \sigma$ gives the values of x in an increasing order, i.e., $i < j$ implies $x_{\sigma(i)} \leq x_{\sigma(j)}$. That means, y_i represents the i-th smallest value among the values listed in x. It is called the *i-th order statistic of x* and is henceforth denoted by $x_{(i)}$. Obviously, $x_{(1)} = x_{\min}$ and $x_{(n)} = x_{\max}$.

2.1 Clause Implementation Phase

The first stage of our competition game is the clause implementation stage. In this phase, each retailer decides on its clause policy. That is, each retailers stipulates the form of the price guarantee it offers its customers. In formal terms, a *clause g_i of retailer i* is a mapping $g_i : \mathbb{R}_+^I \to \mathbb{R}_+$, which specifies the sales price $g_i(q)$ guaranteed by retailer i for any announcement $q := (q_i)_{i \in I}$ of prices. In accordance with the terminology widely used in the Industrial Organization literature, we call $g_i(q)$ the *guaranteed price* of retailer i and profile q the retailers' *advertised prices*.

[5]To see the difference between increasing and upshifting, consider the permutation σ on $I := \{1, 2, 3, 4\}$ defined by $\sigma(1) := 4$, $\sigma(2) := 3$, $\sigma(3) := 1$, $\sigma(4) := 2$. Apparently, σ is upshifting, but not increasing on $\{1, 2\}$. However, it is increasing, but not upshifting on $\{3, 4\}$.

The above definition implicitly assumes that clauses are tied to the advertised prices, but not to the guaranteed ones. In this regard, we follow the approach of Corts (1995) rather than that of Kaplan (2000). Interestingly, as argued in Kaplan (2000), clauses referring to the guaranteed price prove to be collusively effective in static competition models with simultaneous price and clause setting, whereas clauses referring only to the advertised prices fail to be. Nevertheless, Arbatskaya et al. (2004) point out in their empirical study that the majority of clauses in real commercial life are explicitly restricted to advertised price.[6]

Even though we are in line with the approach of Corts (1995), the set of available clauses is substantially different in our setting. While Corts (1995) requires that the price guaranteed by a retailer be based only on two advertised prices, the price advertised by the retailer and the lowest advertised price in the market, we allow that the guaranteed price depends on any advertised price. Therefore, our approach includes clauses directed against some specific competitors.

From now on, we also take for granted that any clause g_i satisfies the two conditions

(G1) $g_i(q) = q_i$ if $q_i \leq \max\{c, q_{\min}\}$,

(G2) $c \leq g_i(q) \leq q_i$ if $q_i > \max\{c, q_{\min}\}$.

and denote the set of the clauses of retailer i satisyfing them by G_i.

Assumption (G1) requires that the guaranteed price correspond to the advertised price whenever the advertised price is the lowest advertised price in the market or not above the marginal costs. This restriction precludes so-called "beat-any-deal" clauses. Such clauses promise the customers to undercut any (and not only any lower) price advertised by some competitor at least by some specified amount or percentage.[7]

Assumption (G2) ensures that if the advertised price exceeds the marginal costs, then the guaranteed price also does it. This requirement can be interpreted as an exit option for the retailer. It precludes that a retailer is forced due to its clause policy to sell the commodity at a price below its marginal cost. A justification of this restriction might be that such loss-making situations are not sustainable so that sooner or later such clause policy will be abandoned. This restriction is in line with several theoretical studies on clause policies which also take for granted that there is a lower bound on the guaranteed prices; for example, Kaplan (2000) assumes that the guaranteed prices are non-negative like any other price.[8]

To bring an arbitrary clause g_i of retailer i in line with Assumptions (G1) and (G2), a transformation is required. Let $t_i : \mathbb{R}_+^{\mathbb{R}_+^I} \times \mathbb{R}_+^I \to \mathbb{R}_+$ be the transformation mapping specified by

$$t_i(g_i, q) := \min\{\max\{c, g_i(q)\}, q_i\}$$

for any clause g_i of retailer i and any profile $q := (q_j)_{j \in I}$ of advertised prices. As can be easily checked, if $g_i(q)$ is redefined as $t_i(g_i, q)$ for any profile q of advertised prices, then g_i satisfies Assumptions (G1) and (G2). To ensure that these two assumptions are fulfilled, it is assumed throughout the paper that any clause has been redefined in such a way.

The simplest clause is the one stipulating that the guaranteed price always corresponds to the advertised price. Such clause is defined by $w_i(q) := q_i$ for any profile q of advertised prices and is termed the *trivial clause* or the *no competition clause*. Due to Assumption (G2), any other clause

[6] It turns out that the collusive effectiveness of clauses referring only to the advertised prices depend on the timing of the retailers' decisions. As demonstrated in Corollary 14 in Trost (2021), such restriction does not impede the collusive effectiveness in the dynamic competition model we consider here.

[7] Indeed, we could weaken Assumption (G1) in order to include such clauses in our analysis. However, this generalization would be pointless as such clauses prove to be collusively ineffective in our model.

[8] We could generalize our analysis by assuming less restrictive lower bounds on the guaranteed prices. However, we decided to refrain from such generalizations as it would make our analysis more tedious without gaining additional insights.

g_i has the property $c \leq g_i(q) < q_i$ for some profile q of advertised prices satisfying $q_i > \max\{c, q_{\min}\}$. Henceforth, we refer to a non-trivial clause as a *competition clause* (CC) and denote the set of all competition clauses available for retailer i by C_i. Obviously, $\{w_i\} = G_i \setminus C_i$.

The presumably most prominent type of a CC is the *meeting competition clause* (MCC), also known as the price-matching guarantee. It entitles the customers to purchase the commodity at the lowest advertised price in the market. Most of the scientific literature on the market effects of CCs focuses on this form. We henceforth denote the MCC of retailer i by m_i and require $m_i(q) := q_{\min}$ for any profile q of advertised prices satisfying $q_i > \max\{c, q_{\min}\}$. The singleton $M_i := \{m_i\}$ contains retailer i's MCC.

A *beating competition clause* (BCC), also known as price-beating guarantee, entitles the customers to purchase the commodity at some price below the lowest advertised price if the retailer fails to advertise the lowest price in the market. More precisely, a beating competition clause b_i requires $c \leq b_i(q) < q_{\min}$ for any profile q of advertised prices satisfying $q_i > \max\{c, q_{\min}\}$. The set of all BCCs available for retailer i is denoted by B_i.

In line with the classification in Arbatskaya (2001) and Arbatskaya et al. (2004), three subgroups of BCCs will be specifically marked: BCCs with lump sum refunds, BCCs with refund factors on the minimum price and BCCs with refund factors on the price differences. We note that the symbols tagging these different types of BCCs are similar to the ones used in those two articles.

- A *BCC with lump sum refund* entitles the customers to purchase the commodity at a price equal to competitors' lowest advertised price minus a fixed amount if the retailer fails to advertise the lowest price in the market. In detail: Let $\mu \in \mathbb{R}_{++}$. A competition clause $b_i^{€,\mu}$ is said to be a BCC with lump sum refund μ if

$$b_i^{€,\mu}(q) := \min\{\max\{c, q_{\min} - \mathbf{1}_{[q>q_{\min}]}(i)\,\mu\}, q_i\}$$

  for any profile q of advertised prices. In the Industrial Organization literature, a lump sum refund is also called the refund depth of the BCC. The set of all BCCs with lump sum refunds available for retailer i is denoted by $B_i^{€}$.

- A *BCC with a refund factor on the minimum price* entitles the customers to purchase the commodity at a price equal to the competitors' lowest advertised price minus a fixed percentage (not greater than 100) of this price if the retailer fails to advertise the lowest price in the market. In detail: Let $\phi \in]0,1]$. A competition clause $b_i^{\%,\phi}$ is said to be a BCC with a refund factor ϕ on the minimum price if

$$b_i^{\%,\phi}(q) := \min\{\max\{c, (1 - \mathbf{1}_{[q>q_{\min}]}(i)\,\phi)q_{\min}\}, q_i\}$$

  for any profile q of advertised prices. The set of all BCCs with refund factors on the minimum price available for retailer i is denoted by $B_i^{\%}$.

- A *BCC with a refund factor on the price difference* entitles the customers to purchase the commodity at a price equal to the advertised price minus a fixed percentage (greater than 100) of the difference between the advertised price and the competitors' lowest advertised price if the retailer fails to advertise the lowest price in the market. In detail: Let $\lambda \in]1, +\infty[$. A price clause $b_i^{\Delta,\lambda}$ is said to be a BCC with a refund factor λ on the price difference if

$$b_i^{\Delta,\lambda}(q) := \min\{\max\{c, q_i - \mathbf{1}_{[q>q_{\min}]}(i)\,\lambda\,(q_i - q_{\min})\}, q_i\}$$

  for any profile q of advertised prices. The set of all BCCs with refund factors on the price difference available for retailer i is denoted by B_i^{Δ}.

Henceforth, a BCC belonging to one of the three subgroups we described above is called a *conventional BCC*. As set forth in the empirical study of Arbatskaya et al. (2004), the MCC and the conventional BCCs might be the types of CCs most widely used in real business life.[9] We refer to these types of CCs as *conventional CCs* from now on. The set consisting of all conventional CCs available for retailer i and the trivial clause w_i is called the set of the *conventional clauses* available for retailer i and is denoted by G_i^{c}.

A clause g_i of retailer i is said to be *symmetric* if $g_i(q) = g_i(q \circ \sigma)$ for any profile q of advertised prices and any permutation σ satisfying $\sigma(i) = i$. That means, the guaranteed price of a symmetric clause depends on the values of the price advertised by the competitors, but not on the distribution of those prices. Consequentially, clauses which refer to the advertised prices of some, but not all competitors are not symmetric. However, as can be easily checked, any conventional CC proves to be symmetric. From now on, we denote the set of all symmetric clauses available for retailer i by G_i^{s}.

A *clause profile* $g := (g_i)_{i \in I}$ summarizes the clause policies adopted in the retail market. For example, profile $w := (w_i)_{i \in I}$ describes the situation in which none of the retailers offers a CC and profile $m := (m_i)_{i \in I}$ describes the situation in which all retailers offer the MCC. Henceforth, the set of all clause profiles is denoted by $G := \times_{i \in I} G_i$, the set of all clause profiles in which all retailers adopt conventional clauses by $G^{\mathrm{c}} := \times_{i \in I} G_i^{\mathrm{c}}$, and the set of all clause profiles in which all retailers adopt symmetric clauses by $G^{\mathrm{s}} := \times_{i \in I} G_i^{\mathrm{s}}$. Note that $G^{\mathrm{c}} \subseteq G^{\mathrm{s}}$.

Let us consider some clause profile $g := (g_i)_{i \in I} \in G$. We define $C(g) := \{i \in I : g_i \in C_i\}$ as the set of retailers which have adopted a CC in clause profile g. Similarly, $M(g) := \{i \in I : g_i \in M_i\}$ and $B(g) := \{i \in I : g_i \in B_i\}$ represent the sets of retailers which have adopted the MCC and a BCC, respectively.

Let us pick some permutation $\sigma \in \Sigma_n$. The clause profile $g^\sigma := g \diamond \sigma$ specified by $g_i^\sigma(q) := g_{\sigma^{-1}(i)}(q \circ \sigma)$ for any profile $q \in \mathbb{R}_+^I$ of advertised prices and any retailer $i \in I$ is referred to as the *σ-variant of clause profile g*. This variant is the clause profile in which the retailers permute their clause policies so that the price guaranteed by retailer i at advertised prices q in clause profile g^σ corresponds to the price guaranteed by retailer $\sigma^{-1}(i)$ at advertised prices $q \circ \sigma$ in clause profile g.[10]

A clause profile $\tilde{g}$ is referred to as an *upshift of clause profile g* whenever there is a permutation σ upshifting on $C(g)$ so that $\tilde{g} := g \diamond \sigma$. That is, upshifts of clause profile g are clause profiles in which any CC offered in clause profile g is adopted by the same or a larger retailer in clause profile $\tilde{g}$. If σ is upshifting and non-constant on $C(g)$, we call $\tilde{g}$ a *proper upshift g*. If σ is the extreme upshift permutation, $\tilde{g}$ is referred to as the *extreme upshift of g*. In this case, the CC offered by the kth largest CC-adopting retailer in clause profile g is taken over by the kth largest retailer in clause profile $\tilde{g}$.

The competition model of this paper adopts some of the peculiarities of the models of Chen (1995) as well as of Hviid and Shaffer (1999). Like Chen (1995), it is assumed that implementing CCs causes one-off costs for the retailers. All retailers implementing CCs incur the same fixed costs in the amount of $f > 0$ regardless of the chosen type of CC. The implementation costs encompass the costs of creating the technical and personnel prerequisites for implementing a CC as well as of making the CC publicly known. As such expenses are largely one-off and more or less the same for any of the CC-adopting retailers, the assumption of fixed and identical implementation costs might be reasonable. Only retailers offering no CCs bear no fixed costs. For the sake of simplification, it is taken for granted that f is discounted to period 0.

[9] A superficial internet search for best price guarantees in retail markets conducted at January 10, 2022, suggested that this claim also held at the time when this paper was completed.

[10] As can be easily checked, this construction is well-defined. Any clause $g_i^\sigma(q)$ in the σ-variant of clause profile g satisfies Assumptions (G1) and (G2) and, thus, g^σ proves to be a clause profile of G.

Like Hviid and Shaffer (1999), it is assumed that exercising CCs might be costly for the customers. All customers making use of a CC incur the same hassle costs in the amount of $z \geq 0$ per purchased unit of the commodity regardless of at which retailer the commodity has been purchased and which type of CC has been offered. The existence of hassle costs might be justified by the real life experience that exercising CCs is usually not a smooth process. In general, the burden of proof rests on the customers. They have to spend time and effort to receive the refund guaranteed by the CC, e.g., for providing enough and sound evidence, seeking out qualified salespersons and raising the issue with them. If one takes for granted that each customer buys one unit of the commodity, our assumption that the hassle costs are measured per unit of the commodity seems plausible. The assumption that the hassle costs are identical among the customers has been made for reasons of simplification.

The *effective purchase price* gives the costs the customer incurs for acquiring a unit of the commodity. This price includes the hassle costs whenever the customer has exercised the retailer's CC. The *effective sales price* is the revenue the retailer earns per sold unit of the commodity. To provide a formal specification of these prices, consider the situation in which retailer i has opted for the clause policy g_i and the retailers in the market advertise prices $q = (q_j)_{j \in I}$. The effective purchase price of the commodity at retailer i is determined by formula

$$q_i^{\mathrm{p}} := g_i^{\mathrm{p}}(q) := q_i + \mathbf{1}_{C_i}(g_i)\, \min\{g_i(q) + z - q_i, 0\}$$

and the effective sales price by formula

$$q_i^{\mathrm{s}} := g_i^{\mathrm{s}}(q) := q_i - \mathbf{1}_{\mathbb{R}_{++}}(q_i - g_i^{\mathrm{p}}(q))\,(q_i - g_i(q)).$$

Obviously, if $g_i = w_i$ (i.e., retailer i does not adopt a CC) or $g_i(q) + z \geq q_i$ (i.e., it is not worthwhile for the customers to make use of retailer i's CC), then both the effective purchase and the effective sales price at retailer i are equal to the advertised price. Otherwise, the effective sales price corresponds to the price guaranteed by the retailers and the effective purchase price is the effective sales price plus the hassle costs.

2.2 Price Competition Phase

Having adopted their competition clause profiles $g := (g_i)_{i \in I}$, the retailers take part in an infinitely repeated Bertrand competition. At each stage $t \in \mathbb{Z}_+$ of this phase, the retailers simultaneously advertise a price for the commodity. We denote the price advertised by retailer i at stage t by q_i^t and the profile listing all prices advertised at stage t by $q^t := (q_i^t)_{i \in I}$. The effective sales and purchase prices at stage t are then given by $q^{\mathrm{s},t} := g^{\mathrm{s}}(q^t)$ and $q^{\mathrm{p},t} := g^{\mathrm{p}}(q^t)$, respectively.

The demand side at stage t is described by *market demand mapping* $D : \mathbb{R}_+ \to \mathbb{R}_+$. Its value $D(p)$ indicates the total quantity of the commodity demanded by the consumers if they have to pay price p per unit of the commodity. The market demand mapping is time invariant and, therefore, not marked with a stage index t. Moreover, we assume that

(D1) D is continuous,

(D2) there is some $\bar{p} > c$ so that $D^{-1}(0) = [\bar{p}, +\infty[$ and D is decreasing on $[0, \bar{p}[$.

These postulates are standard. Price $\bar{p}$ gives the highest amount the consumers are willing to pay for the commodity and is known as the reservation price of the demand side. Due to Assumptions (D1) and (D2), the *monopolistic profit mapping* $\pi : \mathbb{R}_+ \to \mathbb{R}$ given by $\pi(p) := (p-c)D(p)$ is continuous and positive on open interval $]c, \bar{p}[$. Regarding the form of the profit mapping, it is taken for granted that

(D3) π is strictly quasiconcave on $]c, \bar{p}[$.

It follows from ASSUMPTIONS (D1), (D2), and (D3) that there exists a unique p^{m} which maximizes π. We term price p^{m} as the *collusive price* and denote the monopolistic profit attained this price by $\pi^{\mathrm{m}} := \pi(p^{\mathrm{m}})$. A further requirement we impose on our competition model is that

(D4) $D(c) \leq K_{-i}$ for any $i \in I$.

ASSUMPTION (D4) states that the capacities of any coalition of n-1 retailers are sufficient to meet the market demand at price equal marginal costs. If the capacity-constrained retailers took part in a static Bertrand competition game without CCs, then this assumption would entail that (i) the situation in which each retailer charges a price equal to the marginal costs is a Nash equilibrium and (ii) each retailer earns a zero profit in any Nash equilibrium.

Suppose that the consumers face purchase prices $p := (p_i)_{i \in I}$ at period t. Moreover, let r be some additional hypothetical purchase price. The residual market demand at r is defined as the market demand at r not met by the capacities of the retailers undercutting price r. A rationing rule specifies the size of residual market demand $R(r|p)$. In the following, we present three examples of rationing rules, the efficient, the proportional and the perfect one. The former two have been already extensively applied in competition theory.

- The *efficient rationing rule* $R_{\mathrm{e}}(\cdot|\cdot)$ was first used by Levitan and Shubik (1972) and has since appeared, among others, in Kreps and Scheinkman (1983) as well as Osborne and Pitchik (1986). It lays down that consumers with the highest willingness to pay are served first. In formal terms, the residual market demand resulting from efficient rationing is given by

$$R_{\mathrm{e}}(r|p) := \max\{D(r) - K_{[p<r]}, 0\}$$

  for any profile $p \in \mathbb{R}_+^I$ of purchase prices and any hypothetical purchase price $r \in \mathbb{R}_+$.

- The *proportional rationing rule* $R_{\mathrm{p}}(\cdot|\cdot)$ was proposed by Shubik (1959) and have since applied e.g. in Beckmann (1965), Davidson and Deneckere (1986), as well as Allen and Hellwig (1986). It stipulates that each of the consumers have the same probability of being served. The residual market demand resulting from proportional rationing is inductively specified by

$$R_{\mathrm{p}}(r|p) := \begin{cases} D(r) & \text{for any } r \leq p_{(1)}, \\ \max\left\{\frac{R_{\mathrm{p}}(p_{(i)}|p) - K_{[p=p_{(i)}]}}{D(p_{(i)})}, 0\right\} D(r) & \text{for any } p_{(i)} < r \leq p_{(i+1)} \end{cases}$$

  where $p_{(i)}$ is the i-th order statistic of price profile p (i.e., the i-th smallest price in p) and $p_{(n+1)} := +\infty$.[11]

- The *perfect rationing rule* is the opposite extreme of the efficient rationing rule. This rule stipulates that consumers with the lowest willingness to pay are served first. The residual market demand resulting from perfect rationing is inductively specified by

$$R_{\mathrm{l}}(r|p) := \begin{cases} D(r) & \text{for any } r \leq p_{(1)}, \\ \min\left\{D(r), \max\{R_{\mathrm{l}}(p_{(i)}|p) - K_{[p=p_{(i)}]}, 0\}\right\} & \text{for any } p_{(i)} < r \leq p_{(i+1)} \end{cases}$$

  where $p_{(i)}$ is the i-th order statistic of price profile p and $p_{(n+1)} := +\infty$.

Throughout most parts of this paper, we will abstain from assuming a specific rationing rule. Rather, we only take for granted that any *residual market demand mapping* $R : \mathbb{R}_+ \times \mathbb{R}_+^I \to \mathbb{R}_+$ satisfies the property

(R1) $R_{\mathrm{e}}(r|p) \leq R(r|p) \leq R_{\mathrm{l}}(r|p)$.

[11]The proportional rationing rule can also be stated in an explicit form. As can be easily checked, it holds $R_{\mathrm{p}}(r|p) = (\max\{\,1 - \sum_{i \in [p<r]} \frac{k_i}{D(p_i)}, 0\,\})\, D(r)$.

ASSUMPTION (R1) requires that a lower and upper bound of the residual market demand be the residual market demands resulting from efficient and perfect rationing, respectively. As efficient rationing is the most restrictive among all rationing rules, it will be used as a benchmark in the subsequent sections. Without any difficulty, one can verify that ASSUMPTION (R1) entails $R(r|p) = 0$ if $K_{[p<r]} \geq D(p_{\min})$ and $R(r|p) = D(r)$ if $r \leq p_{\min}$. We resort to these two fundamental properties of the residual market demand in our proofs without any explicit reference to ASSUMPTION (R1).

Further assumptions on the underlying rationing rule can be imposed. A residual demand mapping $R(\cdot|\cdot)$ is said to be *regular* whenever it additionally satisfies the properties

(R2) $R(r|\tilde{p}) = R(r|p)$

where $\tilde{p}_i = p_i$ for any $i \in [p < r]$ and $\tilde{p}_i \geq r$ for any $i \in [p \geq r]$,

(R3) $R(r|\tilde{p}) \leq R(r|p)$

where $\tilde{p}_i = p_i$ for any $i \in [p < r]$ and $\tilde{p}_i < r$ for some $i \in [p \geq r]$,

(R4) $R(r|p \circ \sigma^{-1}) \leq R(r|p)$

where $\sigma \in \Sigma_n$ is upshifting on $[p < r]$ and downshifting on $[p \geq r]$.

Let us briefly explain these additional assumptions on the rationing rules: ASSUMPTION (R2) states that the residual market demand at price r is unaffected by price changes in which the prices below r remain constant and the other prices do not fall below r. ASSUMPTION (R3) rules out that the residual market demand at price r increases if some of the retailers lower the price of the commodity from above or equal to r to below r. ASSUMPTION (R4) implies that the residual market demand at price r does not increase if prices below price r are charged by larger retailers and prices above or equal to r are charged by smaller retailers.

The three additional assumptions might be uncontroversial and could be regarded as innocuous. As can be easily checked, any prominent rationing rule such as the efficient, proportional, or perfect rationing rule is regular. Moreover, the following property applies to any regular rationing rule.

Remark 2.1. *Any rationing rule $R(\cdot|\cdot)$ satisfying (R2) and (R4) has the property*

$$R(r|p \circ \sigma^{-1}) \leq R(r|p)$$

where $\sigma \in \Sigma_n$ is upshifting on $[p < r]$.

For some instances, a further property is needed. A rationing rule is said to be *monotone* whenever it satisfies the property

(R5) $R(r|\tilde{p}) \leq R(r|p)$

where $p_i \leq \tilde{p}_i < r$ for any $i \in [p < r]$ and $p_i = \tilde{p}_i$ for any $i \in [p \geq r]$.

Any monotone rationing rule has the characteristic that the residual demand does not increase if some of the retailers charging a price below r increase the prices, but still undercut price r. Without any difficulty, it can be shown that the efficient, proportional, and perfect rationing rule are monotone.

The quantity of the commodity demanded by the consumers from retailer i at stage t and purchase prices $p := (p_i)_{i \in I}$ is derived from the residual market demand. We assume that *demand mapping* $D_i : \mathbb{R}_+^I \to \mathbb{R}_+$ *of retailer* i is specified according to the allocation rule

$$D_i(p) := \frac{k_i}{K_{[p=p_i]}} R(p_i|p)$$

for any profile $p \in \mathbb{R}_+^I$ of purchase prices.

According to this definition, the proportion of the residual market demand directed to retailer i corresponds to i's share of the total capacity of the retailers charging the same price as retailer i. An argument substantiating such demand allocation is that consumers are more likely to meet retailers with higher sales capacities than those with lower sales capacities so that they more likely to buy the commodity from the former retailers than from the later ones. This allocation rule has already been applied in other studies on Bertrand competition with capacity constraints; e.g., in Allen and Hellwig (1986) as well as Osborne and Pitchik (1986).[12]

A retailer i is able to serve demand $D_i(p)$ as long as the demand does not exceed its capacity constraint k_i. The mapping $X_i : \mathbb{R}_+^I \to \mathbb{R}_+$ indicating the quantity

$$X_i(p) := \min\{k_i, D_i(p)\}$$

retailer i is able to sell at profile p of purchase prices is called the *sales mapping of retailer i*.

It follows immediately from Assumption (R1) that if the purchase price of the commodity is the same at any retailer (i.e., $p_i = p_j$ for any $j \in I$), then retailer i's share of the total sales equals κ_i. For this reason, it is justified to interpret κ_i as *retailer i's market share* at price p. Moreover, Assumption (D4) entails that κ_i corresponds to the proportion of the market demand which retailer i serves if $p_i = p_j$ for any $j \in I$ and $p_i \geq c$.

Let us now turn to the situation in which the retailers have implemented clause profile g and advertise prices $q := (q_i)_{i \in I}$ in period t. The profit retailer i earns in this period is given by

$$\pi_i^g(q) := (q_i^{\mathrm{s}} - c) X_i(q^{\mathrm{p}}).$$

Recall that $q_i^{\mathrm{s}} := g_i^{\mathrm{s}}(q)$ is the effective sales price charged by retailer i and $q^{\mathrm{p}} := g^{\mathrm{p}}(q)$ summarizes the effective purchase prices in the market. The mapping $\pi_i^g : \mathbb{R}_+^I \to \mathbb{R}$ specifying retailer i's period profit $\pi_i^g(q)$ for any profile q of advertised prices given that clause profile g have been implemented is referred to as *retailer i's profit mapping under clause profile g*. To simplify the notation, we write π_i instead of π_i^w. Obviously, $\pi_i(q) = (q_i - c)X_i(q)$ for any profile q of advertised prices.

An *outcome* of the competition game summarizes all actions which the retailers have chosen during the course of the game. It is also called a *terminal history* and is represented by a sequence $o := (o_t)_{t=-1}^{\infty} := (g, q^0, q^1, \cdots)$ where g indicates the competition clause policy selected in period $t = -1$ and q^t the prices advertised in period t. The retailers are assumed to discount their future profits by a *common discount factor* $0 \leq \delta < 1$ so that the total profit of any retailer i amounts to

$$\Pi_i(o) := \sum_{t=0}^{\infty} \delta^t \pi_i^g(q^t) - \mathbf{1}_{C_i}(g_i) f.$$

Let $O := G \times \left(\times_{t=0}^{\infty} \mathbb{R}_+^I\right)$ be the set of possible game outcomes. The mapping $\Pi_i : O \to \mathbb{R}$ specifying the total profit of retailer i for any game outcome is called the *total profit mapping of retailer i*.

The common discount factor δ is allowed to take the value of zero. In this case, the retailers do not value future profits so that the competition game transforms itself - in essence - into a two-stage game where the price competition phase consists of only one stage. Obviously, it then holds $\Pi_i(o) := \pi_i^g(q^0)$ for any game outcome $o := (g, q^0, \dots) \in O$. While $\delta > 0$ could be interpreted as a situation in which the end date of the competition is not foreseeable by the retailers, $\delta = 0$ represents the situation in which the end date is commonly known.[13] Our analysis takes into consideration both situations.

[12]Noteworthy, there are also theoretical studies not following this allocation rule. For example, Kreps and Scheinkman (1983) as well as Davidson and Deneckere (1986) assume that the market demand is equally split among the capacity-constrained duopolists whenever both set the same price and each has a capacity meeting at least the half of the market demand. That is, both retailers have the same market share in this case regardless of their shares of the total capacity. The same holds for the model of Tumennasan (2013), which is based on the framework of Kreps and Scheinkman (1983).

[13]For our puposes, it suffices to represent the competition games with finite horizon by two-stage competition games. Resorting to backward induction arguments, one can show that a subgame perfect price policy inducing the collusive price in any of the finitely many stages of the price competition phase exists if, and only if, the collusive price constitutes a Nash equilibrium in the single-stage price competition phase. Therefore, the results obtained in competition games with finite horizon are by nature equal to the ones obtained in the two-stage competition games.

An outcome in which each retailer advertises the collusive price p^{m} in each period of the price competition phase is referred to as a *collusive outcome*. As can be easily checked, if a collusive outcome $o^{\mathrm{m}} := (g, q^0, q^1, \dots)$ is realized, i.e., $q^t_j = p^{\mathrm{m}}$ for any period $t \in \mathbb{Z}_+$ and any retailer $j \in I$, then retailer i earns a total profit in the amount of

$$\Pi_i(o^{\mathrm{m}}) = \frac{1}{1-\delta}\,\kappa_i\,\pi^{\mathrm{m}} \;-\; \mathbf{1}_{C_i}(g_i)\,f.$$

An outcome in which each retailer advertises a price equal to marginal costs c in each period of the price competition phase is referred to as a *competitive outcome*. Obviously, if a competitive outcome $o^{\mathrm{p}} := (g, q^0, q^1, \dots)$ is realized, i.e., $q^t_j = c$ for any period $t \in \mathbb{Z}_+$ and any retailer $j \in I$, then retailer i earns a total profit in the amount of

$$\Pi_i(o^{\mathrm{p}}) = -\,\mathbf{1}_{C_i}(g_i)\,f,$$

i.e., retailer i incurs a total profit of zero in case it has not adopted a CC, and a loss of f otherwise.

2.3 Business Policies

The rules of our extended capacity-constrained Bertrand competition game we detailed in the preceding subsections are summarized by $\Gamma(\delta, f, n, z)$ or, simply, by Greek capital letter Γ whenever no specific reference is made to the parameters of the game. Whenever it is assumed that the rationing rule underlying competition game $\Gamma(\delta, f, n, z)$ belongs to some specific class, we will mark this by a subscript. For example, $\Gamma_{\mathrm{e}}(\delta, f, n, z)$, $\Gamma_{\mathrm{r}}(\delta, f, n, z)$, and $\Gamma_{\mathrm{m}}(\delta, f, n, z)$ denote competition games whose rationing rules are efficient, regular, and monotone, respectively. Moreover, we simply write $\Gamma(\delta, f, n)$ instead of $\Gamma(\delta, f, n, 0)$ whenever exercising CCs is assumed to be hassle-free.

In the remainder of this section, we introduce formal terms which enables us to describe the possible courses of our competition game and the strategies available for the retailers in a compact way. Our notation mainly follows the one suggested in Chapter 6 of the textbook of Osborne and Rubinstein (1994) for multi-stage games with perfect information.

A history up to period t is a sequence enumerating the actions chosen by the retailers until period t. Let h^t be such a history and pick some period $\tau \leq t$ and some retailer $i \in I$. The component $h^t_{i,\tau}$ of h^t indicates the action retailer i has chosen in period τ according to history h^t The n-tuple $h^t_\tau := (h^t_{i,\tau})_{i \in I}$ lists the actions of all retailers in period τ according to history h^t.

We recursively define $H^{-1} := G$ and $H^t := H^{t-1} \times \mathbb{R}^I_+$ for any $t \in \mathbb{Z}_+$. Apparently, H^t consists of all possible histories up to period t. Moreover, we define singleton $H^{-2} := \{\emptyset\}$ where $\emptyset$ stands for the initial history (starting point) of our competition game. The set of all non-terminal histories is denoted by $H := \cup_{t=-2}^{\infty} H^t$. Pick some arbitrary non-terminal history $h \in H^{t_0}$. A history $h \in H^t$ is said to be consistent with h whenever $t \geq t_0$ and $h^t_{i,\tau} = h_{i,\tau}$ for any $i \in I$ and any $\tau \leq t_0$. We denote the set of all non-terminal histories consistent with h by H_h.

A strategy or, synonymously, a *business policy of retailer* i is a complete plan of action. It prescribes the actions retailer i takes for any conceivable history. More precisely, it specifies which competition clause policy retailer i selects at the beginning of the game and, for any history $h^{t-1} \in H^{t-1}$ and $t \in \mathbb{Z}_+$, which price retailer i would advertise in period t if he observed the previous actions h^{t-1}. In formal terms, a business policy of retailer i is described as a mapping $s_i : H \to G_i \cup \mathbb{R}_+$ where $s_i(\emptyset) \in G_i$ and $s_i(h^{t-1}) \in \mathbb{R}_+$ for any $h^{t-1} \in H^{t-1}$ and $t \in \mathbb{Z}_+$. The set of business policies available for retailer i is denoted by S_i.

A *business policy profile* $s := (s_i)_{i \in I}$ lists the business policies chosen by all retailers. We denote the set of these profiles by $S := \times_{i \in I} S_i$. The outcome $o(s) := (o_t(s))_{t=-1}^{\infty}$ induced by business policy profile s is the infinite sequence of actions realized by retailers pursuing these policies. It is also referred to as the terminal history induced by business policy profile s. The actions recorded in this history are recursively specified in the following way: It holds $o_{-1}(s) := s(\emptyset)$ and, for any $t \geq 0$, we have $o_t(s) := s(o^{t-1}(s))$ where $o^{t-1}(s) := (o_r(s))_{r=-1}^{t-1}$.

We denote the subgame of Γ starting after non-terminal history h by Γ^h and the restriction of business policy s_i on H_h by s_i^h. The latter mapping specifies the actions of retailer i only for histories which are consistent with history h. Apparently, the set of these restrictions constitutes the business policy set of retailer i in subgame Γ^h. In line with the above rule of notational simplification, if $h := (g) \in H^{-1}$, we simply write Γ^g and s_i^g instead of $\Gamma^{(g)}$ and $s_i^{(g)}$, respectively. Mapping s_i^g is called the *price policy of retailer i in subgame* Γ^g. With slight abuse of notation, we sometimes express business policy s_i of retailer i by $(g_i, (s_i^{\tilde{g}})_{\tilde{g} \in G})$. Correspondingly, a business policy profile is sometimes expressed by $(g, (s^{\tilde{g}})_{\tilde{g} \in G})$ where $g := (g_i)_{i \in I}$ and $s^{\tilde{g}} := (s_i^{\tilde{g}})_{i \in I}$.

Retailer i is said to follow a *grim trigger price policy in subgame* Γ^g whenever its business policy s_i satisfies

$$s_i^g(h^{t-1}) = \begin{cases} p^{\mathrm{m}} & \text{if either } t = 0 \text{ or } h_{i,t_0}^{t-1} = p^{\mathrm{m}} \text{ for any period } 0 \leq t_0 < t \text{ and any } i \in I, \\ c & \text{otherwise,} \end{cases}$$

for any $h^{t-1} \in H_g^{t-1}$ and any $t \in \mathbb{Z}_+$. The grim trigger price policy states that the retailer advertises the collusive price p^{m} at the beginning of the price competition phase and continues to advertise this price as long as all retailers have advertised the collusive price in any preceding period. However, if the latter is not satisfied, the retailer advertises the competitive price c. We henceforth denote the grim trigger price policy of retailer i in subgame Γ^g by t_i^g. Obviously, if the retailers realize clause profile g and adopt grim trigger price policies $t^g := (t_i^g)_{i \in I}$ in subgame Γ^g, then the collusive outcome $(g, p^{\mathrm{m}}, p^{\mathrm{m}}, \dots)$ results.

Retailer i is said to follow a *competitive price policy in subgame* Γ^g whenever its business policy s_i satisfies

$$s_i^g(h^{t-1}) = c$$

for any $h^{t-1} \in H_g^{t-1}$ and any $t \in \mathbb{Z}_+$. The competitive price policy states that the retailer always advertises a price equal to the marginal costs regardless of the prices advertised in the preceding periods. We henceforth denote the competitive price policy of retailer i in subgame Γ^g by c_i^g. Apparently, if the retailers realize clause profile g and adopt competitive price policies $c^g := (c_i^g)_{i \in I}$ in subgame Γ^g, then the competitive outcome $(g, c, c, \dots)$ results.

3 Tacit Collusion with Competition Clauses

To solve the competition game presented in the last section, the solution concept of subgame perfectness is applied. In doing so, it is taken for granted that the retailers pursue grim trigger policies in the price competition phase whenever these policies constitute a subgame perfect equilibrium and the competitive price policies otherwise. This assumption substantially simplifies the analysis.

Our interest is focused on the subgame perfect business policy profiles inducing collusion. The clause profiles realized in these business policy profiles are called perfectly collusive. In the following subsection, we describe them in a formal way. Afterwards, we introduce so-called critical values for the game parameters which have to be met in order to facilitate tacit collusion. In the third subsection, we apply these critical values to characterize perfectly collusive clause profiles in a compact way. This characterization simplifies the detection of perfectly collusive clause profiles and becomes helpful for solving the issues addressed in the subsequent sections.

3.1 Perfectly Collusive Clause Profiles

To predict the business policies of the retailers, we resort to the solution concept of subgame perfectness. Henceforth, we denote the set of the subgame perfect business policy profiles in competition game Γ by $\mathcal{S}(\Gamma)$. As retailers are assumed to interact infinitely often, non-compliant behavior can be punished by future retaliatory measures of the competitors. To simplify our analysis, we focus only on those subgame perfect business policy profiles in which grim trigger price policies are implemented whenever these policies constitute a subgame perfect equilibrium in the price competition subgame, whereas competitive price policies are implemented otherwise. As is well known, the grim trigger price policies unleash the most severe punishment for defecting from collusion in Bertrand markets.[14] In formal terms, we narrow down the solution set by imposing the restriction

$$\mathcal{S}^{\mathrm{g}}(\Gamma) := \left\{ s \in \mathcal{S}(\Gamma) : s^g = \begin{cases} t^g & \text{if } t^g \text{ is a subgame perfect equilibrium in } \Gamma^g, \\ c^g & \text{otherwise} \end{cases} \right\}.$$

Although restricting the set of solutions to $\mathcal{S}^{\mathrm{g}}$ is a substantial simplification, it is by far not a peculiarity of our analysis. Such simplification has been made in the theory of partial cartels, like in the models of Escrihuela-Villar (2008) as well as Bos and Harrington (2010) to name a few. Moreover, it has also been proposed by Liu (2013) for the analysis of collusively effective competition clause policies. Notably, this restriction does not cause existence problems; without difficulty, one can show that solution set $\mathcal{S}^{\mathrm{g}}(\Gamma)$ is non-empty for any competition game Γ.

As the scope of our analysis is confined to these business policies, we exclusively reserve the term subgame perfectness to business policy profiles belonging to $\mathcal{S}^{\mathrm{g}}(\Gamma)$. A clause profile $\hat{g} := (\hat{g}_i)_{i\in I}$ is said to be *subgame perfect* in Γ whenever there exists a subgame perfect business policy profile $\hat{s} := (\hat{g}, (s^g)_{g\in G})$ in Γ, i.e., $\hat{s} \in \mathcal{S}^{\mathrm{g}}(\Gamma)$. In the course of this paper, several refinements of solution concept $\mathcal{S}^{\mathrm{g}}$ will be presented.

As our primary interest is on the use of CCs for facilitating collusion, we are mainly focused on the subgame perfect clause profiles inducing the collusive outcome. The business policy profiles of $\mathcal{S}^{\mathrm{g}}(\Gamma)$ having this characteristic are referred to as *perfectly collusive*. We henceforth denote this subset of $\mathcal{S}^{\mathrm{g}}(\Gamma)$ by

$$\mathcal{S}^{\mathrm{m}}(\Gamma) := \{s \in \mathcal{S}^{\mathrm{g}}(\Gamma) : \ o(s) \text{ is a collusive price outcome}\}.$$

A clause profile $\hat{g} := (\hat{g}_i)_{i\in I}$ is said to be *perfectly collusive* in Γ whenever there exists some business policy profile $\hat{s} := (\hat{g}, (\hat{s}^g)_{g\in G}) \in \mathcal{S}^{\mathrm{m}}(\Gamma)$. As will be argued later, such clause profiles can be characterized by so-called critical discount factors and critical implementation costs. To provide this characterization, we first specify these critical values in the following subsection.

3.2 Critical Values

The main objective pursued in this subsection is to provide thresholds for the game parameters which have to be met in order to facilitate collusion in the market. More specifically, we compute so-called critical values for the discount factor as well as for the implementation and hassle costs. As will be set forth in this subsection, a clause profile cannot induce collusion if the common discount factor is less than the critical discount factor or the actual costs of implementing or exercising the CC exceed their critical values.

[14] An interesting future research project might be to consider other price strategies than the grim trigger price policy. For example, Lu and Wright (2010) propose the so-called price matching punishment. Such price policy stipulates that if the retailer is currently undersold, then the retailer will choose the current lowest price in the market (and not the marginal costs) as the price for the next period. Such price policy is less severe than the grim trigger price policy, but is viewed by competition economists - e.g., by Lu and Wright (2010) - as a better description of actual pricing behavior.

■ **Critical Discount Factor**

The *critical discount factor of retailer i* at clause profile g is defined as

$$\delta^g_{i,\mathrm{crit}} := \inf \Delta^g_i \quad \text{where} \quad \Delta^g_i := \left\{ \delta \in [0,1[: \frac{1}{1-\delta}\,\kappa_i\,\pi^{\mathrm{m}} \;\geq\; \sup_{q_i \neq p^{\mathrm{m}}} \pi^g_i(q_i, p^{\mathrm{m}}_{-i}) \right\} .$$

In words, the critical discount factor $\delta^g_{i,\mathrm{crit}}$ of retailer i at clause profile g corresponds to the lowest discount factor for which the collusive profit of retailer i does not fall short of the highest one-off profit this retailer is able to attain by defecting from the collusion.

We note that a retailer can defect from collusion in two ways: One is announcing a price below the collusive level, and the other is announcing a price above the collusive level. The first way is referred to as undercutting, the other as "overcutting".[15] To ease the notation, let us denote the supremum of the profits retailer i attains in the former way by $\pi^{\downarrow}_i(g) := \sup_{q_i < p^{\mathrm{m}}} \pi^g_i(q_i, p^{\mathrm{m}}_{-i})$ and in the latter way by $\pi^{\uparrow}_i(g) := \sup_{q_i > p^{\mathrm{m}}} \pi^g_i(q_i, p^{\mathrm{m}}_{-i})$. The supremum of the profits retailer i is able to attain by defecting from the collusive price is henceforth denoted by $\pi^{\updownarrow}_i(g) := \sup_{q_i \neq p^{\mathrm{m}}} \pi^g_i(q_i, p^{\mathrm{m}}_{-i})$. To specify these suprema, we often make use of the following two properties.[16]

Remark 3.1. *Consider $\Gamma(\delta, f, n, z)$. It holds:*

(a)

$$0 \leq \sup_{c < q_i < p^{\mathrm{m}}} \pi^g_i(q_i, p^{\mathrm{m}}_{-i}) = \pi^{\downarrow}_i(g)$$

and, thus, $0 \leq \sup_{c < q_i \neq p^{\mathrm{m}}} \pi^g_i(q_i, p^{\mathrm{m}}_{-i}) = \pi^{\updownarrow}_i(g)$ for any retailer $i \in I$ and any clause profile $g \in G$.

(b)

$$\pi^{\updownarrow}_i(g) \leq (p^{\mathrm{m}} - c)\max\{k_i, D(p^{\mathrm{m}})\}$$

for any retailer $i \in I$ and any clause profile $g \in G$.

Remark 3.1 substantially simplifies the calculation of the highest profit retailer i is able to attain by defection. More specifically, according to part (a), it suffices to take into account only the advertised prices above the marginal costs. Apart from that, parts (a) and (b) state a lower and an upper bound for this profit, respectively.

Based on the suprema $\pi^{\downarrow}_i(g)$ and $\pi^{\uparrow}_i(g)$, we introduce the discount factors

$$\delta^{\downarrow}_i(g) = \inf\left\{ \delta \in [0,1[: \frac{1}{1-\delta}\kappa_i \pi^{\mathrm{m}} \;\geq\; \pi^{\downarrow}_i(g) \right\} \quad \text{and} \quad \delta^{\uparrow}_i(g) = \inf\left\{ \delta \in [0,1[: \frac{1}{1-\delta}\kappa_i \pi^{\mathrm{m}} \;\geq\; \pi^{\uparrow}_i(g) \right\}.$$

Obviously, it holds $\delta^g_{i,\mathrm{crit}} = \max\{\delta^{\uparrow}_i(g), \delta^{\downarrow}_i(g)\}$. The *critical discount factor at clause profile g* is defined as the maximum individual critical discount factor at this profile, i.e.,

$$\delta^g_{\mathrm{crit}} := \max\{\delta^g_{i,\mathrm{crit}} : i \in I\}.$$

It immediately follows from the definition of the individual critical discount factors: If the common discount factor is less than the critical discount factor of clause profile g, then there is at least one retailer in g which earns a higher profit by defecting from the collusion than by sticking with it and, thus, would prefer to abandon the collusion. That means, collusion is not sustainable under clause profile g if the common discount factor falls short of its critical discount factor.

[15] The latter term has been used e.g. by Hviid and Shaffer (1999).

[16] These properties have already been established in Trost (2021). For this reason, we have omitted the proof of this remark. Unlike our setting, Trost (2021) takes for granted that the rationing rule satisfies Assumption (R1) and (R2). However, as can be easily checked, any of the results of Trost (2021) invoked here - e.g., Remark 3.1 - is still valid even if Assumption (R2) is abandoned.

Proposition 3.2. *Consider* $\Gamma(\delta, f, n, z)$*. If* $\delta < \delta^g_{\text{crit}}$*, then clause profile* g *is not perfectly collusive.*

In accordance with the previous notational simplifications, we omit the superscript of the critical discount factors if the clause profile is the trivial one, i.e., $g = w$. The critical discount factor of the trivial clause profile proves to be the greatest among the critical discount factors. Or putting it differently, if CCs are implemented by some retailers, the critical discount factor does not increase. This result is summarized in the following remark and has already been proved in Trost (2021).

Remark 3.3. *Consider* $\Gamma(\delta, f, n, z)$*. It holds*

$$\delta_{i,\text{crit}} = 1 - \max\left\{\kappa_i, \tfrac{D(p^{\text{m}})}{K}\right\} \geq \delta^g_{i,crit}$$

for any retailer $i \in I$ *and any clause profile* $g \in G$ *so that*

$$\delta_{\text{crit}} = 1 - \max\left\{\kappa_1, \tfrac{D(p^{\text{m}})}{K}\right\} \geq \delta^g_{\text{crit}}$$

for any clause profile $g \in G$*.*

If the hassle costs are sufficiently large, it might not be worthwhile for the customers to exercise the CCs. This refusal would entail that a retailer undercutting the collusive price cannot immediately be undersold or at least price matched by any of its competitors - even if some of them offers CCs. For this reason, the highest possible profit a deviant retailer can then earn is the same as in the case without CCs. This in turn implies that the critical discount factor of any clause profile would be equal to that of the trivial one.

Remark 3.4. *Consider* $\Gamma(\delta, f, n, z)$*. If* $z \geq p^m - c$*, then* $\delta^g_{\text{crit}} = \delta_{\text{crit}}$ *for any clause profile* $g \in G$*.*

Next, we aim to provide some upper and lower bounds of critical discount factors resulting from specific non-trivial clause profiles. Our first finding is that the minimum critical discount factor resulting from clause profiles in which only one retailer adopts a CC is positive. In other words, to reach a critical discount factor sufficiently close to zero, more than one retailer has to offer a CC.

Remark 3.5. *Consider* $\Gamma(\delta, f, n, z)$ *and some clause profile* $g \in G$*. If there is some* $i \in I$ *so that* $C(g) = \{i\}$*, then* $\delta^g_{\text{crit}} \geq \delta^g_{i,\text{crit}} \geq 1 - \max\{\kappa_i, \frac{D(p^{\text{m}})}{K}\}$*.*

Lower bounds of critical discount factors resulting from specific clause profiles in which some of the CC-adopting retailers offer a conventional CC are derived next. Hereby, we assume zero hassle costs for reasons of simplicity, i.e., we consider competition games of form $\Gamma(\delta, f, n)$. The subsequent results prove to be helpful in studying the market examples presented in the sections after this.

Remark 3.6. *Consider* $\Gamma(\delta, f, n)$*. It holds:*

(a) *If non-trivial clause profile* g *satisfies* $g_i = m_i$ *for any* $i \in C(g)$ *except for at most one, then*

$$\delta^g_{\text{crit}} \geq 1 - \max\left\{\kappa_{C(g)}, \tfrac{D(p^{\text{m}})}{K}\right\}.$$

In particular, if $g_i = m_i$ *for any* $i \in C(g)$*, then* $\delta^g_{\text{crit}} = 1 - \max\{\kappa_{C(g)}, \frac{D(p^{\text{m}})}{K}\}$*.*

(b) *If non-trivial clause profile* g *satisfies* $g_i = b_i^{\Delta,\lambda_i}$ *for some* $i \in C(g)$*, then*

$$\delta^g_{\text{crit}} \geq 1 - \max\left\{\kappa_i, \tfrac{D(p^{\text{m}})}{K}\right\}.$$

(c) *Suppose the rationing is monotone. If non-trivial clause profile* g *satisfies* $g_i = b_i^{€,\mu_i}$ *for some* $i \in C(g)$ *and clause profile* $\tilde{g}$ *is given by* $\tilde{g} := (g_{-i}, \tilde{g}_i)$ *where* $\tilde{g}_i := b_i^{\%,\frac{\mu_i}{p^{\text{m}}}}$*, then*

$$\delta^g_{\text{crit}} \;\geq\; \delta^{\tilde{g}}_{\text{crit}}.$$

Part (a) of Remark 3.6 provides lower bounds of the critical discount factors resulting from clause profiles in which all CC-adopting retailers except for at most one offer the MCC. In particular, it reveals that the critical discount factor is equal to the total market share of the non CC-adopting retailers, i.e., $1-\kappa_{C(g)}$, if all CC-adopting retailers offer the MCC and their total capacity is not less than the market demand at the monopoly price. In consequence, the following link results: the more retailers offer the MCC, the less the critical discount factor is. The reason is that if a retailer undercuts the collusive price by advertising a price below the collusive price, then any of the MCC-adopting competitors follows suit so that the profit is equally split among them. This in turn implies that the more retailers adopt the MCC, the less the profit the deviating retailer is able to earn.

Part (b) states that the critical discount factor of a clause profile in which one of the retailers offers a BCC with a refund factor on the price difference never falls short of $1-\max\{\kappa_n, \frac{D(p^{\mathrm{m}})}{K}\}$ regardless of which clauses are offered by its competitors. This is due to the fact that such BCCs are a cunning device to circumvent the disciplinary force induced by the CCs of the competitors. They enable the retailer to undercut the collusive price by any sufficiently small amount without triggering their CCs. Indeed, by advertising a price above the collusive level, the retailer induces its customers to exercise the BCC so that the effective price charged by it is less than the collusive one. In contrast, the effective price charged by its competitors remain unchanged at the collusive level as the CCs are applicable only to the advertised prices.

Part (c) rules out that the critical discount factor increases in markets with monotone rationing if one of the CC-adopting retailers switches from a BCC with lump refund μ_i to a BCC with refund factor $\frac{\mu_i}{p^{\mathrm{m}}}$ on the minimum price. This finding can be explained as follows. If one of the other retailers deviates from the collusive price by advertising a lower price, any of the customers of the retailer with the BCC makes use of this option so that the effective price they pay is below the price advertised by the other retailer. However, the effective price charged by the former retailer is closer to this advertised one if it offers the BCC with refund factor on the minimum price. Hence, as the rationing rule is monotone, the profit of the deviating retailer becomes smaller in this case.

■ **Critical Implementation Costs**

The *critical implementation costs of clause profile g at common discount factor δ* is defined as

$$f_{\mathrm{crit}}^{g,\delta} := \begin{cases} \frac{1}{1-\delta}\kappa_i \pi^{\mathrm{m}} & \text{if } C(g) \neq \emptyset \text{ where } i := \min C(g), \\ +\infty & \text{otherwise.} \end{cases}$$

This value gives the collusive profit of the smallest CC-adopting retailer. Obviously, whenever the actual implementation costs f exceed $f_{\mathrm{crit}}^{g,\delta}$, then this retailer would be better off without implementing a CC at the competitive outcome than with implementing a CC at the collusive outcome. This fact leads us to the following statement.

Proposition 3.7. *Consider $\Gamma(\delta, f, n, z)$. If $f > f_{crit}^{g,\delta}$, then clause profile g is not perfectly collusive.*

Define $\tilde{f}^g := \frac{1}{1-\delta} f^{g,\delta}$ whenever clause profile g is non-trivial and $\tilde{f}^g := +\infty$ otherwise. Apparently, $\tilde{f}^g$ corresponds to the annuity from period 0 onward whose present value is equal to $f^{g,\delta}$ at discount factor δ. According to Proposition 3.7, non-trivial clause profile g is not perfectly collusive if $\tilde{f}^g > \kappa_i \pi^{\mathrm{m}}$ holds for $i := \min C(g)$.

■ **Critical Hassle Costs**

To specify a threshold for the hassle costs, we introduce mapping $\bar{\pi}_i(\cdot)$ where

$$\bar{\pi}_i(p) := (p-c)\min\{k_i, D(p)\}$$

for any $p \in \mathbb{R}_+$ and any retailer $i \in I$. Apparently, mapping $\bar{\pi}_i(\cdot)$ gives the profit earned by retailer i if the customers of retailer i pay a price p and the customers of the other retailers a price above p.

Henceforth, we refer to mapping $\bar{\pi}_i$ as the *monopolistic profit mapping of retailer i*. We are interested in the price $\bar{p}_i^\delta$ for which the monopolistic profit of retailer i corresponds to the total (net) surplus retailer i attains at the collusive outcome. The following remark has already been proved in Trost (2021).

Remark 3.8. *Consider $\Gamma(\delta, f, n, z)$. For any retailer $i \in I$ and any common discount factor $\delta < \delta_{i,\mathrm{crit}}$, there is a unique $\bar{p}_i^\delta$ so that $c < \bar{p}_i^\delta < p^m$ and $\bar{\pi}_i(\bar{p}_i^\delta) = \frac{1}{1-\delta}\kappa_i \pi^{\mathrm{m}}$. Moreover, the conditions*

(i) $\bar{p}_i^\beta < \bar{p}_i^\delta$

(ii) $\bar{p}_j^\delta \leq \bar{p}_i^\delta$

are satisfied for any common discount factors $\beta < \delta$ and any retailers $j \leq i$.

Based on price $\bar{p}_i^\delta$, we define

$$\bar{\mu}_i^\delta := p^{\mathrm{m}} - \bar{p}_i^\delta\,, \qquad \bar{\phi}_i^\delta := \frac{p^{\mathrm{m}} - \bar{p}_i^\delta}{p^{\mathrm{m}}}\,, \qquad \bar{z}_i^\delta := \bar{p}_i^\delta - c$$

for any $\delta < \delta_{i,\mathrm{crit}}$ and any $i \in I$. Due to Remark 3.8, these values are positive. Moreover, it implies that $\bar{\mu}_i^\delta$ and $\bar{\phi}_i^\delta$ is decreasing in δ and non-increasing in i, whereas $\bar{z}_i^\delta$ is increasing in δ and non-decreasing in i. We call threshold $\bar{z}_i^\delta$ the *critical hassle costs of retailer i at common discount factor δ*.

Proposition 3.9. *Consider $\Gamma(\delta, f, n, z)$ where $\delta < \delta_{\mathrm{crit}}$. If $z > \bar{z}_1^\delta$, then there is no perfectly collusive clause profile.*

Summing up, Propositions 3.7 and 3.9 provide two necessary conditions for the collusive efficacy of non-trivial clause profiles: It is required that the actual implementation costs f and actual hassle costs z do not exceed their critical values $f_{\mathrm{crit}}^{g,\delta}$ and $\bar{z}_1^\delta$, respectively.

3.3 Alternative Characterization

As will be stated next, perfectly collusive clause profiles can be characterized in an alternative way. Only three conditions are required to specify perfectly collusive clause profiles; two refer to the critical discount factor and the remaining one to the critical implementation costs.

Proposition 3.10. [Trost, 2021, Remark 4] *Consider $\Gamma(\delta, f, n, z)$. A clause profile $\hat{g}$ is perfectly collusive if, and only if, properties*

(M1) $\delta_{\mathrm{crit}}^{\hat{g}} \leq \delta$

(M2) $\delta < \delta_{\mathrm{crit}}^{g}$ *for any $g := (w_i, \hat{g}_{-i}) \in G$ and any $i \in C(g)$,*

(M3) $f \leq f_{\mathrm{crit}}^{\hat{g},\delta}$

are satisfied.

The three conditions of Proposition 3.10 completely characterize the perfectly collusive clause profiles in market $\Gamma(\delta, f, n, z)$. They mirror the conditions of external and internal stability derived in the theory of partial cartels, see e.g. the expositions of D'Aspremont et al. (1983).

Property (M1) represents the *external stability* of the coalition of CC-adopting retailers in a perfectly collusive clause profile: None of the non CC-adopting retailers has an incentive to adopt a CC as the collusive outcome is already reached and, thus, the implementation of a CC would turn out to be a costly business operation without any additional gain for them. Properties (M2) and (M3) ensure the *internal stability* of the coalition of CC-adopting retailers in a perfectly collusive clause profile: None of the CC-adopting retailers prefers repealing the CC. Such withdrawal would induce a competitive outcome; with the consequence that none of them would be better off.

By means of the two stability conditions, perfectly collusive clause profiles can be detected without particular difficulties. We next set up a trivial numerical market example in order to illustrate the use of these conditions. This market example is applied as the running example of the paper.

Example I. *Consider a Bertrand market whose demand side is described by market demand mapping* $D(p) := \max\{1 - p, 0\}$. *The supply side consists of four retailers. Their capacities are given by* $k_1 := \frac{20}{100}$, $k_2 := \frac{50}{100}$, $k_3 := \frac{60}{100}$, *and* $k_4 := \frac{70}{100}$. *Each retailer has marginal costs of zero (i.e.,* $c := 0$*). The common discount factor is* $0 \leq \delta < 1$ *and the implementation costs for CCs satisfy* $f \leq \frac{1}{40}$. *Exercising CCs does not cause hassle costs for any customer (i.e.,* $z := 0$*).*

As can be easily checked, the market demand in this example satisfies ASSUMPTIONS (D1) - (D4). Moreover, we obtain a monopoly price of $p^{\mathrm{m}} = \frac{1}{2}$ so that a monopolistic retailer would earn $\pi^{\mathrm{m}} = \frac{1}{4}$. The total capacity of the four retailers amounts to $K = 2$, which in turn entails market shares of $\kappa_1 = \frac{10}{100}$, $\kappa_2 = \frac{25}{100}$, $\kappa_3 = \frac{30}{100}$, and $\kappa_4 = \frac{35}{100}$. Note that $f \leq \kappa_1 \pi^{\mathrm{m}} \leq f^{g,\delta}_{\mathrm{crit}}$ for any clause profile g and any common discount factor δ.

Let us suppose throughout the remainder of this subsection that the residual demand of this market results from efficient rationing. At first, we consider the trivial clause profile as well as the clause profiles in which all CC-adopting retailers opt for the MCC and examine which of them prove to be perfectly collusive. In doing so, we resort to the characterization of perfectly collusive clause profiles provided in PROPOSITION 3.10.

The critical discount factors can be computed by the remarks derived in the previous subsection. As argued in REMARK 3.3, the critical discount factor of the trivial clause profile amounts to $1 - \frac{D(p^{\mathrm{m}})}{K} = \frac{75}{100}$. According to REMARK 3.6(a), the critical discount factor of a clause profile in which all CC-adopting retailers opts for the MCC is equal to the market share of the non CC-adopting retailers (i.e., $1 - \kappa_J$ where J denotes the coalition of the CC-adopting retailers) as long as the total capacity of the CC-adopting retailers is large enough to serve the market demand at the competitive price. Otherwise, the critical discount factor would be equal to $1 - \frac{D(p^{\mathrm{m}})}{K}$.[17]

The critical implementation costs of these clause profiles (measured in terms of an annuity) have also been specified in the previous subsection. We know that critical implementation costs of the trivial clause profile corresponds to $+\infty$ and of the non-trivial clause profile where retailer i is the smallest MCC-adopting retailer to $\kappa_i \pi^{\mathrm{m}}$.

Applying the internal and external stability conditions of PROPOSITION 3.10, we can figure out the range of the common discount factors and implementation costs for which the trivial clause profile and the clause profiles with only MCC-adopting retailers become perfectly collusive. Indeed, as the actual implementation costs are assumed to not exceed $\frac{1}{40} = \kappa_1 \pi^{\mathrm{m}}$, PROPERTY (M3) is satisfied for any of those clause profiles and, thus, it suffices to take only account of PROPERTIES (M1) and (M2).

Apparently, the trivial clause profile becomes perfectly collusive at common discount factors not less than $\frac{75}{100}$. This result is a corollary of the PERFECT FOLK THEOREM. For such large discount factors, the grim trigger threat suffices to uphold collusive behavior. The adoption of a CC would be a pointless operation in this case; it would only cause costs without any additional benefits.

Let us now consider the clause profiles in which the only CC-adopting retailer offers the MCC. The following statements result immediately from PROPERTIES (M1) and (M2): If the smallest or the second smallest retailer adopt the MCC, then collusion is not enforceable regardless of the value of the common discount factor. However, it becomes possible if the largest or the second largest retailer adopts the MCC. In the former case, collusion results for common discount factors belonging to the right-open interval $[\frac{65}{100}, \frac{75}{100}[$; in the later case, collusion results for common discount factors belonging to the right-open interval $[\frac{70}{100}, \frac{75}{100}[$.

[17]Remarkably, one can infer from REMARKS 3.3 and 3.6(a) that these critical discount factors result regardless of which rationing rule is assumed.

Let us turn to the clause profiles in which the largest retailers as the only CC-adopting retailers offer the MCC. We conclude from PROPERTIES (M1) and (M2) that the clause profiles in which the two largest retailers and the three largest retailers adopt the MCC are perfectly collusive whenever the common discount factor belongs to the right-open intervals $[\frac{35}{100}, \frac{65}{100}[$, and $[\frac{10}{100}, \frac{35}{100}[$, respectively. The clause profile in which all retailers adopt the MCC becomes perfectly collusive at any common discount factor less than $\frac{10}{100}$.

The previous results are contained in FIGURE II. It lists the trivial clause profile as well as any clause profile in which all CC-adopting retailers opts for the MCC. The bars illustrate the ranges of the implementation costs (measured in terms of an annuity) and common discount factors in which they do not exceed their critical values. The red subareas comprise the values of the implementation costs and common discount factors for which the clause profiles are perfectly collusive.

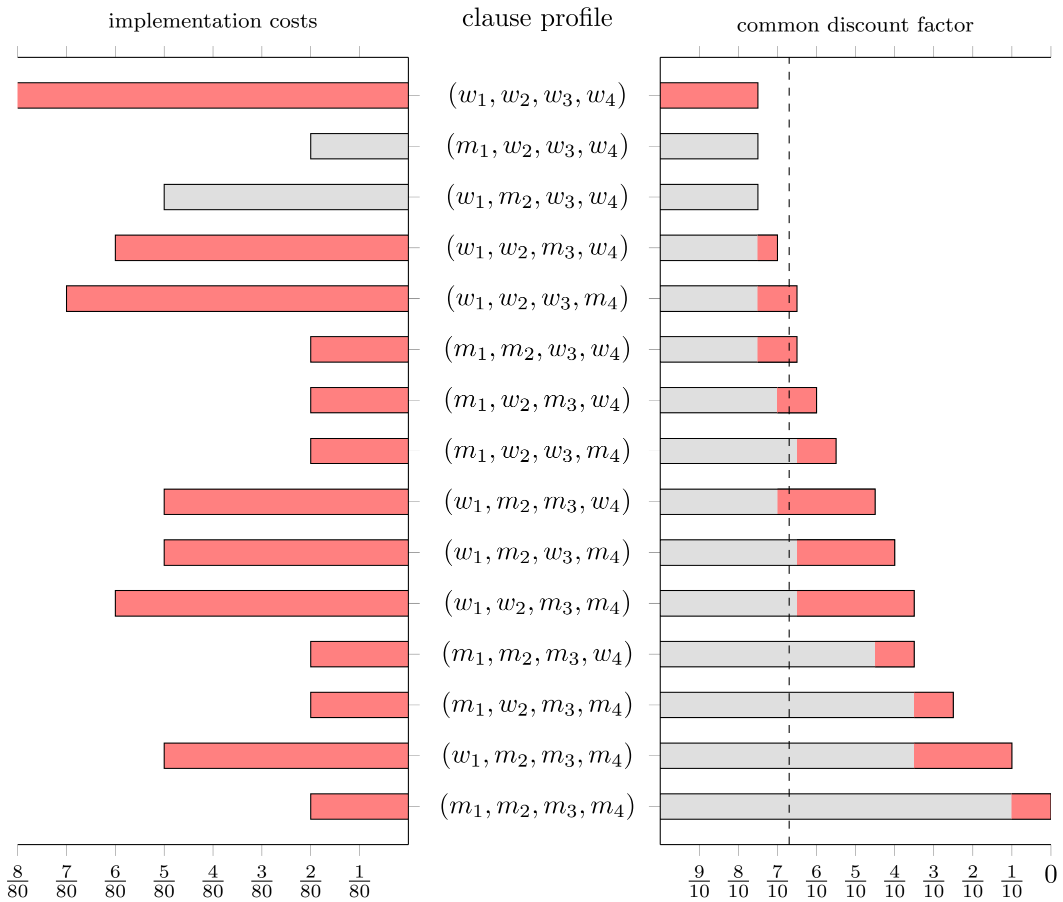

Figure II: Perfectly collusive coalitions of MCC-adopting retailers in EXAMPLE I

The striking feature of the results depicted in FIGURE II is that collusion is enforceable at any common discount factor. Indeed, if the common discount factor is sufficiently large, it suffices for collusion that only few of the retailers offer the MCC. This finding is in stark contrast to the claims of earlier theoretical studies on the collusive effectiveness of CCs such as the ones of Doyle (1988) and Corts (1995). The latter argue that a necessary condition for such effectiveness is that all retailers in the markets have to adopt MCCs. The reason for this difference is that unlike their competition models, it is taken here for granted that the clauses as binding commitments are implemented before the retailers announce the prices and that competition takes place as repeated interaction.

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It is noteworthy that FIGURE II does not provide a complete list of all perfectly collusive clause profiles. As demonstrated next, other clause profiles in which retailers adopt alternative forms of CCs also prove to be perfectly collusive at some common discount factors. FIGURE III lists as examples the clause profiles in which the largest, the second largest, and both retailers as the only the CC-adopting retailers offer the BCC with lump sum refund $p^{\mathrm{m}} - c = \frac{1}{2}$. As usual, the bars illustrate the ranges in which the implementation costs and common discount factors do not exceed their critical values (i.e., satisfy PROPERTIES (M1) and (M3), respectively). The clause profiles are perfectly collusive at values belonging to the red subareas.

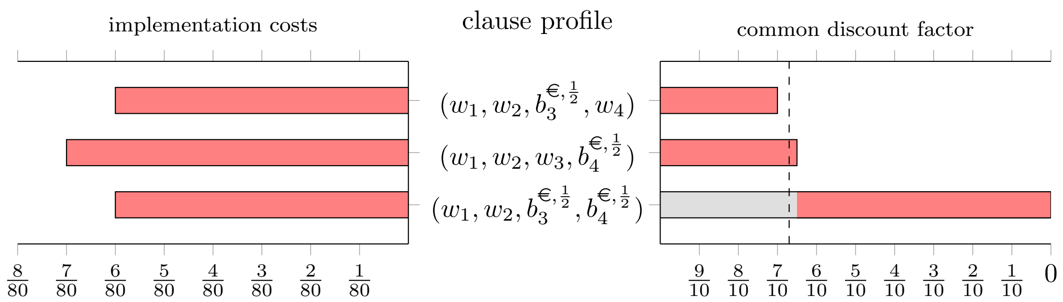

Figure III: Perfectly collusive coalitions of BCC-adopting retailers in EXAMPLE I

As can be easily verified, if the largest retailer as the only CC-adopting retailer offers the BCC with lump sum refund $\frac{1}{2}$, the critical discount factor of each of its competitors equals zero. However, the incentive of the largest retailer to defect from the collusion is the same as if none of the retailers had adopted a CC. Therefore, the critical discount factor of this clause profile corresponds to the critical discount factor of the largest retailer in the trivial clause profile and, thus, $\frac{65}{100}$ due to REMARK 3.3. Similar arguments apply to the clause profile in which the second largest retailer offers such BCC. This clause profile has a critical discount factor of $\frac{70}{100}$ due to REMARK 3.3. In the case that the two largest retailers as the only CC-adopting retailers offer such BCCs, the critical discount factor of each retailer becomes zero so that the critical discount factor of the clause profile equals zero.

Remarkably, the clause profile in which the largest retailer as the only CC-adopting retailer offers the BCC with lump sum refund $\frac{1}{2}$ is perfectly collusive at the same common discount factors as if the largest retailer had adopted the MCC. This equivalence also emerges if the second largest retailer is the CC-adopting retailer. However, it vanishes if both retailers as the only CC-adopting retailers offer such BCCs. The range of the common discount factors for which this clause profile becomes perfectly collusive corresponds to the right-open interval $[0, \frac{65}{100}[$ and, thus, is substantially larger than the range resulting from adopting the MCC.

The latter finding is intriguing. It implies that partial adoption of CCs suffices for collusion even though the common discount factor is small. This becomes possible if the retailers choose suitable forms of CCs; e.g., the BCC with lump sum refund $\frac{1}{2}$ in the market of EXAMPLE I.

A further result of our analysis is that there exist several clause profiles being perfectly collusive for some common discount factors. Suppose for example that the common discount factor takes the value of $\frac{2}{3}$. This situation is illustrated by the dashed lines in FIGURES II and III. One can see that collusion can be induced by clause profiles (w_1, w_2, w_3, m_4), (m_1, m_2, w_3, w_4), (m_1, w_2, m_3, w_4), (w_1, m_2, m_3, w_4), and $(w_1, w_2, w_3, b_4^{€,\frac{1}{2}})$. Moreover, other (conventional) clause profiles not depicted in these figures might also be perfectly collusive at this common discount factor.[18]

[18]For example, any clause profile in which the largest retailer as the only CC-adopting retailers chooses some conventional BCC is perfectly collusive at common discount factor equal to $\frac{2}{3}$.

In the face of this plethora of solutions, a closer inspection might be imperative. Indeed, some of those perfectly collusive profiles in our market example seem to be more reasonable than others. While clause profiles (w_1, w_2, w_3, m_4) and $(w_1, w_2, w_3, b_4^{€,\frac{1}{2}})$ consist of only one CC-adopting retailer, the clause profiles (m_1, m_2, w_3, w_4), (m_1, w_2, m_3, w_4), (w_1, m_2, m_3, w_4) have two CC-adopting retailers. As concerted practices might be easier to realize the less the number of involved agents, it might be justified to view the former two clause profiles as more reliable predictions. We address this issue in the subsequent section in detail by introducing additional criteria to single out the most reasonable perfectly collusive clause profiles.

We conclude this subsection by stating a sufficient condition for the existence of perfectly collusive clause profiles. It turns out that if there is some clause profile g in Γ whose critical discount factor does not exceed the common discount factor and whose critical implementation costs are not below the actual implementation costs, then there is some perfectly collusive clause profile in Γ whose CC-adopting retailers are a subset of the CC-adopting retailers of g.

Proposition 3.11. *Consider $\Gamma(\delta, f, n, z)$. If a clause profile g satisfies properties (M1) and (M3), then there is some perfectly collusive clause profile $\hat{g} := (g_J, w_{-J})$ where $J \subseteq C(g)$.*

4 Robustly Collusive Clause Profiles

The main concern of this paper is to find out whether there exists a systematic pattern in the adoption of CCs. For this purpose, we aim at detecting the most plausible coalition of CC-adopting retailers. In doing so, the solution concept proposed in the previous section is substantially refined. Being more precise, we impose two additional, lexicographically ordered selection criteria on the set of perfectly collusive clause profiles.

The primary one is cost-efficiency. It requires that the perfectly collusive clause profile have the lowest total implementation costs among the perfectly collusive clause profiles. The secondary one is resilience. It requires that the clause profile be the most resilient regarding decreases in the common discount factor and increases in the implementation costs among the cost-efficient clause profiles. Perfectly collusive clause profiles satisfying both criteria are said to be robustly collusive.

In the subsequent subsection, this refinement concept will be introduced in a formal way. It is applied throughout the remaining part of this paper in order to single out the most plausible spreading pattern of CCs. The fundamental results of our analysis are stated in the subsection after the next. It turns out that the retailers with the largest capacities prove to be the ones which are most incentivized to adopt CCs.

In the last subsection, our results are illustrated by the market example we already discussed in the previous section. This example is also applied to point to one of the limitations of our results: In specific market situations, none of the conventional clause profiles proves to be robustly collusive. As conventional clause profiles are widely used in real commercial life, this finding might challenge the view that CCs are adopted as devices facilitating collusion. The two sections after this one deal with this objection.

4.1 Definition

In order to provide an unambiguous statement about the coalition of retailers most inclined to adopt CCs for the sake of collusion, we introduce a refinement of solution concept $\mathcal{S}^{\mathrm{m}}(\cdot)$. This refinement is imposed to single out the most plausible perfectly collusive business policy profiles and is based on efficiency and resilience considerations.

The primary criterion is a cost-efficiency criterion; only business policy profiles of solution set $\mathcal{S}^{\mathrm{m}}$ with the lowest total implementation costs are selected. The *total implementation costs* are the sum of the implementation costs of all (CC-adopting) retailers, i.e., if $s := (g, (s^{\tilde{g}})_{\tilde{g}\in G})$ denotes the business policy profile chosen by the retailers, then the total implementation costs of s are $|C(g)|\, f$. The business policy profiles of $\mathcal{S}^{\mathrm{m}}(\Gamma)$ having the lowest total implementation costs are termed *cost-efficient*. We denote the set consisting of those business policy profiles by

$$\mathcal{S}^{\mathrm{e}}(\delta, f, n, z) := \{s \in \mathcal{S}^{\mathrm{m}}(\delta, f, n, z) : \ |C(o_{-1}(s))| \leq |C(o_{-1}(\tilde{s}))| \text{ for any } \tilde{s} \in \mathcal{S}^{\mathrm{m}}(\delta, f, n, z)\} .$$

Putting it differently, solution set $\mathcal{S}^{\mathrm{e}}(\delta, f, n, z)$ consists of the perfectly collusive business policy profiles with the lowest number of CC-adopting retailers.

Solution set $\mathcal{S}^{\mathrm{e}}$ is further narrowed down by a resilience criterion. It is required that there be no other cost-efficient business policy profile with more permissible thresholds regarding the common discount factor or the implementation costs, i.e., with a lower critical discount factor or a higher critical implementation costs. Cost-efficient business policy profiles fulfilling this criterion are said to be *robustly collusive*, and we denote the set consisting of those business policy profiles by

$$\mathcal{S}^{\mathrm{r}}(\delta, f, n, z) := \{s \in \mathcal{S}^{\mathrm{e}}(\delta, f, n, z) \ : \ \text{if } \mathcal{S}^{\mathrm{e}}(\delta, f, n, z) \cap \mathcal{S}^{\mathrm{e}}(\tilde{\delta}, \tilde{f}, n, z) \neq \emptyset \ \text{ for some } \tilde{\delta} \leq \delta \text{ and } \tilde{f} \geq f, \\ \text{then } s \in \mathcal{S}^{\mathrm{e}}(\tilde{\delta}, \tilde{f}, n, z)\}.$$

Henceforth, a clause profile g is said to be *robustly collusive* in competition game $\Gamma(\delta, f, n, z)$ whenever there exists a robustly collusive business profile $s := (g, (s^{\tilde{g}}))_{\tilde{g}\in G})$ in $\Gamma(\delta, f, n, z)$.

Our refinement concept can be motivated at least in an informal way. As surveyed e.g. in Buccirossi (2008), competition theory generally identifies two problems regarding the formation of tacit collusion, the coordination problem and the enforcement problem. The first one refers to the difficulty of finding a mutual understanding among the firms to act in a concerted way. The second one refers to the issue that such agreements also have to be sustainable. Even if the firms have agreed on some coordinated actions, none of them should have an incentive afterwards to deviate unilaterally from the agreement.

The coordination problem might be easier to overcome the less the number of retailers which have to implement practices facilitating collusion. This link is captured by our primary selection criterion, the cost-efficiency criterion. The enforcement issue might be easier to resolve, the more resilient the agreement with respect to adverse demand or cost shocks. Such shocks might raise the incentive of the retailers to defect, entailing an increase in their critical discount factors or a decrease in their critical implementation costs. Our secondary criterion, the resilience criterion, is aimed to capture the capability of the agreement to absorb these changes.[19] The motivation backing our refinement concept is summarized in FIGURE IV.

Let us consider two perfectly collusive clause profiles g and $\tilde{g}$ in competition game $\Gamma(\delta, f, n, z)$. We say that g is *at least as collusive as* $\tilde{g}$ in $\Gamma(\delta, f, n, z)$ whenever either (*i*) $|C(g)| < |C(\tilde{g})|$ or (*ii*) $|C(g)| = |C(\tilde{g})|$ as well as $\delta^{g}_{\mathrm{crit}} \leq \delta^{\tilde{g}}_{\mathrm{crit}}$ and $f^{g,\delta}_{\mathrm{crit}} \geq f^{\tilde{g},\delta}_{\mathrm{crit}}$ are satisfied. If g is at least as collusive as $\tilde{g}$, but the converse does not hold, than g is said to be *more collusive* than $\tilde{g}$ in $\Gamma(\delta, f, n, z)$. Note that the binary relation "at least as collusive as" and its asymmetric part "more collusive than" refer only to the perfectly collusive clause profiles in $\Gamma(\delta, f, n, z)$.

Let $\tilde{G}$ be some subset of the perfectly collusive clause profiles in competition game $\Gamma(\delta, f, n, z)$. A clause profile $g \in \tilde{G}$ is called the *most collusive in* $\tilde{G}$ whenever it is at least as collusive as any other

[19]It turns out that our results are independent of the order of the subordinate criteria. We assume throughout the paper that a clause profile is more resilient whenever it has both a lower critical discount factor and a higher critical implementation costs. If one splits this criterion into two criteria (e.g., resilience to increases in the critical discount factor becomes the secondary criterion and resilience to decreases in the critical implementation costs the tertiary one), none of our results is changed.

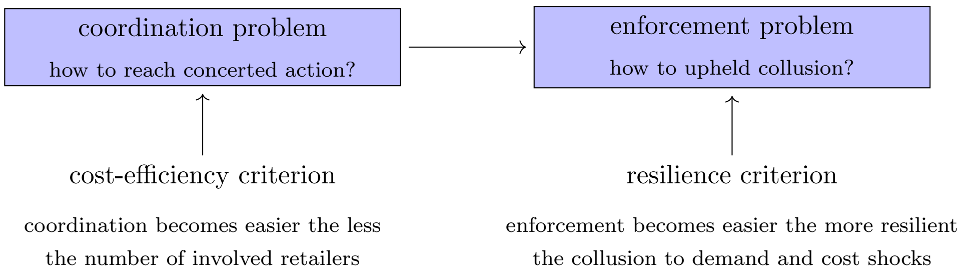

Figure IV: Motivation for the refinement criteria

clause profile of $\tilde{G}$. Apparently, clause profile g proves to be robustly collusive in $\Gamma(\delta, f, n, z)$ if, and only if, it is perfectly collusive and at least as collusive as any other perfectly collusive clause profile in $\Gamma(\delta, f, n, z)$.

Interestingly, by means of Propopsition 3.2 and the characterization of perfectly collusive clause profiles provided in Proposition 3.10, the Perfect Folk Theorem by Friedman (1971) can be transferred to our extended Bertrand competition game: Whenever the common discount factor is sufficiently close to 1, collusive behavior in the market is enforceable without adopting CCs.

Theorem 4.1. *Consider $\Gamma(\delta, f, n, z)$. The trivial clause profile w is robustly collusive if, and only if, $\delta_{crit} \leq \delta < 1$. Moreover, no other clause profile is robustly collusive at such common discount factors.*

As set forth in Theorem 4.1, the grim trigger threats of the competitors are too weak to enforce collusive behavior whenever the common discount factor is less than δ_{crit}. This finding is where our further analysis steps in. We examine whether the (partial) adoption of CCs might facilitate collusion for such common discount factors. In particular, we are interested to find out which of the retailers might be most incentivized to adopt CCs for the sake of collusion. The efficiency and resilience criteria ensure that the coalition of CC-adopting retailers is the same for any robustly collusive clause profile so that an unambiguous result occurs.

4.2 Spreading Pattern of Competition Clauses

To specify the coalition of the CC-adopting retailers in the robustly collusive clause profiles, we resort to the efficient rationing rule as the benchmark. According to Assumption (R1), efficient rationing is the most restrictive on residual market demand among all rationing rules. To simplify the following formal expositions, we introduce specific notation whenever efficient rationing is assumed to underlie the market.

Consider such competition game $\Gamma_{\mathrm{e}}(\delta, f, n, z)$. We henceforth denote the profit retailer i earns at advertised prices $q := (q_j)_{j\in I}$ under clause profile $g := (b_J^{€, p^{\mathrm{m}}-c}, w_{-J})$ (i.e., a coalition $J \subseteq I$ of retailers has implemented the BCC with lump sum refund $p^{\mathrm{m}} - c$, whereas the other retailers have not implemented a CC) by

$$\hat{\pi}_i^J(q) := \pi_i^g(q) = (q_i^{\mathrm{s}} - c)\, \min\left\{k_i, \frac{k_i}{K_{[q^{\mathrm{p}}=q_i^{\mathrm{p}}]}} R_{\mathrm{e}}(q_i^{\mathrm{p}}|q^{\mathrm{p}})\right\}$$

where $q^{\mathrm{p}} := g^{\mathrm{p}}(q)$ and $q^{\mathrm{s}} := g^{\mathrm{s}}(q)$ denote the effective purchase prices and sales prices at clause profile g, respectively. The critical discount factor of retailer i at clause profile g is denoted by $\hat{\delta}_{i,\mathrm{crit}}^J$ and the critical discount factor of clause profile g by $\hat{\delta}_{\mathrm{crit}}^J := \max\{\hat{\delta}_{i,\mathrm{crit}}^J : i \in I\}$. Obviously, $\hat{\delta}_{\mathrm{crit}}^{\emptyset} = \delta_{\mathrm{crit}}$.

As will be argued next, $\hat{\delta}_{\mathrm{crit}}^J$ proves to be the greatest lower bound of the critical discount factors resulting from clause profiles in which coalition J consists of all CC-adopting retailers. Remarkably, this result holds regardless of which rationing rule applies to the market.

Remark 4.2. *Consider* $\Gamma(\delta, f, n, z)$. *For any* $J \subseteq I$, *it holds*

$$\hat{\delta}^{J}_{crit} = \inf\{\delta^{g}_{crit} \,:\, g \in G \text{ satisfying } C(g) = J\}\ .$$

Moreover, if $\hat{\delta}^{\sigma(J)}_{crit} = \hat{\delta}^{J}_{crit}$ *for some* $\sigma \in \Sigma_n$ *upshifting and non-constant on* J, *then there exists some* $g \in G$ *so that* $C(g) = \sigma(J)$ *and* $\delta^{g}_{crit} = \hat{\delta}^{\sigma(J)}_{crit}$.

As shown in the subsequent lemma, a specific form of monotonicity in the set $\{\hat{\delta}^{J}_{\text{crit}} : J \subseteq I\}$ of those critical discount factors appears: If the BCC with lump refund $p^{\text{m}} - c$ is adopted by larger retailers, then the critical discount factor does not increase. In formal terms, $\hat{\delta}^{\sigma(J)}_{\text{crit}} \leq \hat{\delta}^{J}_{\text{crit}}$ for any $J \subseteq I$ and any $\sigma \in \Sigma_n$ upshifting on J.

Indeed, the subsequent lemma is more general than this statement. It establishes this property for any regular rationing rule and any symmetric clause profile. Putting it differently, it states that in Bertrand markets with a regular rationing rule, any upshift of a symmetric clause profile g induces a critical discount factor which does not exceed that of g.

Lemma 4.3. *Consider* $\Gamma_r(\delta, f, n, z)$. *For any* $g \in G^s$ *and any* $\sigma \in \Sigma_n$ *upshifting on* $C(g)$, *it holds*

$$\delta^{g\diamond\sigma}_{crit} \leq \delta^{g}_{crit}\ .$$

In particular, the weak inequality turns into equality if $0 = \delta^{g}_{crit}$ *or* $\delta^{g\diamond\sigma}_{crit} = \delta_{crit}$.

We exemplify Lemma 4.3 by the market of Example I and the conventional clause profiles $(w_1, b_2^{€,\frac{1}{2}}, b_3^{€,\frac{1}{2}}, w_4)$ and $(w_1, w_2, b_3^{€,\frac{1}{2}}, b_4^{€,\frac{1}{2}})$. The former clause profile is the one in which the second and the third largest retailer adopt the BCC with lump sum $p^{\text{m}} - c$, while the latter clause profile is the one in which this BCC is adopted by the two largest retailers. Their critical discount factors are specified for the efficient, proportional, and perfect rationing rule; see the bars in the first three rows of Figure V which give the ranges of the common discount factors not falling short of the critical discount factors. It is known that any of those rationing rules satisfies Assumptions (R2) - (R4) and, thus, is regular. In line with Lemma 4.3, the critical discount factor of the former clause profile does not fall short of the critical discount factor of the latter clause profile for any of them.

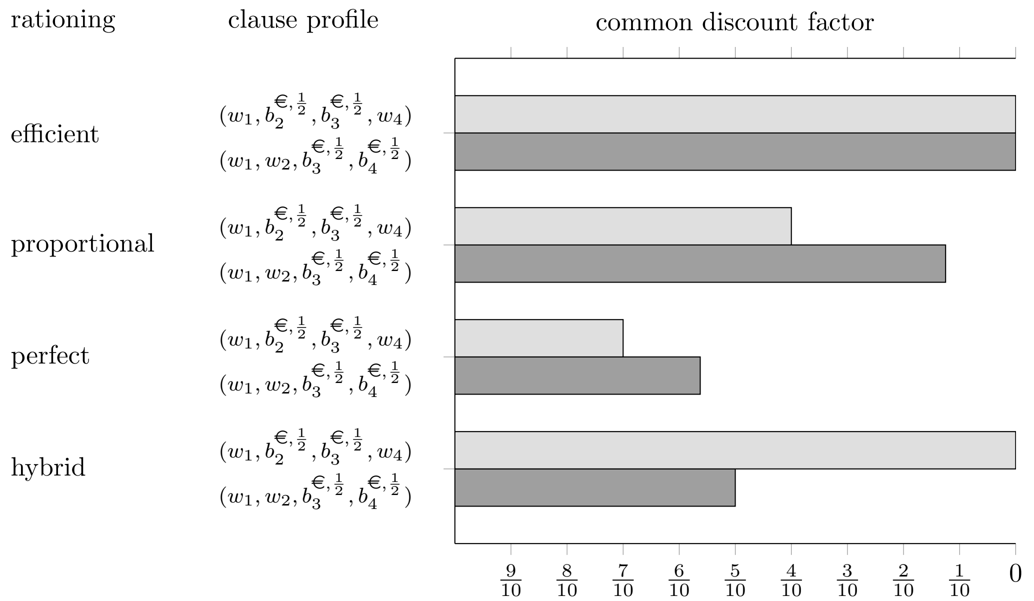

Figure V: Conventional CCs in Example I under different rationing rules

We point to two important limitations of LEMMA 4.3. First, it has been derived from the premise that the rationing rule is regular. Second, it refers only to symmetric clause profiles. We will demonstrate by counterexamples that none of these limitations can be abandoned; that means, LEMMA 4.3 can be generalized neither to arbitrary rationing rules nor to to arbitrary clause profiles. This shortcoming becomes the starting point of the next lemma which establishes a weaker version of monotonicity in the discount factors if the rationing rule is not regular or the clause profile not symmetric.

To understand the first objection we raised above, let us suppose for the time being that the rationing in the market of EXAMPLE I proceeds according to a hybrid rationing rule. The specific characteristic of this rationing rule is that it differentiates between retailers. More specifically, the customers of the largest retailer are assumed to be the (remaining) consumers with the lowest willingness to pay, whereas the customers of the other retailers are assumed to be the (remaining) consumers with the highest willingness to pay. In other words, the perfect rationing rule applies to the largest retailer, whereas the efficient rationing rule applies to the other retailers.[20] The hybrid rationing rule turns out to be monotone, but fails to be regular. In detail, it satisfies ASSUMPTIONS (R2), (R3) and (R5), but violates ASSUMPTION (R4).

As the hybrid rationing rule is not regular, the preconditions of LEMMA 4.3 are not satisfied, with the consequence that this lemma is not anymore applicable. Indeed, as recorded in the fourth row of FIGURE V, the prediction of this lemma becomes inadequate: Under the hybrid rationing rule, the critical discount factor of the clause profile in which the second and third largest retailers adopt the BCC with lump sum refund $p^{\mathrm{m}} - c = \frac{1}{2}$ is less than the one of the clause profile in which the two largest retailers adopt such BCC. We infer from this observation that the conclusion of LEMMA 4.3 breaks down by abandoning the regularity assumption.

To see the second objection, let us consider the non-conventional clause profile $g := (w_1, g_2, g_3, w_4)$ where the CCs adopted by the retailers $i \in \{2, 3\}$ are defined as follows: Both offer a lump sum refund $p^{\mathrm{m}} - c = \frac{1}{2}$ whenever their advertised prices are undercut by some of their competitors different from retailer 4. However, if only retailer 4 undersells them, they do not offer a refund so that their advertised prices corresponds to their actual ones in this case.[21] Let $\tilde{g} := g \diamond \sigma$ be the upshift of g where $\sigma \in \Sigma_n$ satisfies $\sigma(4) := 2$ and $\sigma(2) := 4$. That means, compared to clause profile g, retailers 2 and 4 have changed their roles in clause profile $\tilde{g}$.

The critical discount factors of the two clause profiles under efficient, proportional, perfect, and hybrid rationing are depicted in FIGURE VI. As the bars on the right side in this figure give the ranges of the common discount factors not falling short of them, the critical discount factors correspond to numerical values of the right ends of these bars. We recognize that the critical discount factors of g are less than those of its proper upshift $\tilde{g}$ for any of the four rationing rules. This finding underscores the above claim that the conclusion of LEMMA 4.3 cannot readily be upheld for asymmetric clause profiles.

To sum up, there are instances in which the the critical discount factor of an upshift of a clause

[20] In formal terms, the residual market demand resulting from the hybrid rationing rule is given by

$$R_{\mathrm{h}}(r|p) := \begin{cases} \max\left\{\min\{D(r), D(p_n) - k_n\} - K_{[p<r]\setminus\{n\}}, 0\right\} & \text{if } p_n < r \\ \max\left\{D(r) - K_{[p<r]}, 0\right\} & \text{otherwise} \end{cases}$$

for any prices $p \in \mathbb{R}_+^I$ and $r \in \mathbb{R}_+$.

[21] In formal terms, competition clause g_i is defined by

$$g_i(q) := \begin{cases} \max\{q_{\min} - \frac{1}{2}, 0\} & \text{if } q_j < q_i \text{ for some } j \in I \setminus \{i, 4\} \\ q_i & \text{otherwise} \end{cases}$$

for any profile $q \in \mathbb{R}_+^I$ of advertised prices and any retailer $i \in \{2, 3\}$.

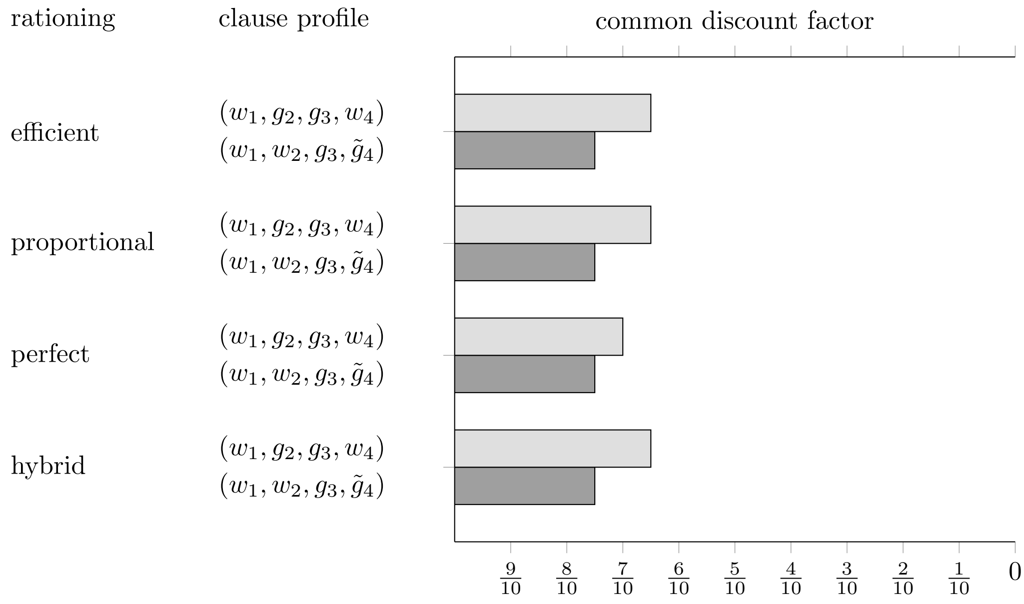

Figure VI: Non-conventional CCs in Example I under different rationing

profile exceed the one of this clause profile. However, according to Lemma 4.3, this can only be observed in markets in which the rationing rule is not regular or the clause profile not symmetric.

Nevertheless, as will be argued next, such an increase in the critical discount factor can be ruled out even for those markets as long as the CCs in the upshift are modified in an appropriate way. Putting it differently, it will be shown in the subsequent lemma that any upshift of some clause profile g induces - at least after some modifications of its CCs - a critical discount factor not exceeding the one of g. Moreover, if the permutation underlying the upshift of g induces an upshift of $b := (b^{€,\frac{1}{2}}_{C(g)}, w_{-C(g)})$ having a lower critical discount factor than b, then the upshift of g is even modifiable in such a way so that the critical discount factor becomes smaller than the one of g.

Lemma 4.4. *Consider $\Gamma(\delta, f, n, z)$. For any $g \in G$ and any $\sigma \in \Sigma_n$ upshifting on $C(g)$, there exists some $\tilde{g} \in G$ so that $C(\tilde{g}) = \sigma(C(g))$ and*

$$\delta^{\tilde{g}}_{crit} \leq \delta^{g}_{crit} \,.$$

Moreover, the weak inequality becomes strict if $\hat{\delta}^{\sigma(C(g))}_{crit} < \hat{\delta}^{C(g)}_{crit}$.

The latter two lemmata and Remark 6.3 are essential for resolving the fundamental issue of this paper. They imply that perfectly collusive clause profiles in which not the largest retailers adopt CCs are never robustly collusive.

Proposition 4.5. *Consider $\Gamma(\delta, f, n, z)$ with $\delta < \delta_{crit}$. For any perfectly collusive clause profile g satisfying $\min C(g) < n + 1 - |C(g)|$, there exists some perfectly collusive clause profile $\tilde{g}$ being more collusive than g and satisfying $\min C(\tilde{g}) = n + 1 - C(\tilde{g})$.*

An immediate consequence of this proposition is that the robustly collusive clause profiles have in common that the largest retailers are the ones adopting CCs. This salient characteristic of the robustly collusive clause profiles is summarized in the following corollary.

Corollary 4.6. *Consider $\Gamma(\delta, f, n, z)$ with $\delta < \delta_{crit}$. There exists some $k \in I$ so that any robustly collusive clause $\hat{g}$ satisfies $C(\hat{g}) = \{k, k + 1, \ldots, n\}$.*

4.3 Market Example

In this subsection, we aim to exemplify the propositions derived in the previous subsection. For this purpose, the market of Example I is revisited and the spreading pattern of the CCs in its robustly collusive clause profiles are examined. Apart from underscoring the previous propositions, this market example is also used to point to one of the limitations of these propositions.

According to the formula provided in Remark 3.3, the critical discount factor of the trivial clause profile in this market is $\delta_{\text{crit}} = \frac{75}{100}$. We therefore conclude from Theorem 4.1 that the trivial clause profile is the only robustly collusive clause profile if, and only if, the common discount factor δ does not fall short of $\frac{75}{100}$. Noteworthy, this holds regardless of which rationing rule applies to the market.

Suppose now until further notice that the residual demand in the market is determined by efficient rationing. We aim to find out whether robustly collusive clause profiles exist under this rationing rule even though the common discount factor is less than $\frac{75}{100}$. Note, whenever such clause profiles can be detected, we can also specify the coalition of retailers most willing to implement CCs in order to facilitate collusion.

Let us begin by examining the clause profiles in which only one retailer implements a CC. As we know from Remark 3.5, the critical discount factors of clause profiles in which retailer i is the only CC-adopting retailer do not fall short of $1 - \max\{\kappa_i, \frac{D(\pi^{\text{m}})}{K}\}$. Consequentially, $1 - \kappa_4 = \frac{65}{100}$ becomes a lower bound of the critical discount factors resulting from such clause profiles. Indeed, this value proves to be their greatest lower bound as such critical discount factor could be realized by the MCC adopted by the largest retailer as proved in Remark 3.6(a). Remarkably, the MCC is not the only CC inducing such critical discount factor. Without any difficulty, it can be shown that any conventional BCC adopted by the largest retailer induces this critical discount factor.

As collusive behavior in the market is established without any CCs at common discount factors not less than $\frac{75}{100}$, we conclude that any clause profile in which the largest retailer as the only CC-adopting retailer offers the MCC or some other conventional CC is robustly collusive for any common discount factor less than $\frac{75}{100}$, but not less than $\frac{65}{100}$. In contrast, perfectly collusive clause profiles in which the only CC-adopting retailer is not the largest retailer in the market never become robustly collusive. Compared to the former clause profiles, they have both a larger critical discount factor and lower critical implementation costs.

Moreover, as the critical discount factor of any clause profile in which only one of the retailers offers a CC does not fall short of $\frac{65}{100}$, it is necessary that more than one retailer have to adopt CCs in order to induce collusive behavior at common discount factor less than $\frac{65}{100}$. To figure out robustly collusive clause profiles for such common discount factors, we therefore consider the clause profiles in which at least two retailers adopt CCs.

Simple calculations establish that if the two largest retailers adopt BCCs with lump sum refunds or refund factors on the minimum price, then the critical discount factor becomes zero in our market example. Some of the perfectly collusive clause profiles in which a different pair of retailers (i.e., at least one of them is neither the largest nor the second largest retailer) implement CCs might also induce such critical discount factor; e.g., the clause profile in which the second and third largest retailers implement such BCCs. However, compared to the former clause profiles, all of them have lower critical implementation costs.

Bringing together the latter two results, we conclude that as long as the common discount factor is less than $\frac{65}{100}$, the two largest retailers are the only CC-adopting retailers in any robustly collusive clause profile. For example, as set forth above, the clause profiles in which the two largest retailers as the only CC-adopting retailers offer BCCs with lump sum refunds or refund factors on the minimum price prove to be robustly collusive for such common discount factors. Figure VII includes these results.

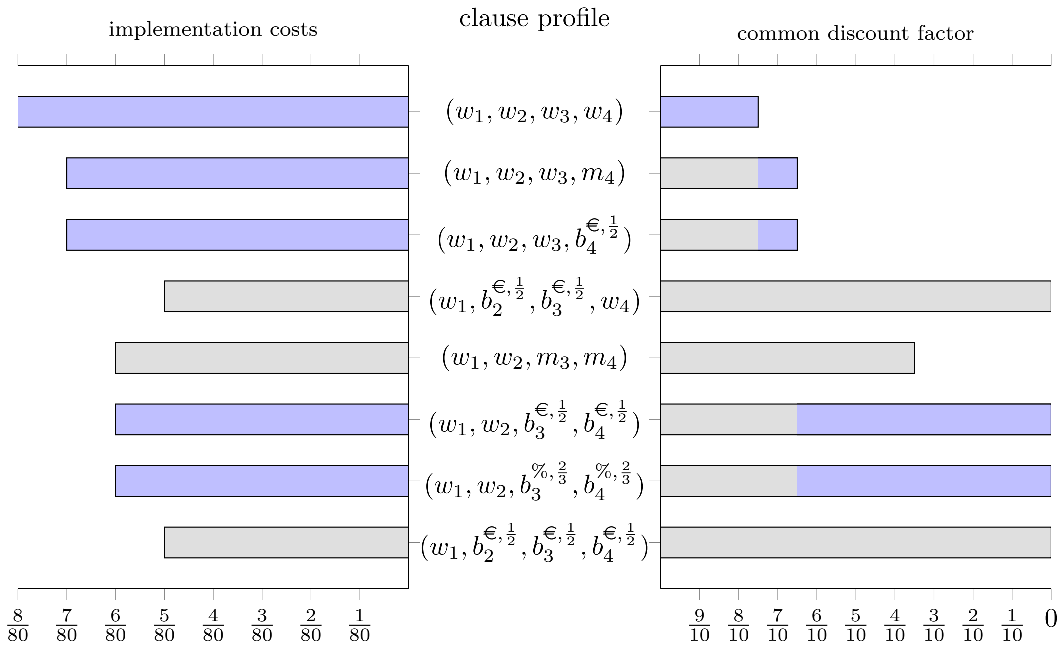

Figure VII: Robustly collusive clause profiles under efficient rationing

Indeed, several clause profiles are listed in FIGURE VII. As usual , the bars on the left give the ranges of the implementation costs (measured in terms of an annuity) not exceeding their critical values and the bars on the right the ranges of the common discount factors not falling short of their critical values in the market of EXAMPLE I under efficient rationing. The ranges of the implementation costs and common discount factors in which the clause profiles prove to be robustly collusive in this market are printed in blue.

The key insight gained by COROLLARY 4.6 is that the largest retailers are the ones most inclined to implement CCs. For the market of EXAMPLE I, we obtain the following implementation pattern: None of the retailers adopts a CC if the common discount factor is not less than $\frac{75}{100}$; only the largest retailer adopts a CC if the common discount factor is less than $\frac{75}{100}$, but not less than $\frac{65}{100}$; and only the two largest retailers adopt CCs if the common discount factor is less than $\frac{65}{100}$. As demonstrated by this example, partial adoption of CCs might sometimes suffice for facilitating collusion. The spreading pattern of the CCs in this market is depicted in FIGURE VIII.

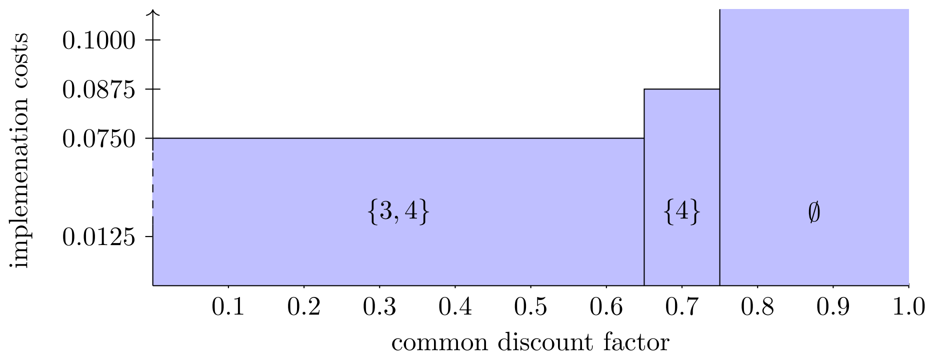

Figure VIII: Robustly collusive coalitions of CC-adopting retailers under efficient rationing

A further remarkable feature of the example is that conventional clause profiles are among the robustly collusive clause profiles for any common discount factor; for example, the perfectly collusive clause profiles in which the largest retailers as the only CC-adopting retailers offer the BCC with lump sum refund $p^{\mathrm{m}} - c = \frac{1}{2}$ prove to be robustly collusive. However, as will be argued next, this finding is fragile and does not hold for any rationing rule.

To substantiate the latter claim, we now suppose that the residual market demand is determined by perfect rationing. The critical implementation costs and critical discount factors resulting from specific clause profiles under this rationing rule are depicted in Figure IX. Apparently, some of the critical discount factors differ from the ones obtained under efficient rationing; compare the right ends of the bars on the left in Figure IX with those in Figure VII.

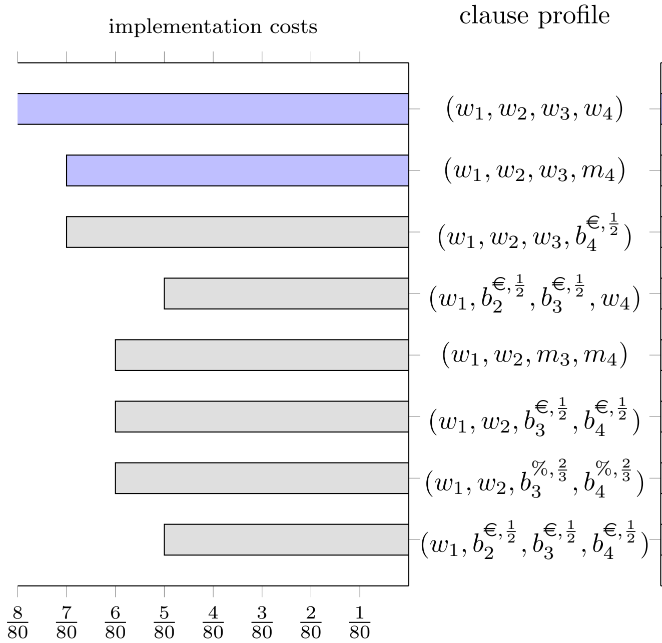

Figure IX: Robustly collusive clause profiles under perfect rationing

Applying the same arguments like in the case of efficient rationing, one can confirm that the value $\frac{65}{100}$ constitutes a lower bound of the critical discount factors resulting from clause profiles with only one CC-adopting retailer. Even more, this value turns out to be the greatest lower bound of those critical discount factors as it is induced e.g. by the MCC adopted by the largest retailer as set forth in Remark 3.6(a). Nevertheless, in contrast to the case of efficient rationing, such critical discount factor does not result from any BCC.

Take for example the clause profile in which the largest retailer as the only CC-adopting retailer offers the BCC with lump sum refund $p^{\mathrm{m}} - c = \frac{1}{2}$. Simple calculations show that the critical discount factor of this clause profile equals $\frac{75}{100}$ and, thus, corresponds to the one resulting from the trivial clause profile. This result is due to the fact that under perfect rationing, BCCs with sufficiently large lump sum refunds do not weaken the incentive of the smallest retailer to defect from the collusion.

To understand this point, it is necessary to recall that under perfect rationing, the consumers with the lowest willingness to pay shop at the cheapest retailer. This entails that each competitor of the largest retailer is able to reach out the customers with higher willingness to pay by slightly undercutting the collusive price. In particular, the smallest retailer is incentivized to do this as it could then operate at full capacity. More specifically, by such undercutting, the smallest retailer could attain any profit below $(p^{\mathrm{m}} - c)k_1 = \frac{1}{10}$. It follows that the critical discount factor of clause profiles in which the largest retailer adopts a BCC with a sufficiently large lump sum refund (e.g., with refund $p^{\mathrm{m}} - c$) correspond to that of the trivial clause profile. In consequence, those clause profiles are not perfectly collusive for any common discount factor under perfect rationing.

Summing up, there exist clause profiles which are robustly collusive under perfect rationing for common discount factors less than $\frac{75}{100}$, but not less than $\frac{65}{100}$. These clause profiles have in common that the largest retailer is the only CC-adopting retailer like in the case of efficient rationing. Interestingly, the largest retailer could choose a conventional CC such as the MCC in order to facilitate collusion. However, unlike the case of efficient rationing, not any BCC proves to be such an effective device.[22]

Let us now turn to market situations in which the common discount factor is less than $\frac{65}{100}$. Our previous results imply that at least two of the four retailers have to implement CCs in any clause profile being robustly collusive at such common discount factors. However, as will be argued next, none of the robustly collusive clause profiles is conventional. Being more precise, it turns out that both CC-adopting retailers offer non-conventional CCs in any clause profile being robustly collusive at such common discount factors.

This claim will be proven in two steps. First, we will argue that any clause profile in which at least one of the two CC-adopting retailers offers a conventional CC has a positive critical discount factor. Afterwards, we will construct a clause profile in which two CC-adopting retailers offer non-conventional CCs so that the critical discount factor becomes zero. The above claim immediately results from these findings. That means, if a clause profile is robustly collusive at a common discount factor less than $\frac{65}{100}$, then none of the CC-adopting retailers offers a conventional CC.

We notice that due to Remarks 3.6(a) and 3.6(b), the critical discount factor of any clause profile in which at least one of the two CC-adopting retailers offers the MCC or the BCC with a refund factor on the price difference are positive. Therefore, it remains to consider the clause profiles in which at least one of them offers a BCC with a lump sum refund or a BCC with a refund factor on the minimum price. Indeed, as the perfect rationing rule is monotone, it suffices according to Remark 3.6(c) to examine only the latter case.

Let us denote the two CC-adopting retailers by i and j where i represents a retailer offering a BCC with refund factor ϕ_i on the minimum price. When specifying the refund factor, retailer i has to take account of two incentives of defection. On one side, a refund factor has to be chosen so that underselling its competitors by announcing a price above the collusive one is not worthwhile. On the other side, the refund factor has to deter the competitors - in particular, the other CC-adopting retailer - from announcing a price below the collusive one.

Due to the first incentive, a necessary condition for a critical discount factor being equal to zero is that the one-off profit retailer i attains by announcing a price above the collusive price must not exceed the one-period profit retailer i earns at the collusive outcome. That means, refund factor ϕ_i has to satisfy

$$\bar{\pi}_i((1-\phi_i)p^{\mathrm{m}}) = ((1-\phi_i)p^{\mathrm{m}} - c)\min\{k_i, D((1-\phi_i)p^{\mathrm{m}})\} \leq \kappa_i \pi^{\mathrm{m}}$$

and, thus, $(1-\phi_i)p^{\mathrm{m}} \leq \bar{z}_i^0$. As $i \leq 4$ and $\bar{z}_i^0 \leq \bar{z}_4^0 = \frac{1}{8}$ due to Remark 3.8, we obtain $\phi_i \geq \frac{3}{4}$. We conclude that retailer i has never an incentive in this market example to undersell its competitors by announcing a price above the collusive one if the refund factor on the minimum price takes a value of at least $\frac{3}{4}$.

[22]It might be interesting to specify all conventional CCs which prove to be robustly collusive at such critical discount factors. However, as our primary objective is to figure out spreading patterns in the adoption of CCs, we have abstained from this exercise.

The second incentive we described above implies a further necessary condition. It is required for a critical discount factor being equal to zero that the one-off profit a competitor of i attains by announcing a price below the collusive price must not exceed the one-period profit the competitor earns at the collusive outcome. For example, if competitor j announces a price $q_j < p^{\mathrm{m}}$, its profit has to be less than or equal to its one-period profit at the collusive outcome. Consequently, the perfect rationing rule entails that refund factor ϕ_i has to satisfy

$$(q_j - c)\min\{k_j, \min\{\max\{D((1-\phi_i)q_j) - k_i, 0\}, D(q_j)\}\} \leq \kappa_j \pi^{\mathrm{m}} .$$

Suppose retailer j in our market example deviates by announcing price $q_j := \frac{8}{20}$. It is then required $\frac{8}{20}\min\{k_j, \min\{\max\{\frac{12}{20} + \frac{8}{20}\phi_i - k_i, 0\}, \frac{12}{20}\}\} \leq \frac{1}{4}\kappa_j$. If $1 \leq i < j \leq 4$, we obtain $\frac{64}{400}\phi_i \leq \frac{8}{20}\min\{k_j, \min\{\max\{\frac{12}{20} + \frac{8}{20}\phi_i - k_i, 0\}, \frac{12}{20}\}\}$ and $\frac{1}{4}\kappa_j \leq \frac{35}{400}$. This in turn entails $\frac{64}{400}\phi_i \leq \frac{35}{400}$ and, thus, $\phi_i \leq \frac{35}{64}$. If $1 \leq j < i \leq 4$, we obtain $\min\{\frac{32}{400}, -\frac{16}{400} + \frac{64}{400}\phi_i\} \leq \frac{8}{20}\min\{k_j, \min\{\max\{\frac{12}{20} + \frac{8}{20}\phi_i - k_i, 0\}, \frac{12}{20}\}\}$ and $\frac{1}{4}\kappa_j \leq \frac{30}{400}$. This in turn entails $\frac{64}{400}\phi_i \leq \frac{46}{400}$ and, thus, $\phi_i \leq \frac{46}{64}$. We conclude that retailer j has never an incentive in this market example to deviate from collusion by announcing a price $\frac{8}{20}$ if the refund factor on the minimum price takes a value of at most $\frac{46}{64}$.

Apparently, the latter condition $\phi_i \leq \frac{46}{64}$ is at odds with the former condition $\phi_i \geq \frac{48}{64}$. For this reason, it is impossible to design a conventional clause profile with two CC-adopting retailers and a critical discount factor equal to zero.[23] Nevertheless, as will be discussed in our last step of reasoning, there exist a non-conventional clause profile having both characteristics. We already know from our previous findings that both retailers have to adopt non-conventional CC in such clause profiles.

Let us consider the non-conventional clause profile in which the two largest retailers offer BCCs with the same splitting refund factors on the minimum price: the refund factor is equal to 1 whenever some of their competitors advertises a lower price, but the lowest price of them is not less than p^{m}, and equal to $\frac{1}{5}$ whenever some of their competitors advertises a lower price and this price is less than p^{m}.[24] As can be easily checked, the critical discount factor of this clause profile is zero and, thus, lower than any of those resulting from conventional clause profiles with two CC-adopting retailers. Hence, none of the conventional clause profiles is robustly collusive in the market of Example I under perfect rationing whenever the common discount factor is less than $\frac{65}{100}$.

In contrast, the non-conventional clause profile specified above proves to be robustly collusive at such common discount factors. The reason for this becomes immediately clear: As already known, all clause profiles being perfectly collusive at common discount factors below $\frac{65}{100}$ have at least two CC-adopting retailers. For this reason, none of them has lower total implementation costs than the non-conventional clause profile constructed above. Additionally, their critical implication costs cannot exceed the ones of this clause profile. Finally, as the critical discount factor of the non-conventional clause profile is equal to zero, none of them has a lower critical discount factor.

Let us recapitulate the core finding of the previous discussion: There exist market constellations so that conventional clause profiles are not among the robustly collusive ones. As was demonstrated above, this occurs e.g. in the market of Example I under perfect rationing and at sufficiently small common discount factors.

[23] Indeed, it can be established that the lowest critical discount factor resulting from some conventional clause profile with two CC-adopting retailers is equal to $\frac{25}{100}$ under perfect rationing. Such critical discount factor can be induced e.g. by the clause profile in which the two largest retailers adopt the BCC with refund factor $\frac{2}{3}$ on the minimum price. Proposition 6.1, which will be presented in Section 6, turns out to be helpful in deriving this result.

[24] In formal terms, this competition clause is defined by

$$g_i(q) := \begin{cases} 1_{\{q \in \mathbb{R}_+^I : q_{\min} < p^{\mathrm{m}}\}}(q)\frac{4}{5}q_{\min} & \text{if } q_j < q_i \text{ for some } j \in I, \\ q_i & \text{otherwise} \end{cases}$$

for any profile $q \in \mathbb{R}_+^I$ of advertised prices.

Our next objective is to look at relevant market regimes in which such finding could be ruled out *per se*, i.e., in which some of the conventional clause profiles are robustly collusive regardless of the underlying rationing rule and the value of the common discount factor. For this purpose, we will study two market regimes in the subsequent sections: markets with dominant retailers and markets with regular rationing.

The former markets refer to markets with retailers whose capacities are large enough to serve the market demand at the competitive price. One of our main results is that robust collusion is always enforceable by conventional clause profiles in markets with at least two dominant retailers. Noteworthy, to facilitate collusion in such markets, it suffices that only the two largest retailers adopt CCs.

Additional assumptions about the underlying rationing rule hold in markets with regular rationing. These assumptions have already been discussed in Section 2.2 and might be quite uncontroversial as they are satisfied by any prominent rationing rule such as the efficient or proportional rationing rule. They imply that the spreading pattern of the CCs in the clause profiles being the most collusive among the conventional clause profiles corresponds to the one observed for the robustly collusive clause profiles; namely, the largest retailers are the ones adopting the CCs.

Nevertheless, these additional assumptions prove to be too weak to ensure that those clause profiles are also robustly collusive. To establish this characteristic, further assumptions about the underlying rationing rule are required. We restrict ourselves to verifying this characteristic for the special case of markets with efficient rationing.

5 Market Regime: Dominant Retailers

The first market scenario we examine is the one with dominant retailers. In this paper, a retailer is said to be *dominant* whenever it has the capacity to serve the market demand at the competitive price (i.e., at the price equal to the marginal costs). It turns out that for such markets, there exist conventional clause profiles being robustly collusive as long as the hassle and implementation costs are sufficiently small. The proof of this claim is constructive: For any common discount factor and any sufficiently small hassle and implementation costs, we construct an example of a conventional clause profile being robustly collusive.

Theorem 5.1. *Consider $\Gamma(\delta, f, n, z)$ with $z \leq \bar{z}_1^{\delta}$. It holds*

(a) *Suppose $k_n \geq D(c)$. The conventional clause profile $\hat{g} := (w_{-n}, b_n^{€, p^m - c})$ is robustly collusive and, thus, any robustly collusive clause profile has the property that the only CC-adopting retailer is the largest retailer if and only if*

$$(i) \quad 1 - \kappa_n \leq \delta < \delta_{crit} \qquad \text{and} \qquad (ii) \quad f \leq \frac{1}{1-\delta} \kappa_n \pi^m .$$

(b) *Suppose $k_{n-1} \geq D(c)$. The conventional clause profile $\hat{g} := (w_{J_{n-2}}, b_{n-1}^{€, p^m - c}, b_n^{€, p^m - c})$ is robustly collusive and, thus, any robustly collusive clause profile has the property that the only CC-adopting retailers are the two largest retailers if and only if*

$$(i) \quad \delta < 1 - \kappa_n \qquad \text{and} \qquad (ii) \quad f \leq \frac{1}{1-\delta} \kappa_{n-1} \pi^m .$$

According to Theorem 5.1, some of the robustly collusive clause profiles are conventional in markets with at least two dominant retailers; for example, the conventional clause profile in which the CC-adopting retailers choose the BCC with lump sum refund $p^{\mathrm{m}} - c$. Moreover, this theorem states that at most the two largest (dominant) retailers adopt CCs in any robustly collusive clause profile. Remarkably, this holds regardless of which rationing rule underlies the residual market demand.[25]

The latter finding is in stark contrast to results of earlier theoretical studies such as those of Doyle (1988) and Corts (1995). Those scholars argue that all retailers have to adopt CCs in order to facilitate collusion. In markets with dominant retailers, partial adoption of CCs suffices for collusion if there are more than two retailers. Noteworthy, this holds even though the retailers do not value future profits, i.e., even though the common discount factor is equal to zero and, thus, our competition model becomes a two-stage game by its nature.

The results of THEOREMS 4.1 and 5.1 are visualized in FIGURE X. The blue areas give the ranges of the common discount factors and implementation costs in which the coalitions of the CC-adopting retailers described in these theorems become robustly collusive.

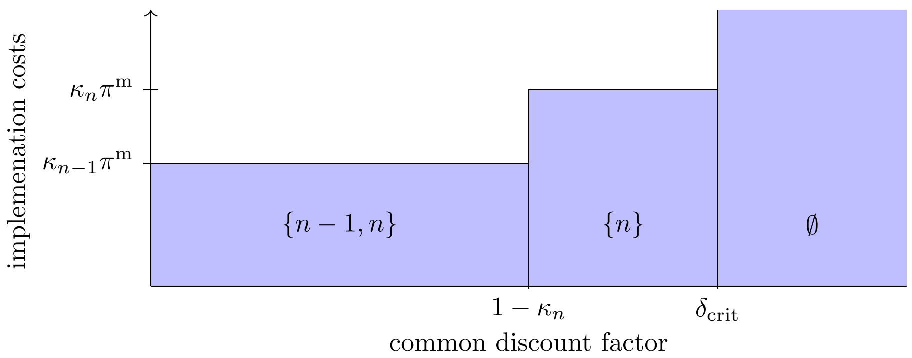

Figure X: Robustly collusive coalitions of CC-adopting retailers under dominant retailers

We illustrate the core message of THEOREM 5.1 by means of a market example.

Example II. *Consider a Bertrand market whose demand side is described by market demand mapping $D(p) := \max\{1-p, 0\}$. The supply side consists of four retailers whose capacities are $k_1 := \frac{1}{10}$, $k_2 := \frac{3}{10}$, $k_3 := \frac{10}{10}$, and $k_4 := \frac{11}{10}$. Each retailer produces at marginal costs of zero (i.e., $c = 0$). The common discount factor of the retailers is $0 \leq \delta < 1$ and the implementation costs for CCs are $f \leq \frac{1}{1-\delta}\frac{1}{10}$. The customers incur hassle costs $z \leq \frac{1}{10}$ if they exercise CCs.*

Apparently, there are two dominant retailers in this market example: retailers 3 and 4. As can be easily checked, the market capacity amounts to $K = \frac{25}{10}$ so that the market shares of the retailers are $\kappa_1 = \frac{4}{100}$, $\kappa_2 = \frac{12}{100}$, $\kappa_3 = \frac{40}{100}$, and $\kappa_4 = \frac{44}{100}$. Note that the conditions $f \leq \frac{1}{1-\delta}\kappa_3\pi^{\mathrm{m}}$ and $z \leq \bar{z}_1^0 = \frac{1}{10}$ are satisfied.

Simple calculations confirm that the critical discount factor of the trivial clause profile is equal to $\delta_{\text{crit}} = \frac{8}{10}$ in the market of EXAMPLE II. According to THEOREM 4.1, the trivial clause profile is robustly collusive for any common discount factor $\frac{8}{10} \leq \delta < 1$. It follows from THEOREM 5.1(a) that the conventional clause profile in which the largest retailer adopts the BCC with lump sum refund $p^{\mathrm{m}} - c = \frac{1}{2}$ is robustly collusive for any common discount factor $1 - \kappa_4 = \frac{56}{100} \leq \delta < \frac{80}{100}$. Due to THEOREM 5.1(b), the conventional clause profile in which the two largest retailers adopt such BCC is robustly collusive for any common discount factor $0 \leq \delta < \frac{56}{100}$. These results are depicted in FIGURE

[25] We note that the conditions stated in THEOREM 5.1 are sufficient, but not necessary for this result. As argued in SECTION 4, there are markets (e.g., the market of EXAMPLE I under efficient rationing) not satisfying these conditions, but nevertheless have the characteristic that no more than the two largest retailers have to adopt CCs in order to induce collusion in the market. Future research could seek out conditions on the capacities of the retailers being weaker than those of THEOREM 5.1, but still ensuring this characteristic.

XI. Like in the previous figures, the ranges of the implementation costs and common discount factors in which the clause profiles become robustly collusive are printed in blue.

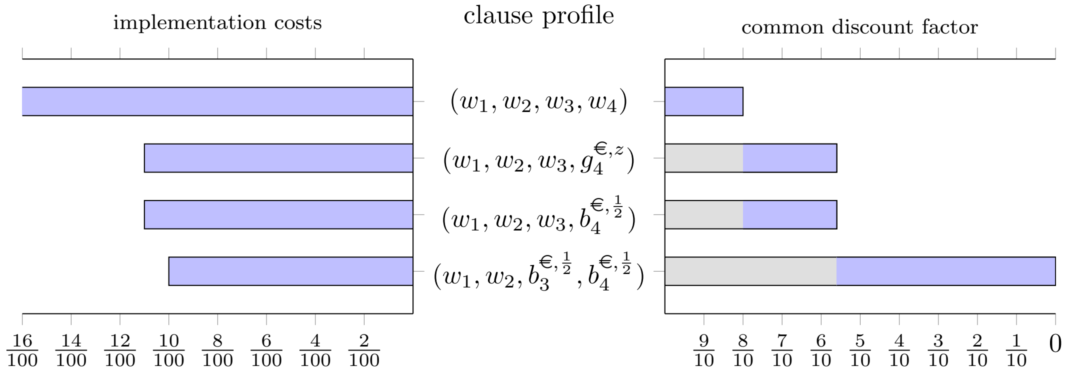

Figure XI: Robustly collusive clause profiles under dominant retailers

Propositions of this and the previous section enable us to state an existence result for markets with dominant retailers. Indeed, it immediately follows from Theorems 4.1 and 5.1 that for any common discount factor and any sufficiently small hassle and implementation costs, collusion can be facilitated by conventional clause profiles in which no more than the two largest (dominant) retailers adopt CCs. This existence result is summarized in the following corollary.

Corollary 5.2. *Consider $\Gamma(\delta, f, n, z)$ with at least two dominant retailers. If the implementation and hassle costs satisfy $f \leq \kappa_{n-1}\pi^m$ and $z \leq \bar{z}_1^0$, respectively, then for any common discount factor $0 \leq \delta < 1$ and any number of retailers n, solution set $\mathcal{S}^r(\delta, f, n, z)$ contains a conventional business policy profile in which at most the two largest dominant retailers adopt CCs.*

We have established by Theorem 5.1 that there exist conventional clause profiles being robustly collusive in markets with at least two dominant retailers. However, this theorem fails to provide a comprehensive list of all clause profiles being both conventional and robustly collusive. To see this deficiency, revisit the market of Example II and consider the clause profile in which the largest retailer as the only CC-adopting retailer chooses the CC with lump sum refund z (i.e., in the case of no hassle costs, the largest retailers adopts the MCC).

As can be easily checked, the critical discount factor and the critical implementation costs of this clause profile are identical to the ones resulting from the clause profile in which the largest retailer chooses the BCC with lump sum refund $\frac{1}{2}$. In consequence, the former clause profile proves to be another conventional clause profile being robustly collusive for common discount factors $\frac{56}{100} \leq \delta < \frac{80}{100}$. A worthwhile task might be to provide an overview of all forms of conventional clause profiles being robustly collusive. However, as the main concern of this paper is to figure out the spreading pattern of the CCs, we leave this task for future research.

6 Market Regime: Regular Rationing

In this section, we allow for any market structure, but impose additional requirements on the rationing rules. As pointed out in Corollary 4.6, the largest retailers offer CCs in any robustly collusive clause profile. However, it turned out in some instances such as the market of Example I under perfect rationing that none of the robustly collusive clause profile is conventional. The issue addressed in this section is whether the latter finding is avoidable by some additional, but innocuous assumptions regarding the rationing rules.

But even if robust collusion is not realizable by conventional clause profiles, it might be worthwhile to know whether the spreading pattern of CCs observed for robustly collusive clause profile also holds for the subclass of the clause profiles being both perfectly collusive and conventional. The focus on conventional clause profiles could be motivated by the argument that retailers favor such CCs for reasons of transparency and easier handling.[26] However, as will be demonstrated later, the answer to this question becomes negative if no further assumptions regarding the rationing rules are imposed. To substantiate this claim, we will consider a rationing rule under which clause profiles with smaller CC-adopting retailers prove to be more collusive than clause profiles with larger CC-adopting retailers, but the same number of CC-adopting retailers.

Nevertheless, it will be shown in the subsequent subsection that this can definitely be ruled out as soon as a regular rationing rule is assumed to underlie the market. That means, the spreading pattern of the CCs in the conventional clause profiles being the most collusive among the conventional clause profiles then corresponds to that we already observed for the robustly collusive clause profiles: The largest retailers are the ones adopting the CCs. The restriction to regular rationing rules might be largely uncontroversial as any prominent rationing rule such as the efficient, proportional, and perfect rationing rule fulfill them.

Despite of its relevance for the spreading pattern of conventional CCs, regularity does not imply that the most collusive ones among the conventional clause profiles prove to be robustly collusive, as we have already observed for the market of Example I under perfect rationing. Putting it differently, regularity of the rationing rule proves to be too weak to ensure that robust collusion can be established by conventional clause profiles. However, as will be argued in the subsection after the next, if efficient rationing underlies the market, this becomes possible.

6.1 Spreading Pattern of Conventional Competition Clauses

The first issue we address in this subsection is whether the spreading pattern of CCs figured out for robustly collusive clause profiles also occurs for the most collusive among the conventional clause profiles. However, it turns out that this does not always hold, as will be demonstrated by means of the market of Example I. For the time being, we suppose that the rationing in this market proceeds according to the hybrid rationing rule, which we introduced in Section 4.2. It is known that the hybrid rationing rule is monotone, but fails to be regular.

The critical implementation costs and critical discount factors of specific conventional clause profiles in this market are depicted by the right ends of the bars on the left and right in Figure XII, respectively. As usual, the ranges of the implementation costs and common discount factors in which these clause profiles become robustly collusive are printed in blue.

We will demonstrate next that whenever the common discount factor is sufficiently low, then the largest retailer does not offer a CC in any of the clause profiles which are most collusive among the conventional ones. Apparently, this finding departs from the spreading pattern of CC we derived for the robustly collusive clause profiles.

According to Theorem 4.1, the trivial clause profile is the only robustly collusive clause profile in this market at common discount factors not less than $\frac{80}{100}$. Moreover, Remarks 3.5 and 3.6(a) ensure that the clause profile in which the largest retailer as the only CC-adopting retailers offers the MCC belongs to the robustly collusive clause profiles at common discount factors not less than $\frac{65}{100}$, but below $\frac{80}{100}$. Bringing these results together, we conclude that for any discount factor not falling

[26]This issue could be incorporated in the competition model by assuming heterogeneous implementation costs. Due to their lower degree of complexity, the implementation costs of conventional CCs might be significantly lower than the implementation costs of non-conventional CCs.

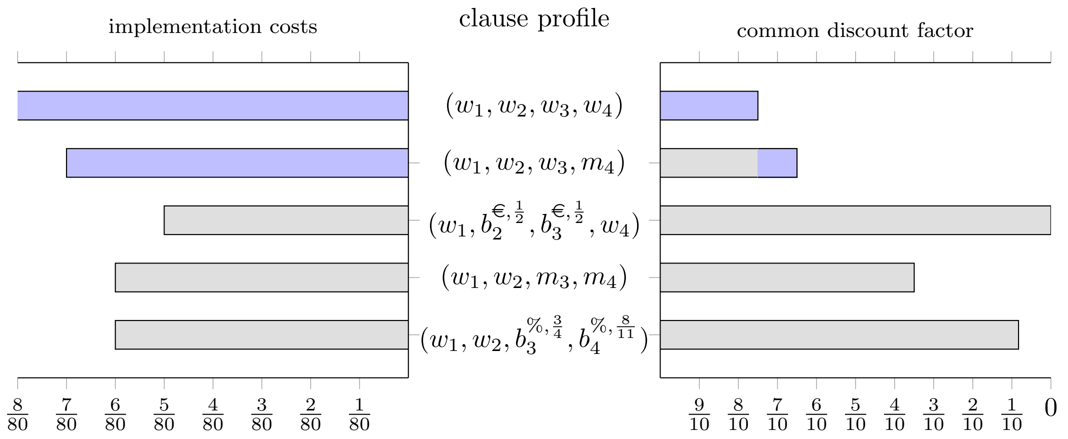

Figure XII: Robustly collusive clause profiles under hybrid rationing

short of $\frac{65}{100}$, there exists a conventional clause profile being robustly collusive. Hence, the spreading pattern of CCs described in Corollary 4.6 holds at least for these instances.

Let us now consider a common discount factor less than $\frac{65}{100}$. The above results entail that at least two retailers have to adopt CCs in any clause profile being perfectly collusive at such common discount factors. Suppose there are two CC-adopting retailers. Moreover, it is assumed that both offer conventional CCs and one of them is the largest retailer. We aim to figure out a lower bound of the critical discount factors induced by such clause profiles.

If the largest retailer implements a BCC with refund factor on the price difference, then the critical discount factor of the clause profile is not less than $\frac{65}{100}$ due to Remark 3.6(b). If it offers the MCC, then the critical discount factor of the clause profile is not less than $\frac{35}{100}$ due to Remark 3.6(a). Indeed, according to his remark, this critical discount factor can be induced e.g. by the clause profile in which both retailers offer the MCC. It remains to specify the lowest critical discount factor of the clause profiles in which each of the two offers a BCC with a lump refund or a BCC with a refund factor on the minimum price. As the hybrid rationing rule is monotone, Remark 3.6(c) is applicable. Due to this remark, it suffices to focus only on the cases where both retailers offer BCCs with refund factors on the minimum price.

Consider the conventional clause profile $g := (w_{-\{i,4\}}, b_i^{\%,\phi_i}, b_4^{\%,\phi_4})$ where i denotes the other retailer offering the BCC. As can be easily checked, it holds $\delta^g_{k,\text{crit}} = 0$ for any retailer $k \neq i, 4$ so that $\delta^g_{\text{crit}} = \min\{\delta_i^{\downarrow}(g), \delta_i^{\uparrow}(g), \delta_4^{\downarrow}(g), \delta_4^{\uparrow}(g)\} \geq \min\{\delta_i^{\downarrow}(g), \delta_4^{\uparrow}(g)\}$. In the following, we aim to specify the refund factor $\hat{\phi}_4$ which minimizes $\min\{\delta_i^{\downarrow}(g), \delta_4^{\uparrow}(g)\}$.

Resorting to Assumption (D1) and standard arguments, we conclude that $\pi_i^{\downarrow}(g)$ and $\pi_4^{\uparrow}(g)$ are continuous on $\phi_4 \in]0,1]$. This in turn implies that the discount factors $\delta_i^{\downarrow}(g)$ and $\delta_4^{\uparrow}(g)$ are continuous on $\phi_4 \in]0,1]$. It follows from Assumption (D2) and the hybrid rationing rule that $\pi_i^{\downarrow}(g)$ is positive and strictly increasing on $\phi_4 \in]0,1]$. Consequentially, there exists some $\theta_4^{\downarrow} \in [0,1]$ so that $\delta_i^{\downarrow}(g)$ is equal to 0 on $]0, \theta_4^{\downarrow}]$ and positive and strictly increasing on $]\theta_4^{\downarrow}, 1]$. Moreover, Assumption (D2) ensures that $\pi_4^{\uparrow}(g)$ is strictly decreasing on $\phi_4 \in]0,1]$ and equal to zero if $\phi_4 = 1$. Consequentially, there exists some $\theta_4^{\uparrow} \in [0,1]$ so that $\delta_4^{\uparrow}(g)$ is positive and strictly decreasing on $]0, \theta_4^{\uparrow}[$ as well as equal to zero on $[\theta_4^{\uparrow}, 1]$. Simple calculations confirm $\theta_4^{\downarrow} = 1 - \frac{9}{100\kappa_i}$ and $\theta_4^{\uparrow} = \frac{3}{4}$. This entails that there exists a unique $\hat{\phi}_4 \in]\theta_4^{\downarrow}, \theta_4^{\uparrow}]$ so that $\delta_i^{\downarrow}(g) = \delta_4^{\uparrow}(g)$.

As can be easily seen, the larger retailer i, the greater refund factor $\hat{\phi}_4$ and, thus, the less discount factor $\delta_i^{\downarrow}(g) = \delta_4^{\uparrow}(g)$. Therefore, the latter discount factor is minimized whenever $i = 3$ (i.e., retailer 3 is the other CC-adopting retailer).

By simple calculations, we obtain $\hat{\phi}_4 = \frac{8}{11}$ and, thus, $\delta_4^{\downarrow}(g) = \delta_4^{\uparrow}(g) = \frac{1}{12}$ if clause profile g is of the form $(w_{-\{3,4\}}, b_3^{\%,\phi_3}, b_4^{\%,\frac{8}{11}})$. It follows from $\delta_{\text{crit}}^g \geq \min\{\delta_4^{\downarrow}(g), \delta_4^{\uparrow}(g)\}$ that $\frac{1}{12}$ is a lower bound of the critical discount factor of such clause profiles. This finding along with the above results implies that $\frac{1}{12}$ is a lower bound of all critical discount factors resulting from conventional clause profiles in which one of the two CC-adopting retailers is the largest retailer.

Indeed, it turns out that this lower bound is realizable by some clause profile. Suppose that retailer 3 is retailer i in the above clause profiles. Whenever it offers a BCC with refund factor $\phi_3 \geq \frac{8}{11}$ on the minimum price, then $\delta_3^{\uparrow}(g) \leq \frac{1}{12}$ and $\delta_4^{\downarrow}(g) \leq \frac{1}{12}$ and, thus, $\delta_{\text{crit}}^g = \frac{1}{12}$. That is, any clause profile g of the form $(w_{-\{3,4\}}, b_3^{\%,\phi_3}, b_4^{\%,\frac{8}{11}})$ satisfying $\phi_3 \geq \frac{8}{11}$ induces critical discount factor $\delta_{\text{crit}}^g = \frac{1}{12}$.

However, such clause profiles do not induce the lowest critical discount factor among the conventional clause profiles with two CC-adopting retailers. To see this, consider the conventional clause profile $\tilde{g} := (w_1, b_2^{€,\frac{1}{2}}, b_3^{€,\frac{1}{2}}, w_4)$. As can be easily checked, it holds $\delta_{\text{crit}}^{\tilde{g}} = 0$. In consequence, the critical discount factor of clause profile $\tilde{g}$ is lower than the critical discount factors resulting from conventional clause profiles in which one of the two CC-adopting retailers is the largest retailer. Thereby, we have shown that if the common discount factor is below $\frac{65}{100}$, then none of the conventional clause profiles in which the largest retailer offers a CC is the most collusive among the conventional clause profiles. That means, the spreading pattern we observed for the robustly collusive clause profiles cannot be confirmed for the subclass of perfectly collusive and conventional clause profiles.

Admittedly, the example of the hybrid rationing rule is quite contrived. It might be hard to justify the unequal treatment of the retailers inherent in this rationing rule. Our next task is to examine whether the usual spreading pattern of CCs reappears if less plausible rationing rules such as the hybrid rationing rule are ruled out a priori.

From now on, it is taken for granted that the rationing rule is regular. We already know that the hybrid rationing rule does not belong to this type of rationing rule, but more prominent rationing rules such as the efficient, proportional or perfect rationing rule do. Given this additional assumption, we are able to specify the spreading pattern of the conventional CCs by means of Lemma 4.3. It follows from this lemma that for any conventional and perfectly collusive clause profile in which not any CC is adopted by one of the largest retailers, we can find some more collusive conventional clause profile in which the CCs are adopted only by the largest retailers. This result is summarized in the following proposition.

Proposition 6.1. *Consider $\Gamma_r(\delta, f, n, z)$ with $\delta < \delta_{crit}$. For any conventional and perfectly collusive clause profile g satisfying $\min C(g) < n + 1 - |C(g)|$, there exists some conventional clause profile $\hat{g}$ being more collusive than g and satisfying $\min C(\hat{g}) = n + 1 - C(\hat{g})$.*

An immediate consequence of Proposition 6.1 is that any clause profile being most collusive among the conventional clause profiles has the characteristic that the largest retailers are the only ones which adopt the CCs. That means, the spreading pattern of the CCs we derived for robustly collusive clause profiles in the last section also occurs even though the retailers are confined to select conventional CCs.

Corollary 6.2. *Consider $\Gamma_r(\delta, f, n, z)$ with $\delta < \delta_{crit}$. There exists some $k \in I$ so that any clause profile $\hat{g}$ being the most collusive among the conventional clause profiles satisfies $C(\hat{g}) = \{k, k + 1, \ldots, n\}$.*

The merit of Corollary 6.2 is to provide insights on the spreading pattern of conventional CCs. However, by no means, the corollary guarantees that conventional clause profiles belong to the robustly collusive ones. Indeed, we already noticed for the market of Example I under perfect rationing that none of the conventional clause profiles are robustly collusive for common discount factors less than $\frac{65}{100}$.

Recall, to substantiate this claim, we considered the clause profile in which the two largest retailers offer BCCs with splitting lump sum refunds: $p^{\mathrm{m}} - c = \frac{1}{2}$ is repaid whenever some of the competitors advertises a lower price, but the lowest price of the competitors is not less than p^{m}, but only $\frac{1}{5}$ whenever some competitor advertises a lower price and this price is less than p^{m}. It turned out that the critical discount factor of this clause profile equals zero.

On the other hand, we argued that any conventional clause profile with two CC-adopting retailers induces a positive critical discount factor. As at least two retailers have to adopt CCs in any perfectly collusive clause profile at common discount factors less than $\frac{65}{100}$, the above non-conventional clause profile proves to be robustly collusive for any of those common discount factors, but the conventional clause profiles do not.

The result of this market example collides with the empirical finding that in real commercial life retailers rather offer CCs in form of conventional clause profiles. For this reason, it might be worthwhile to find out whether the infimum of the critical discount factors resulting from conventional clause profiles (substantially) deviates from the one of the robustly collusive clause profiles. By means of Proposition 6.1, it becomes a less tedious exercise to specify the greatest lower bound of those discount factors. According to this proposition, it suffices to focus only on the clause profiles in which the two largest retailers adopt conventional CCs.

We conclude from Remarks 3.6(a) and 3.6(b) that the critical discount factors of clause profiles in which one of the two retailers offers the MCC or a BCC with a refund factor on the price difference does not fall short of $\frac{35}{100}$. Indeed, as stated in Remark 3.6(a), this lower bound is realized if both retailers offer the MCC.

It remains to examine whether lower critical discount factors results for clause profile in which each of the two retailers adopts either a BCC with a lump sum refund or with a refund factor on the minimum price. Again, this analysis can substantially be simplified due to Remark 3.6(c). As the critical discount factor does not increase if a retailer goes over from the former type of the BCC to (a suitable version of) the latter type, it suffices to consider only the clause profiles in which both retailers adopt BCCs with refund factors on the minimum price.

Let $g := (w_1, w_2, b_3^{\%,\phi_3}, b_4^{\%,\phi_4})$ be a conventional clause profile of such form. As the total capacity of the two largest retailers exceed the market demand at the competitive price, the critical discount factors $\delta^g_{i,\mathrm{crit}}$ of the two smallest retailers $i \in \{1, 2\}$ are equal to zero regardless of which refund factors the two largest retailers choose. Hence, $\delta^g_{\mathrm{crit}} = \max\{\delta^{\uparrow}_3(g), \delta^{\downarrow}_4(g), \delta^{\uparrow}_4(g), \delta^{\downarrow}_3(g)\}$.

Let us now examine the incentives of the two largest retailers $i, j \in \{3, 4\}$ to defect from collusion. It follows from Assumption (D1) and standard arguments that $\pi^{\uparrow}_i(g)$ and $\pi^{\downarrow}_j(g)$ are continuous on $\phi_i \in]0, 1]$. This in turn implies that the discount factors $\delta^{\uparrow}_i(g)$ and $\delta^{\downarrow}_j(g)$ are continuous on $\phi_i \in]0, 1]$. Moreover, Assumption (D2) entails that $\pi^{\uparrow}_i(g)$ is strictly decreasing on $\phi_i \in]0, 1]$ and equal to zero if $\phi_i = 1$. Hence, there exists some $\theta^{\uparrow}_i \in [0, 1]$ so that $\delta^{\uparrow}_i(g)$ is positive and strictly decreasing on $]0, \theta^{\uparrow}_i[$ and equal to zero on $[\theta^{\uparrow}_i, 1]$. Due to Assumption (D2) and perfect rationing, $\pi^{\downarrow}_j(g)$ is strictly increasing on $\phi_i \in [0, 1]$. Hence, there exists some $\theta^{\downarrow}_i \in [0, 1]$ so that $\delta^{\downarrow}_i(g)$ is equal to 0 on $]0, \theta^{\downarrow}_i]$ and positive and strictly increasing on $]\theta^{\downarrow}_i, 1]$. Simple calculations confirm $\theta^{\uparrow}_3 = \frac{3}{4}$ and $\theta^{\downarrow}_3 = \frac{19}{35}$ as well as $\theta^{\uparrow}_4 = \frac{3}{4}$ and $\theta^{\downarrow}_4 = \frac{7}{10}$.

These results ensure that there exists a unique $\hat{\phi}_i \in]\theta^{\downarrow}_i, \theta^{\uparrow}_i[$ so that $\delta^{\uparrow}_i(g) = \delta^{\downarrow}_j(g)$. It turns out that these equalities hold whenever $\hat{\phi}_3 = \frac{2}{3}$ and $\hat{\phi}_4 = \frac{8}{11}$. Let us define clause profile $\hat{g} := (w_1, w_2, b_3^{\%,\frac{2}{3}}, b_4^{\%,\frac{8}{11}})$. Our above argumentation implies $\max\{\delta^{\uparrow}_i(g), \delta^{\downarrow}_j(g)\} > \max\{\delta^{\uparrow}_i(\hat{g}), \delta^{\downarrow}_j(\hat{g})\}$ for any $\phi_i \neq \hat{\phi}_i$ and any different $i, j \in \{3, 4\}$. Therefore, δ^g_{crit} is minimized at $g = \hat{g}$. As can be easily verified, it holds $\delta^{\uparrow}_3(\hat{g}) = \delta^{\downarrow}_4(\hat{g}) = \frac{1}{4}$ and $\delta^{\downarrow}_4(\hat{g}) = \delta^{\uparrow}_3(\hat{g}) = \frac{1}{12}$ so that critical discount factor $\delta^{\hat{g}}_{\mathrm{crit}} = \delta^{\hat{g}}_{3,\mathrm{crit}} = \delta^{\hat{g}}_{4,\mathrm{crit}} = \frac{1}{4}$ results.

We point out that there are also other clause profiles of the form $g := (w_1, w_2, b_3^{\%,\phi_3}, b_4^{\%,\phi_4})$

inducing such critical discount factor. To see this, we note that $\delta_4^{\downarrow}(\hat{g}) \leq \frac{1}{4}$ and $\delta_3^{\uparrow}(\hat{g}) \leq \frac{1}{4}$ whenever $\frac{2}{3} \leq \phi_4 \leq \frac{4}{5}$. Therefore, any clause profile $g := (w_1, w_2, b_3^{\%,\frac{2}{3}}, b_4^{\%,\phi_4})$ satisfying $\frac{2}{3} \leq \phi_4 \leq \frac{4}{5}$ has critical discount factor $\delta_{\text{crit}}^{g} = \frac{1}{4}$; take for example clause profile $(w_1, w_2, b_3^{\%,\frac{2}{3}}, b_4^{\%,\frac{2}{3}})$, which has already been recorded in the seventh row of FIGURE IX.

Summing up, we obtain the following result: The minimum critical discount factor of conventional clause profiles with two CC-adopting retailers in the market of EXAMPLE I under perfect rationing is equal to $\frac{1}{4}$. As there are non-conventional clause profiles with the same number of CC-adopting retailers, but with critical discount factors equal to 0, the minimum critical discount factor of conventional clause profiles with two CC-adopting retailers is greater than the minimum critical discount factor of the non-conventional clause profiles with two CC-adopting retailers. This results explains why none of the conventional clause profiles proves to be robustly collusive for common discount factors less than $\frac{65}{100}$ in this market example.

The latter finding leaves us with the question whether the collusive superiority of non-conventional clause profiles is also observable for other regular rationing rules. Interestingly, a different result occurs if the residual market demand in the market of EXAMPLE I is specified by efficient rationing. Under such rationing, the critical discount factor becomes zero if the two largest retailers adopt the BCC with lump sum refunds $p^{\text{m}} - c$ (see the sixth row in FIGURE VII) and, thus, there exists a conventional clause profile which is robustly collusive for any common discount factors less than $\frac{65}{100}$.

It remains to prove whether this result also holds in general. That is, we have to examine whether efficient rationing ensures that for any common discount factor, there exists some clause profile which is both conventional and robustly collusive. The subsequent subsection is devoted to this issue.

6.2 Special Case of Efficient Rationing

Let us turn to the special cases in which the residual demand of the Bertrand markets is specified by efficient rationing. It has been suggested above that in such markets, there exist conventional clause profiles being robustly collusive as long as the hassle and implementation are sufficiently small. To prove the claim, we proceed constructively: For any common discount factor and any sufficiently small hassle and implementation costs, we set up an example of a clause profile which is both conventional and robustly collusive. The construction is based on the following two insights.

Remark 6.3. *It holds:*

(a)

$$\hat{\delta}_{crit}^{\tilde{J}} \leq \hat{\delta}_{crit}^{J}$$

for any $J \subseteq \tilde{J} \subseteq I$.

(b) *If $\delta < \delta_{crit}$ and $z \leq \bar{z}_1^{\delta}$, then*

$$\hat{\delta}_{crit}^{J} \leq \delta$$

for any non-empty $J \subseteq I$ satisfying $\hat{\delta}_{j,crit}^{J} \leq \delta$ where $j := \max J$.

To understand REMARK 6.3, consider the set of the clause profiles in which all CC-adopting retailers offers the BCC with lump sum refund $p^{\text{m}} - c$. Part (a) states that under efficient rationing the critical discount factor does not increase the more retailers adopt the BCC with lump sum refund $p^{\text{m}} - c$. Part (b) states that under efficient rationing and for sufficiently small hassle costs, the critical discount factor of such clause profile does not exceed some threshold whenever the critical discount factor of its largest CC-adopting retailer does not exceed this threshold. These two properties entail that clause profiles in which all CC-adopting retailers choose the BCC with lump sum refund $p^{\text{m}} - c$ become robustly collusive in Bertrand markets with efficient rationing.

Theorem 6.4. *Consider* $\Gamma_e(\delta, f, n, z)$ *where* $\delta < \delta_{crit}$ *and* $z \leq \bar{z}_1^{\delta}$. *The non-trivial clause profile* $\hat{g} := (w_{J_k}, b_{-J_k}^{€,p^m-c})$ *(i.e., the non-trivial clause profile in which the* $n-k$ *largest retailers adopt the BCC with lump sum refund* $p^m - c$*, while the other retailers do not adopt CCs) is robustly collusive if and only if the conditions*

(i) $\delta_{n,crit}^{\hat{g}} \leq \delta$

(ii) $\delta < \delta_{n,crit}^{g}$ *where* $g := (w_{J_{k+1}}, b_{-J_{k+1}}^{€,p^m-c})$.

(iii) $f \leq \frac{1}{1-\delta}\, \kappa_{k+1}\, \pi^m$.

are satisfied.

Notably, an additional merit of Theorem 6.4 is that it simplifies the specification of the number of CC-adopting retailers in the robustly collusive clause profiles. According to this theorem, it suffices for accomplishing this task to calculate the critical discount factors of the largest retailer. For better understanding, this approach is explained by the market of Example I.

As hassle costs do not prevail in this market, $z \leq \bar{z}_1^{\delta}$ is trivially satisfied for any common discount factor δ. Moreover, the implementation costs satisfy condition (iii) of Theorem 6.4 due to assumption $f \leq \frac{1}{40} \leq \frac{1}{1-\delta}\frac{1}{40}$. Let us suppose that efficient rationing applies to the residual market demand and consider the non-trivial clause profiles $\hat{g} := (w_1, w_2, b_3^{€,\frac{1}{2}}, b_4^{€,\frac{1}{2}})$ and $g := (w_1, w_2, w_3, b_4^{€,\frac{1}{2}})$. As can be easily verified, the critical discount factors of the largest retailer, i.e., retailer 4, at these clause profiles are equal to 0 and $\frac{65}{100}$, respectively. Hence, we conclude from Theorem 6.4 that $\hat{g}$ is robustly collusive if, and only if, $0 \leq \delta < \frac{65}{100}$.

Theorems 4.1 and 6.4 ensure for markets with efficient rationing that for any common discount factor and any sufficiently small hassle and implementation costs, there exists a conventional clause profile being robustly collusive. That means, retailers have not to resort to non-conventional clause profiles to facilitate robust collusion in such market settings. This existence result is reproduced in the following corollary.

Corollary 6.5. *Consider* $\Gamma_e(\delta, f, n, z)$. *If the implementation and hassle costs satisfy* $f \leq \kappa_1 \pi^m$ *and* $z \leq \bar{z}_1^0$, *respectively, then solution set* $\mathcal{S}^r(\delta, f, n, z)$ *contains a conventional business policy profile for any common discount factor* $0 \leq \delta < 1$ *and any number of retailers* n.

While Corollary 6.5 ensures the existence of conventional clause profiles inducing robust collusion, Corollary 4.6 characterizes the pattern of the clause profiles inducing robust collusion. Interestingly, analogous results can be found in the theory of price leadership.

To explain the formation of price leadership, Deneckere and Kovenock (1992) as well as Ishibashi (2008) use competition models very similar to ours. Like us, both consider price competition games with infinite horizon and capacity-constrained retailers under efficient rationing. They show that in such market environments, price leadership results where the retailer with the largest capacity is the one taking on the role of the price leader.[27]

However, it is noteworthy that their results and ours rely on a different assumption about the size of the total (market-wide) capacity. They suppose that there is a retailer so that the total capacity minus the capacity of this retailer is not large enough to serve the market demand at the marginal costs, i.e., $D(c) > K_{-i}$ is satisfied for some retailer i.[28] Apparently, this condition violates Assumption (D4) on which our results are based.

[27] As shown in Furth and Kovenock (1993), this results holds even in Bertrand markets with heterogeneous products.

[28] If $D(c) > K_{-i}$ for some retailer i, then the one-stage Bertrand competition would not end in a non-profit equilibrium, i.e., not end in a situation in which customers pay a price equal to the marginal costs so that the retailers earn zero profits. For more details about the (mixed) Nash equilibrium in this case, see the expositions of Kreps and Scheinkman (1983) as well as Osborne and Pitchik (1986).

Interestingly, if the capacity constraints in the models of Deneckere and Kovenock (1992) and Ishibashi (2008) satisfy Assumption (D4), then the formation of the price leadership becomes inconclusive. Moreover, price leadership might fail to be an effective facilitating practice in this case. For example, consider the duopoly model of Deneckere and Kovenock (1992). As stated there in the (a) parts of Theorems 1 - 3, none of the retailers benefits from taking on the price leader position. A price leader earns a zero profit regardless of which price it charges. Moreover, even if there were a price leader, the price follower might also not benefit from the sequential price setting. Indeed, if the leader opts for a price equal to the marginal costs, then the follower also earns a zero profit.

Nevertheless, as argued in Corollary 6.5, collusion can be enforced in this case by the adoption of CCs whenever the hassle and implementation costs are sufficiently low. Due to this finding, our paper can be viewed as complementary to those of Deneckere and Kovenock (1992) as well as Ishibashi (2008).

7 Concluding Remarks

The fundamental issue this paper has aimed to address is whether there is a systematic pattern in the adoption of CCs if the retailers use them in order to facilitate tacit collusion. To accomplish this task, we have adopted the competition model proposed in Trost (2021). This model is similar to the ones applied in the cartel formation literature, in particular to the model of Bos and Harrington (2010).

In line with the literature on cartel formation, our competition model is a multi-stage game with perfect information and split in two phases. The game begins with the clause implementation phase. In this phase, the retailer decide about their clause policies. Afterwards, the price competition phase starts. This phase consists of an infinitely repeated Bertrand game. That is, the retailers compete in prices in each stage of this phase. The effective prices of each stage are the prices resulting from the prices announced by the retailers in this stage and the clause policies chosen in the implementation phase.

A crucial assumption of our competition model is that the clause profiles are chosen before the price competition takes place. That means, each retailer is informed about the clause policies of any of its competitors when announcing the price of the merchandise. This assumption is substantial. The idea behind it is that CCs are regarded by us as binding and lasting commitments of the retailers towards its customers. Putting it differently, CCs are viewed as less flexible than prices.

The distinctive characteristic of the retailers is their sales capacities. Their market shares result from their capacities according to a proportional rule: If all retailers choose the same price, the market share of a retailer is given by its share of the total (market-wide) capacity. This allocation rule has enabled us to describe the patterns in the adoption of CCs by market shares.

The model has been solved by a refinement of the concept of subgame perfectness which consists of two lexicographically ordered selection criteria. The primary criterion is that the number of the CC-adopting retailers has to be minimal. This criterion can be justified by the argument that concerted practices are easier to realize the less retailers are involved. Apparently, it ensures that collusion is reached with the minimal total implementation costs.

The secondary criterion is resilience. It requires that the clause profile have to be most resilient regarding decreases in the common discount factor and increases in the clause implementation costs. By this criterion, we have singled out the clause profiles inducing the most sustainable collusion, i.e, the ones which are the most resilient against unexpected temporary adverse demand shocks and most protected against deficits due to unexpected additional implementation costs.

As a result of the refined solution concept, a specific pattern in the adoption of CCs occurs: The largest retailers are the ones offering CCs (see Corollary 6.2). This finding is the main result of our paper and can be explained as follows. The smallest retailers are the ones which benefit most from defecting from collusion as their profits at the collusive price are relatively small due to their small market shares. To prevent their defection, the largest retailers adopt CCs. This business practice reduces the profits the smaller retailer would earn by undercutting the collusive price.

Remarkably, it turns out that for some market instances, none of the robustly collusive clause profiles is of the conventional form, i.e., there are CC-adopting retailers which do not offer a CC of a form usually observed in real commercial life. This discrepancy has motivated us to reconsider the assumptions of the competition model. We have aimed to figure out whether this finding occurs only at specific market circumstances and, thus, could rather be viewed as a side issue. In doing so, we have considered two relevant market regimes.

The first regime consists of markets with dominant retailers (i.e., retailers with capacities large enough to serve the whole market at the competitive price). We have established for markets with at least two dominant retailers that the above finding is resolved. That is, robust collusion can always be induced by specific conventional clause profiles in such markets. Indeed, it suffices that only the two largest retailer adopt CCs (see Corollary 5.2).

The other regime consists of markets which a regular rationing rule underlies. However, this restriction proves to be too weak to resolve the above finding. Nevertheless, it turns out that the most collusive conventional clause profile in those markets has the usual spreading pattern of CCs, i.e., the largest retailers are the ones adopting conventional CCs (see Corollary 6.2). Finally, we have demonstrated that the above finding vanishes if the special case of efficient rationing is assumed (see Corollary 6.5). Figure XIII provides a compact overview of the main results of the paper.

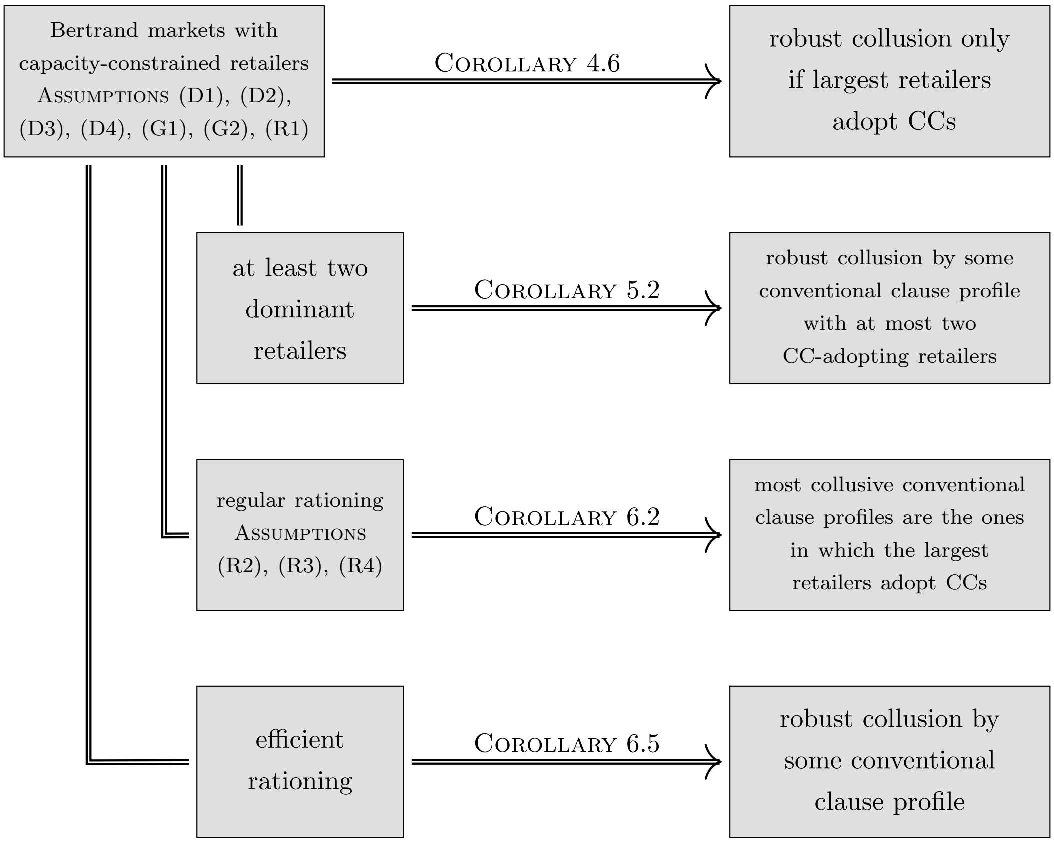

Figure XIII: Overview about the main results

Remarkably, there is an analogy between the spreading pattern of CCs derived in this paper and the pattern of price leadership derived by Deneckere and Kovenock (1992) as well as Ishibashi (2008). The latter establish that if the timing of the price setting is endogenous, the retailer with the largest capacity becomes the price leader. However, this result relies on capacity constraints violating Assumption (D4), i.e., the sum of the capacities of all retailers except for one is assumed to be less than the market demand at the competitive price.

On the other side, it has been shown that if this assumption is satisfied in their models, price leadership might fail to be an effective facilitating practice. Due to this considerations, our results could be regarded as complementary to the ones of Deneckere and Kovenock (1992) as well as Ishibashi (2008). Summing up, collusion can be induced by CCs in cases where price leadership fails to do.

According to our findings, the spreading pattern of CCs might be informative about collusive intentions in the market. Our analysis suggests that dominant retailers are usually the ones adopting CCs to enforce collusion. They abuse CCs as a tool to exercise their market power. The policy implication of our findings becomes obvious: If it is observed that CCs are adopted by the largest or dominant retailers, antitrust agencies should be alerted and might induce an in-depth analysis of the market to trace collusive behavior.

Several modifications of the competition model might be desirable in order to check the robustness of our results. First, one could drop the assumption that the consumers are indifferent between the retailers. A more realistic approach might be that the retailers are spatially or vertically differentiated. Second, the timing structure of the retailers' decision could be altered. Instead of assuming that the clauses are irrevocably fixed at the beginning of the game, one could consider a setting in which the clauses are valid only for one period and are stipulated by the retailers at the beginning at each stage game. Moreover, elements of incomplete knowledge could be incorporated in the model; for example, some of the consumers might be non-shoppers and unaware about some retailers, or the retailers might face demand and costs shocks.

A further suggestion for modification relates to the punishment policies the retailers apply at the collusive outcome. We have assumed throughout the paper that all retailers use the grim trigger price policies at collusion. However, other punishment policies seem to be more plausible or realistic. For example, Lu and Wright (2010) proposes so-called price matching punishments, which stipulate that if a retailer is undercut, then it will choose the current lowest price in the market as the price for the next period. It might be a worthwhile task to figure out whether our results are substantially affected if all (or some, e.g., the fringe) retailers pursue such price policies.[29]

Another interesting project of future research might be to examine which forms of CCs are most suitable in order to facilitate collusion. In this paper, we have mainly focused on the spreading pattern of CCs. Not much has been said so far about the form of the CC. In the literature, different point of views regarding the collusive efficacy of BCCs has been taken. While Baye and Kovenock (1994) and Chen (1995) view such forms as effective to facilitate collusion, Corts (1995) and Hviid and Shaffer (1999) deny it. An examination of the different (conventional) forms of the BCCs based on our general framework might shed some light on this so far unresolved issue.

[29] Unlike grim trigger punishments, price matching punishments do not induce collusion in homogeneous Bertrand markets regardless of the value of the common discount factor; see Proposition 1 in Lu and Wright (2010). Therefore, the adoption of CCs might belong to the practices facilitating tacit collusion even though the common discount factor is close to 1.

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Proofs

Proof of Remark 2.1. Let $p \in \mathbb{R}_+^I$ be an arbitrary profile of prices and $r \in \mathbb{R}_+$ some arbitrary hypothetical price. It is assumed that permutation $\sigma \in \Sigma_n$ is upshifting on $[p < r]$. A further permutation $\tau \in \Sigma_n$ is specified. We stipulate that $\tau(i) := i$ for any $i \in [p < r]$. To complete the specification of τ for the remaining elements of I, we proceed recursively. Let $\ell := |[p \geq r]|$. We define

$$i_1 := \min\,[p \geq r] \qquad \text{and} \qquad j_1 := \min\,\sigma([p \geq r]) \qquad \text{and} \qquad \tau(i_1) := \sigma^{-1}(j_1)$$

and, for any $1 < k \leq \ell$,

$$i_k := \min\,[p \geq r] \setminus J_{i_{k-1}} \quad \text{and} \quad j_k := \min\,\sigma([p \geq r]) \setminus J_{j_{k-1}} \quad \text{and} \quad \tau(i_k) := \sigma^{-1}(j_k).$$

As can be easily checked, it holds $i_k \geq j_k$ for any $1 \leq k \leq \ell$.

Let us define $\tilde{\sigma} := \sigma \circ \tau$. By construction, $\tilde{\sigma}([p < r]) = \sigma([p < r])$. Hence, $\tilde{\sigma}$ is upshifting on $[p < r]$. Consider some $i \in [p \geq r]$. Then, there exists some $k \in I$ so that $i = i_k$. As $\tilde{\sigma}(i_k) = j_k$ and $j_k \leq i_k$, we obtain $\tilde{\sigma}(i) \leq i$. That is, $\tilde{\sigma}$ is downshifting on $[p \leq r]$.

Let us define $\tilde{p} = p \circ \tilde{\sigma}$. It follows from Assumption (R4) that $R(r|\tilde{p}) \leq R(r|p)$. Moreover, we conclude from Assumption (R2) that $R(r|\tilde{p} \circ \tau^{-1}) = R(r|\tilde{p})$. Bringing to mind that $\sigma = \tilde{\sigma} \circ \tau^{-1}$, we obtain the desired result $R(r|p \circ \sigma) \leq R(r|p)$. □

Proof of Proposition 3.2. As $\delta < \delta^g_{\text{crit}}$, we have $\pi^{\updownarrow}_i(g) > \frac{1}{1-\delta}\kappa_i \pi^{\text{m}}$. Hence, there exists some price $\tilde{q}_i$ so that $\pi^g_i(\tilde{q}_i, \pi^{\text{m}}_{-i}) > \frac{1}{1-\delta}\kappa_i \pi^{\text{m}}$. Let us consider some business policy profile $s = (g, s^{\hat{g}})_{\hat{g} \in G})$ where $s^g := t^g$ (i.e., if clause profile g has been realized, grim trigger policies are implemented by the retailers). Moreover, we define business policy profile $\tilde{s} := (\tilde{s}_i, s_{-i})$ where $\tilde{s}_i(h) := s_i(h)$ for any $h \in H \setminus \{g\}$ and $\tilde{s}_i(g) := \tilde{q}_i$. That is, the only difference between s and $\tilde{s}$ is that retailer i advertises a different price (more precisely, p^{m} in s and $\tilde{q}_i$ in $\tilde{s}$) in the period after clause profile g has been implemented. By construction, it holds

$$\Pi_i(o(\tilde{s})) = \pi^g_i(\tilde{q}_i, \pi^{\text{m}}_{-i}) - \mathbf{1}_{C_i}(g)\, f > \frac{1}{1-\delta}\kappa_i \pi^{\text{m}} - \mathbf{1}_{C_i}(g)\, f = \Pi_i(o(s))\ .$$

This strict inequality implies that s is not subgame perfect. As this holds for any business policy profile $s := (g, (s^{\hat{g}})_{\hat{g} \in G})$ satisfying $s^g = t^g$, we conclude that g is not perfectly collusive. □

Proof of Remark 3.4. Consider some arbitrary clause profile g. Moreover, let us suppose that price profile $q := (q_1, p^{\text{m}}_{-1})$ satisfying $c < q_1 < p^{\text{m}}$ has been advertised by the retailers. As $z \geq p^{\text{m}} - c$, we observe $g^{\text{p}}(q) = g^{\text{s}}(q) = q$ (i.e., the effective purchase and sales prices correspond to the advertised prices) and, thus, $\pi^g_i(q) = (q_1 - c)\min\{k_1, D(q_1)\}$ by Assumption (R1). Indeed, Assumption (D1) ensures $\sup_{c<q_1<p^{\text{m}}} \pi^g_i(q) = (p^{\text{m}} - c)\min\{k_1, D(p^{\text{m}})\}$. It then follows from Remark 3.1(b) that $\pi^{\updownarrow}_i(g) = (p^{\text{m}} - c)\min\{k_1, D(p^{\text{m}})\}$. Hence, $\delta^g_{1,\text{crit}} = 1 - \max\{k_1, \frac{D(p^{\text{m}})}{K}\}$. Applying Remark 3.3, we finally obtain $\delta^g_{\text{crit}} = \delta_{\text{crit}}$. □

Proof of Remark 3.5. Suppose retailer i is the only retailer offering a CC. If this retailer undercuts the collusive price, none of its competitors immediately go along with it. Due to Assumptions (R1) and (D1), the supremum of the profits retailer i is able to attain by such defections amounts to $\pi^{\downarrow}_i(g) = (p^{\text{m}} - c)\min\{k_i, D(p^{\text{m}})\}$. It follows from Remark 3.1(*b*) that $\pi^{\updownarrow}_i(g) = (p^{\text{m}} - c)\min\{k_i, D(p^{\text{m}})\}$. This in turn implies that the critical discount factor of retailer i at clause profile g is equal to $\delta^g_{i,\text{crit}} = 1 - \max\{\kappa_i, \frac{D(p^{\text{m}})}{K}\}$. Hence, if retailer i is the only CC-adopting retailer, the critical discount factor at clause profile g is not below $1 - \max\{\kappa_i, \frac{D(\pi^{\text{m}})}{K}\}$. □

Proof of Remark 3.6.

(a) Let j be one of the CC-adopting retailers satisfying $C(g) \setminus (M(g) \cup \{j\}) = \emptyset$. That is, if there is a CC-adopting retailer not offering the MCC, then j represents this retailer. Otherwise j is one of the MCC-adopting retailers.

Suppose retailer j defects from the collusion by announcing a price $c < q_j < p^{\mathrm{m}}$. Apparently, any of its CC-adopting competitors follows suit and charges the same effective price. The effective prices charged by the non CC-adopting retailers remain at the collusive level. Due to Assumption (R1), we observe $X_j(g^{\mathrm{p}}(q_j, p^{\mathrm{m}}_{-j}) = \min\{k_j, \frac{k_j}{K_{C(g)}} D(q_j)\}$ and, thus, $\pi^g_j(q_j, p^{\mathrm{m}}_{-j}) = (q_j - c) \min\{k_j, \frac{k_j}{K_{C(g)}} D(q_j)\}$. It follows from Assumption (D1) that $\sup_{c<q_j<p^{\mathrm{m}}} \pi^g_j(q_j, p^{\mathrm{m}}_{-j}) = (p^{\mathrm{m}} - c) \min\{k_j, \frac{k_j}{K_{C(g)}} D(p^{\mathrm{m}})\}$. Hence, we obtain the weak inequality $\pi^{\updownarrow}_j(g) \leq (p^{\mathrm{m}} - c) \min\{k_j, \frac{k_j}{K_{C(g)}} D(p^{\mathrm{m}})\}$. This in turn entails $\delta^g_{j,\mathrm{crit}} \geq 1 - \max\{\kappa_{C(g)}, \frac{D(p^{\mathrm{m}})}{K}\}$ and, thus, $\delta^g_{\mathrm{crit}} \geq 1 - \max\{\kappa_{C(g)}, \frac{D(p^{\mathrm{m}})}{K}\}$ as claimed.

Suppose $C(g) = M(g)$ from now on. Pick some $i \in C(g)$. As was established in the last paragraph, we observe $\sup_{c<q_i<p^{\mathrm{m}}} \pi^g_i(q_i, p^{\mathrm{m}}_{-i}) = (p^{\mathrm{m}} - c) \min\{k_i, \frac{k_i}{K_{C(g)}} D(p^{\mathrm{m}})\}$. By Remark 3.1(a), it holds $\pi^{\downarrow}_i(g) = (p^{\mathrm{m}} - c) \min\{k_i, \frac{k_i}{K_{C(g)}} D(p^{\mathrm{m}})\}$. As $\pi^{\uparrow}_j(g) = 0$, we obtain $\pi^{\updownarrow}_i(g) = (p^{\mathrm{m}} - c) \min\{k_i, \frac{k_i}{K_{C(g)}} D(p^{\mathrm{m}})\}$ and, thus, $\delta^g_{i,\mathrm{crit}} = 1 - \max\{\kappa_{C(g)}, \frac{D(p^{\mathrm{m}})}{K}\}$.

To complete the proof, consider some $i \in I \setminus C(g)$. If retailer i defects from collusion by announcing a price $c < q_i < p^{\mathrm{m}}$, then any of the MCC-adopting retailers follows suit and charges the same effective price. This along with Assumption (R1) implies $X_i(g^{\mathrm{p}}(q_i, p^{\mathrm{m}}_{-i})) = \min\{k_i, \frac{k_i}{K_{C(g)}+k_i} D(q_i)\}$ so that $\pi^g_i(q_i, p^{\mathrm{m}}_{-i}) = (q_i - c) \min\{k_i, \frac{k_i}{K_{C(g)}+k_i} D(q_i)\}$. Resorting to Assumption (D1) and Remark 3.1(a), we obtain $\pi^{\downarrow}_i(g) = (p^{\mathrm{m}} - c) \min\{k_i, \frac{k_i}{K_{C(g)}+k_i} D(p^{\mathrm{m}})\}$. As $\pi^{\uparrow}_i(g) = 0$, it holds $\pi^{\updownarrow}_i(g) = (p^{\mathrm{m}} - c) \min\{k_i, \frac{k_i}{K_{C(g)}+k_i} D(p^{\mathrm{m}})\}$ and, thus, $\delta^g_{i,\mathrm{crit}} = 1 - \max\{\kappa_{C(g)\cup\{i\}}, \frac{D(p^{\mathrm{m}})}{K}\}$. Bringing all results together, one concludes that $\delta^g_{\mathrm{crit}} = \max\{\delta^g_{i,\mathrm{crit}} : i \in I\} = 1 - \max\{\kappa_{C(g)}, \frac{D(p^{\mathrm{m}})}{K}\}$.

(b) Let i be a retailer adopting a BCC with refund factor λ_i on the price difference. By advertising a price above the collusive level, retailer i pushes its customers to exercise the BCC so that the effective price charged by the retailer is below the collusive one. However, this does not apply to the effective price charged by its competitors. As the CCs are assumed to be applicable only to the advertised prices, the effective prices charged by them remain at the collusive level. That is, if $q_i > p^{\mathrm{m}}$, then $q^{\mathrm{p}}_i = \max\{p^{\mathrm{m}} - \lambda_i(q_i - p^{\mathrm{m}}), c\}$ and $q^{\mathrm{p}}_j = p^{\mathrm{m}}$ for any $j \neq i$. Due to Assumption (R1), it holds $X_i(q^{\mathrm{p}}) = \min\{k_i, D(q^{\mathrm{p}}_i)\}$. As there are no hassle costs, we obtain $q^{\mathrm{s}} = q^{\mathrm{p}}$ and, thus, $\pi^g_i(q) = (q^{\mathrm{p}}_i - c) \min\{k_i, D(q^{\mathrm{p}}_i)\}$.

In consequence, a retailer adopting a BCC with a refund factor on the price difference is able to undersell its competitors by an effective price slightly below the collusive level if it advertises a price slightly above the collusive price. Indeed, Assumption (D1) ensures $\pi^{\uparrow}_i(q) = (p^{\mathrm{m}} - c) \min\{k_i, D(p^{\mathrm{m}})\}$. It follows from Remark 3.1(b) that $\pi^{\updownarrow}_i(q) = (p^{\mathrm{m}} - c) \min\{k_i, D(p^{\mathrm{m}})\}$. This in turn entails $\delta^g_{i,\mathrm{crit}} = 1 - \max\{\kappa_i, \frac{D(p^{\mathrm{m}})}{K}\}$ and, thus, $\delta^g_{\mathrm{crit}} \geq 1 - \max\{\kappa_i, \frac{D(p^{\mathrm{m}})}{K}\}$.

(c) Consider a competition game $\Gamma_{\mathrm{m}}(\delta, f, n)$. Let i be the retailer which offers a BCC with lump refund sum refund μ_i in clause profile $g := (b^{€,\mu_i}_i, g_{-i})$ and a BCC with the refund factor $\phi_i := \frac{p^{\mathrm{m}} - \mu_i}{p^{\mathrm{m}}}$ on the minimum price in clause profile $\tilde{g} := (b^{\%,\phi_i}_i, g_{-i})$. We will show subsequently that $\delta^g_{i,\mathrm{crit}} = \delta^{\tilde{g}}_{i,\mathrm{crit}}$ and $\delta^g_{j,\mathrm{crit}} \geq \delta^{\tilde{g}}_{j,\mathrm{crit}}$ for any $j \neq i$. Apparently, these results verify our claim $\delta^g_{\mathrm{crit}} \geq \delta^{\tilde{g}}_{\mathrm{crit}}$.

Suppose retailer i deviates from collusion by announcing a price below the collusive one, i.e. $q_i < p^{\mathrm{m}}$. As $g_{-i}(q_i, p^{\mathrm{m}}_{-i}) = \tilde{g}_{-i}(q_i, p^{\mathrm{m}}_{-i})$ by assumption, we obtain $g(q_i, p^{\mathrm{m}}_{-i}) = \tilde{g}(q_i, p^{\mathrm{m}}_{-i})$. It follows $X_i(g^{\mathrm{p}}(q_i, p^{\mathrm{m}}_{-i})) = X_i(\tilde{g}^{\mathrm{p}}(q_i, p^{\mathrm{m}}_{-i}))$ and, thus, $\pi^g_i(q_i, p^{\mathrm{m}}_{-i}) = \pi^{\tilde{g}}_i(q_i, p^{\mathrm{m}}_{-i})$. Hence, we obtain $\pi^{\downarrow}_i(g) = \pi^{\downarrow}_i(\tilde{g})$. This in turn implies $\delta^{\downarrow}_{i,\mathrm{crit}}(g) = \delta^{\downarrow}_{i,\mathrm{crit}}(\tilde{g})$.

Suppose retailer i deviates from collusion by announcing a price above the collusive one, i.e., $q_i > p^{\mathrm{m}}$. For both clause profiles, the effective price charged by retailer i is below the collusive one.

Indeed, it holds

$$g_i(q_i, p^{\mathrm{m}}_{-i}) = \min\{p^{\mathrm{m}} - \mu_i, c\} = \min\{(1 - \tfrac{\mu_i}{p^{\mathrm{m}}})p^{\mathrm{m}}, c\} = \tilde{g}_i(q_i, p^{\mathrm{m}}_{-i}) \ .$$

As $g_{-i}(q_i, p^{\mathrm{m}}_{-i}) = p^{\mathrm{m}} = \tilde{g}_{-i}(q_i, p^{\mathrm{m}}_{-i})$ by assumption, we observe $g(q_i, p^{\mathrm{m}}_{-i}) = \tilde{g}(q_i, p^{\mathrm{m}}_{-i})$. It follows that $X_i(g^{\mathrm{p}}(q_i, p^{\mathrm{m}}_{-i})) = X_i(\tilde{g}^{\mathrm{p}}(q_i, p^{\mathrm{m}}_{-i}))$ and, thus, $\pi^g_i(q_i, p^{\mathrm{m}}_{-i}) = \pi^{\tilde{g}}_i(q_i, p^{\mathrm{m}}_{-i})$. This in turn implies $\pi^{\uparrow}_i(g) = \pi^{\uparrow}_i(\tilde{g})$ and, thus, $\delta^{\uparrow}_{i,\mathrm{crit}}(g) = \delta^{\uparrow}_{i,\mathrm{crit}}(\tilde{g})$. Resorting to the result of the previous paragraph, we finally obtain $\delta^g_{i,\mathrm{crit}} = \delta^{\tilde{g}}_{i,\mathrm{crit}}$.

Consider now an arbitrary retailer $j \neq i$. Suppose retailer j deviates from collusion by announcing a price below the collusive price, i.e., $q_j < p^{\mathrm{m}}$. In this case, we observe $g_j(q_j, p^{\mathrm{m}}_{-j}) = q_j = \tilde{g}_j(q_j, p^{\mathrm{m}}_{-j})$ and $g_k(q_j, p^{\mathrm{m}}_{-j}) = \tilde{g}_k(q_j, p^{\mathrm{m}}_{-j})$ for any $k \neq i, j$. Moreover, it holds

$$g_i(q_j, p^{\mathrm{m}}_{-j}) = \min\{q_j - \mu_i, c\} \leq \min\{q_j - \tfrac{q_j}{p^{\mathrm{m}}}\mu_i, c\} = \tilde{g}_i(q_j, p^{\mathrm{m}}_{-j}) \ .$$

If $q_j > c$, then $\tilde{g}_i(q_j, p^{\mathrm{m}}_{-j}) < q_j$. As there are no hassle costs and the rationing rule underlying the market is assumed to be monotone, it follows $X_j(g^{\mathrm{p}}(q_j, p^{\mathrm{m}}_{-j})) \geq X_j(\tilde{g}^{\mathrm{p}}(q_j, p^{\mathrm{m}}_{-j}))$ and, thus, $\pi^g_j(q_j, p^{\mathrm{m}}_{-j}) \geq \pi^{\tilde{g}}_j(q_j, p^{\mathrm{m}}_{-j}) \geq 0$. This in turn implies $\sup_{c<q_j<p^{\mathrm{m}}} \pi^g_j(q_j, p^{\mathrm{m}}_{-j}) \geq \sup_{c<q_j<p^{\mathrm{m}}} \pi^{\tilde{g}}_j(q_j, p^{\mathrm{m}}_{-j}) \geq 0$. Applying Remark 3.1(a), we obtain $\pi^{\downarrow}_j(g) \geq \pi^{\downarrow}_j(\tilde{g})$. Hence, $\delta^{\downarrow}_{j,\mathrm{crit}}(g) \geq \delta^{\downarrow}_{j,\mathrm{crit}}(\tilde{g})$.

Suppose retailer j deviates from collusion by announcing a price above the collusive price, i.e., $q_j > p^{\mathrm{m}}$. By Assumption (G1), it holds $g_j(q_j, p^{\mathrm{m}}_{-j}) = \tilde{g}_j(q_j, p^{\mathrm{m}}_{-j})$ and $g_k(q_j, p^{\mathrm{m}}_{-j}) = p^{\mathrm{m}} = \tilde{g}_k(q_j, p^{\mathrm{m}}_{-j})$ for any $k \neq i$. As there are no hassle costs, we obtain $X_j(g(q_j, p^{\mathrm{m}}_{-j})) = X_j(\tilde{g}(q_j, p^{\mathrm{m}}_{-j}))$ so that $\pi^g_j(q_j, p^{\mathrm{m}}_{-j}) = \pi^{\tilde{g}}_j(q_j, p^{\mathrm{m}}_{-j})$. It follows $\pi^{\uparrow}_j(g) = \pi^{\uparrow}_j(\tilde{g})$ and, thus, $\delta^{\uparrow}_{j,\mathrm{crit}}(g) = \delta^{\uparrow}_{j,\mathrm{crit}}(\tilde{g})$. Resorting to the result of the previous paragraph, we obtain $\delta^g_{j,\mathrm{crit}} \geq \delta^{\tilde{g}}_{j,\mathrm{crit}}$. As $\delta^g_{i,\mathrm{crit}} = \delta^{\tilde{g}}_{i,\mathrm{crit}}$ for any $i \neq j$, it holds $\delta^g_{\mathrm{crit}} \geq \delta^{\tilde{g}}_{\mathrm{crit}}$. □

Proof of Proposition 3.7. As $f > f^{g,\delta}_{\mathrm{crit}}$, we have $f^{g,\delta}_{\mathrm{crit}} < +\infty$. Hence, $C(g) \neq \emptyset$ and, thus, $f > \frac{1}{1-\delta}\kappa_i\pi^{\mathrm{m}}$ where $i := \min C(g)$. Let us consider some business policy profile $s := (g, (s^{\hat{g}})_{\hat{g}\in G})$ where $s^g := t^g$ (i.e., if clause profile g has been realized, grim trigger price policies are implemented by the retailers). Moreover, we define business policy profile $\tilde{s} := (\tilde{s}_i, s_{-i})$ where $\tilde{s}_i(h) := s_i(h)$ for any $h \in H \setminus \{\emptyset\}$ and $\tilde{s}_i(\emptyset) := w_i$. That is, the only difference between s and $\tilde{s}$ is that retailer i implements a different clause at the beginning. More precisely, it chooses non-trivial clause g_i in s and trivial clause w_i in $\tilde{s}$. By construction, it holds

$$\Pi_i(o(\tilde{s})) = \frac{1}{1-\delta}\kappa_i\pi^{\mathrm{m}} > \frac{1}{1-\delta}\kappa_i\pi^{\mathrm{m}} - f = \Pi_i(o(s)) \ .$$

This strict inequality implies that s is not subgame perfect. As this holds for any business policy profile $s := (g, (s^{\hat{g}})_{\hat{g}\in G})$ satisfying $s^g = t^g$, we conclude that g is not perfectly collusive. □

Proof of Proposition 3.9. Pick an arbitrary clause profile $g \in G$. Note that $\delta < \delta_{\mathrm{crit}}$ is assumed. Therefore, Proposition 3.2 ensures that the trivial clause profile w is not perfectly collusive. Moreover, if $z \geq p^{\mathrm{m}} - c$, we obtain $\delta^g_{\mathrm{crit}} = \delta_{\mathrm{crit}} > \delta$. It follows from Proposition 3.2 that g is not perfectly collusive at such hassle costs. Due to these results, it is justified to suppose from now on that g is non-trivial and z satisfies $\bar{z}^{\delta}_1 < z < p^{\mathrm{m}} - c$.

Let us pick some price $\tilde{q}_1 \in]c + \bar{z}^{\delta}_1, c + z[$. As $\tilde{q}_1 > c + \bar{z}^{\delta}_1$, we conclude from Assumption (D3) and the definition of $\bar{z}^{\delta}_1$ that $\bar{\pi}_1(\tilde{q}_i) > \bar{\pi}_1(c + \bar{z}^{\delta}_1) = \frac{1}{1-\delta}\kappa_1\pi^{\mathrm{m}}$. As $\tilde{q}_1 < c + z < p^{\mathrm{m}}$, we observe $g^{\mathrm{s}}_1(\tilde{q}_1, p^{\mathrm{m}}_{-1}) = g^{\mathrm{p}}_1(\tilde{q}_1, p^{\mathrm{m}}_{-1}) = \tilde{q}_1$. Moreover, due to Assumption (G2), it holds $g^{\mathrm{p}}_k(\tilde{q}_1, p^{\mathrm{m}}_{-1}) \geq c + z$ for any $k \neq i$ so that $X_1(g^{\mathrm{p}}(\tilde{q}_1, p^{\mathrm{m}}_{-1})) = \min\{k_1, D(\tilde{q}_1)\}$ and, thus, $\pi^g_1(\tilde{q}_1, p^{\mathrm{m}}_{-1}) = \bar{\pi}_1(\tilde{q}_1)$. Hence, we obtain $\pi^g_1(\tilde{q}_1, p^{\mathrm{m}}_{-1}) > \frac{1}{1-\delta}\kappa_1\pi^{\mathrm{m}}$.

Let us consider some business policy profile $s := (g, (s^{\hat{g}})_{\hat{g}\in G})$ where $s^g := t^g$ (i.e., if clause profile g has been realized, grim trigger policies are implemented by the retailers). Moreover, we define business policy profile $\tilde{s} := (\tilde{s}_1, s_{-1})$ where $\tilde{s}_1(h) := s_1(h)$ for any $h \in H \setminus \{g\}$ and $\tilde{s}_1(g) := \tilde{q}_1$. That is, the only difference between business policy profiles s and $\tilde{s}$ is that retailer 1 advertises a different price immediately after clause profile g has been implemented. More precisely, it advertises the monopoly price p^{m} in period 0 under s and price $\tilde{q}_i$ in period 0 under $\tilde{s}$. By construction of s and $\tilde{s}$, it holds

$$\Pi_1(o(\tilde{s})) = \pi_1^g(\tilde{q}_1, p_{-1}^{\mathrm{m}}) - \mathbf{1}_{C_1}(g)\, f > \frac{1}{1-\delta}\kappa_1 \pi^{\mathrm{m}} - \mathbf{1}_{C_1}(g)\, f = \Pi_1(o(s)) \, .$$

This strict inequality implies that s is not subgame perfect. As this applies to any business policy profile $s := (g, (s^{\hat{g}})_{\hat{g} \in G})$ satisfying $s^g = t^g$, we conclude that g is not perfectly collusive. □

Proof of Proposition 3.11. Consider a clause profile g satisfying Properties (M1) and (M3), i.e., $\delta_{\mathrm{crit}}^g \leq \delta$ and $f_{\mathrm{crit}}^{g,\delta} \geq f$. We prove our claim by induction on the cardinality of $C(g)$. Suppose $|C(g)| = 0$. That is, $g = w$. As $\delta_{\mathrm{crit}} \leq \delta$ and $f_{\mathrm{crit}} \geq f$, it follows immediately from Proposition 3.10 that clause profile $\hat{g} := g$ is perfectly collusive.

Consider now the case $\ell := |C(g)|$ where $1 \leq \ell \leq n$. The induction premise is that for any clause profile $\tilde{g}$ satisfying $|C(\tilde{g})| = \ell - 1$ as well as $\delta_{\mathrm{crit}}^{\tilde{g}} \leq \delta$ and $f_{\mathrm{crit}}^{\tilde{g},\delta} \geq f$, there is a perfectly collusive clause profile $\hat{g} := (\tilde{g}_J, w_{-J})$ satisfying $\hat{g} = \tilde{g}$ or $J \subset C(\tilde{g})$. Suppose $\delta_{\mathrm{crit}}^{\tilde{g}} > \delta$ for any $\tilde{g} := (w_i, g_{-i})$ where $i \in C(g)$. In this case, we infer from Proposition 3.10 that $\hat{g} := g$ is perfectly collusive. Suppose $\delta_{\mathrm{crit}}^{\tilde{g}} \leq \delta$ for some $\tilde{g} := (w_j, g_{-j})$ where $j \in C(g)$. As g violates Property (M2), it is not perfectly collusive. However, we are able to pick some $j \in C(g)$ so that clause profile $\tilde{g} := (w_j, g_{-j})$ satisfies $\delta_{\mathrm{crit}}^{\tilde{g}} \leq \delta$. Besides that, it holds $|C(\tilde{g})| = \ell - 1$ and $f \leq f_{\mathrm{crit}}^{g,\delta} \leq f_{\mathrm{crit}}^{\tilde{g},\delta}$. The induction premise ensures that there exists a perfectly collusive clause profile $\hat{g} := (g_J, w_{-J})$ satisfying $J \subseteq C(\tilde{g}) \subset C(g)$. □

Proof of Theorem 4.1. Obviously, the definition of robustly collusive clause profiles immediately entails that the trivial clause profile w is the only robustly collusive clause profile if, and only if, it is perfectly collusive. For this reason, it remains to prove that w is perfectly collusive if, and only if, $\delta \geq \delta_{\mathrm{crit}}$.

Suppose first $\delta \geq \delta_{\mathrm{crit}}$. As can be easily checked, w then satisfies Properties (M1) - (M3). It follows from Proposition 3.10 that w is perfectly collusive at this common discount factor. Suppose now w is perfectly collusive. Remark 3.4 rules out $\delta < \delta_{\mathrm{crit}}$. Hence, $\delta \geq \delta_{\mathrm{crit}}$ as claimed. □

Proof of Remark 4.2. Let us consider a competition game $\Gamma(\delta, f, n, z)$ to which rationing rule $R(\cdot|\cdot)$ applies. Moreover pick some arbitrary coalition $J \subseteq I$ of retailers. To simplify the following expositions, we define $b := (b_J^{€, p^{\mathrm{m}}-c}, w_{-J})$. That is, b represents the clause profile in which any retailer of coalition J offers a BCC with lump sum refund $p^{\mathrm{m}} - c$ while the other retailers do not offer CCs. Recall that $\hat{\pi}_i^J(q)$ denotes the profit retailer i earns under efficient rationing if clause profile b has been implemented and price profile q has been advertised and $\hat{\delta}_{\mathrm{crit}}^J$ denotes the critical discount factor of clause profile b under efficient rationing.

In the following, we prove that $\hat{\delta}_{\mathrm{crit}}^J$ is the infimum of the critical discount factors induced by clause profiles in which coalition J constitutes the set of the CC-adopting retailers. Obviously, Remark 3.3 ensures that this claim holds for $J = \emptyset$. Due to Remark 3.4, it is also true for hassle costs $z \geq p^{\mathrm{m}} - c$. For these reasons, we take for granted $J \neq \emptyset$ and $z < p^{\mathrm{m}} - c$ from now on.

We proceed in two steps. Our first objective is to demonstrate that $\delta_{\mathrm{crit}}^g \geq \hat{\delta}_{\mathrm{crit}}^J$ for any clause profile g satisfying $C(g) = J$. That is, $\hat{\delta}_{\mathrm{crit}}^J$ proves to be a lower bound of the critical discount factors induced by those clause profiles. Afterwards, we show that for any $\epsilon > 0$ there exists some clause profile g satisfying $C(g) = J$ so that $\hat{\delta}_{\mathrm{crit}}^J + \epsilon > \delta_{\mathrm{crit}}^g$. Putting it differently, any value greater than $\hat{\delta}_{\mathrm{crit}}^J$ is not a lower bound of the critical discount factors induced by those clause profiles.

Step 1: $\delta_{crit}^g \geq \hat{\delta}_{crit}^J$ for any clause profile $g \in G$ satisfying $C(g) = J$

To accomplish this step, let us consider some arbitrary clause profile g having the property $C(g) = J$. We pick some retailer i and suppose that profile $q := (q_i, p^{\mathrm{m}}_{-i})$ of advertised prices has been realized. In the following, we compare the profits of $\pi_i^g(q)$ and $\hat{\pi}_i^J(q)$.

- If $q_i > p^{\mathrm{m}}$, then Assumptions (G2) and (R1) ensure that $\pi_i^g(q) \geq 0 = \hat{\pi}_i^J(q)$ regardless of whether $i \in J$ or $i \notin J$.
- If $c + z < q_i < p^{\mathrm{m}}$, then Assumption (R1) ensures

$$\begin{aligned}
\pi_i^g(q) &= (q_i - c)\ \min\left\{k_i, \frac{k_i}{K_{[g^{\mathrm{P}}(q)=q_i]}}\ R(q_i|g^{\mathrm{P}}(q))\right\} \\
&\geq (q_i - c)\ \min\left\{k_i, \max\{R(q_i|g^{\mathrm{P}}(q)) - K_{[g^{\mathrm{P}}(q)=q_i]\setminus\{i\}}, 0\}\right\} \\
&\geq (q_i - c)\ \min\left\{k_i, \max\{D(q_i) - K_{J\setminus\{i\}}, 0\}\right\} \\
&= (q_i - c)\ \min\left\{k_i, \frac{k_i}{K_{[b^P(q)=q_i]}} R_{\mathrm{e}}(q_i|b^{\mathrm{P}}(q))\right\} \\
&= \hat{\pi}_i^J(q)\ .
\end{aligned}$$

- If $q_i = c + z$, then Assumption (G2) and (R1) ensure

$$\begin{aligned}
\pi_i^g(q) &= (q_i - c)\ \min\left\{k_i, \frac{k_i}{K_{[g^{\mathrm{P}}(q)=q_i]}}\ R(q_i|g^{\mathrm{P}}(q))\right\} \\
&= (q_i - c)\ \min\left\{k_i, \frac{k_i}{K_{[g^{\mathrm{P}}(q)=q_i]}}\ D(q_i)\right\} \\
&\geq (q_i - c)\ \min\left\{k_i, \frac{k_i}{K_{J\cup\{i\}}}\ D(q_i)\right\} \\
&= (q_i - c)\ \min\left\{k_i, \frac{k_i}{K_{[b^P(q)=q_i]}} R_{\mathrm{e}}(q_i|b^{\mathrm{P}}(q))\right\} \\
&= \hat{\pi}_i^J(q)\ .
\end{aligned}$$

- If $0 \leq q_i < c + z$, then Assumptions (G2) and (R1) ensure

$$\begin{aligned}
\pi_i^g(q) &= (q_i - c)\ \min\left\{k_i, \frac{k_i}{K_{[g^{\mathrm{P}}(q)=q_i]}}\ R(q_i|g^{\mathrm{P}}(q))\right\} \\
&= (q_i - c)\ \min\{k_i, D(q_i)\} \\
&= (q_i - c)\ \min\left\{k_i, \frac{k_i}{K_{[b^P(q)=q_i]}} R_{\mathrm{e}}(q_i|b^{\mathrm{P}}(q))\right\} \\
&= \hat{\pi}_i^J(q)\ .
\end{aligned}$$

We have shown by the above results that $\pi_i^g(q) \geq \hat{\pi}_i^J(q)$ for any $q := (q_i, p^{\mathrm{m}}_{-i}) \in \mathbb{R}^I_+$. Hence, it holds $\sup_{q_i \neq p^{\mathrm{m}}} \pi_i^g(q_i, p^{\mathrm{m}}_{-i}) \geq \sup_{q_i \neq p^{\mathrm{m}}} \hat{\pi}_i^J(q_i, p^{\mathrm{m}}_{-i})$. This in turn entails $\delta^g_{i,\mathrm{crit}} \geq \hat{\delta}^J_{i,\mathrm{crit}}$. As retailer i has been arbitrarily selected, we obtain $\delta^g_{\mathrm{crit}} \geq \hat{\delta}^J_{\mathrm{crit}}$. That is, δ^J_{crit} proves to be a lower bound of the critical discount factors induced by the clauses profile in which coalition J constitutes the set of CC-adopting retailers. This holds regardless of which rationing rule underlies the competition game.

Step 2: For any $\epsilon > 0$, there exists some $g \in G$ so that $C(g) = J$ and $\delta^g_{crit} \leq \hat{\delta}^J_{crit} + \epsilon$.

To prove this claim, we choose some arbitrary $\epsilon > 0$. Without loss of generality, we suppose $\epsilon < 1 - \hat{\delta}^J_{\mathrm{crit}}$. Moreover, we define $\eta := (\frac{1}{1-\hat{\delta}^J_{\mathrm{crit}}-\epsilon} - \frac{1}{1-\hat{\delta}^J_{\mathrm{crit}}})\kappa_1 \pi^{\mathrm{m}}$. Obviously, it holds $\eta > 0$. Pick some retailer $j \in I$ and some profile $q := (q_j, p^{\mathrm{m}}_{-j})$ of advertised prices satisfying $c + z < q_j < p^{\mathrm{m}}$. As market demand is continuous due to Assumption (D1), there exists some $c < \hat{q}_j(q_j) < q_j - z$ so that

$$D(\tilde{q}_j(q_j) + z) - K_{J\setminus\{j\}} < D(q_j) - K_{J\setminus\{j\}} + \frac{\eta}{q_j - c}\ .$$

This strict inequality implies

$$\min\left\{\max\left\{D(\tilde{q}_j(q_j)+z)-K_{J\setminus\{j\}},0\right\},D(q_j)\right\}\le\max\left\{D(q_j)-K_{J\setminus\{j\}},0\right\}+\frac{\eta}{q_j-c}\ .$$

Next, we construct clause profile $g := (g_i)_{i\in I}$. Any retailer $i \notin J$ is assumed to adopt the trivial CC, i.e., $g_i := w_i$. Any retailer $i \in J$ is assumed to adopt the CC specified by

$$g_i(q) := \begin{cases} \hat{q}_j(q_j) & \text{if } q=(q_j,p^{\mathrm{m}}_{-j}) \text{ and } c+z<q_j<p^{\mathrm{m}} \text{ for some } j\in I\setminus\{i\} \\ b_i(q) & \text{otherwise.} \end{cases}$$

Apparently, clause profile g has the desired property $C(g) = J$. Apart from that, we will demonstrate below that it also satisfies the condition $\delta^g_{\mathrm{crit}} \le \hat{\delta}^J_{\mathrm{crit}} + \epsilon$.

To verify this condition, consider some arbitrary retailer $i \in I$ and some profile $q := (q_i, p^{\mathrm{m}}_{-i})$ of advertised prices. We obtain the following results:

- If $q_i > p^{\mathrm{m}}$ or $q_i \le c + z$, then the constructions of g and b along with Assumption (R1) entail both $R(q_i|g^{\mathrm{p}}(q)) = R_{\mathrm{e}}(q_i|b^{\mathrm{p}}(q))$ and $K_{[g^{\mathrm{p}}(q)=q_i]} = K_{[b^{\mathrm{p}}(q)=q_i]}$. Hence, $\pi^g_i(q) = \hat{\pi}^J_i(q)$.
- If $c + z < q_i < p^{\mathrm{m}}$, then the constructions of g and b along with Assumption (R1) entail

$$\begin{aligned} R(q_i|g^{\mathrm{p}}(q)) &\le R_{\mathrm{l}}(q_i|g^{\mathrm{p}}(q)) \\ &= \min\left\{\max\left\{D(\tilde{q}_i(q_i)+z)-K_{J\setminus\{i\}},0\right\},D(q_i)\right\} \\ &\le \max\left\{D(q_i)-K_{J\setminus\{i\}},0\right\}+\frac{\eta}{q_i-c}\ . \\ &\le R_{\mathrm{e}}(q_i|b^{\mathrm{p}}(q))+\frac{\eta}{q_i-c} \end{aligned}$$

  Moreover, it holds $K_{[g^{\mathrm{p}}(q)=q_i]} = k_i = K_{[b^{\mathrm{p}}(q)=q_i]}$ so that $\pi^g_i(q) \le \hat{\pi}^J_i(q) + \eta$.

According to the above results, we have $\pi^g_i(q) \le \hat{\pi}^J_i(q) + \eta$ for any $q := (q_i, p^{\mathrm{m}}_{-i}) \in \mathbb{R}^I_+$. Hence, it holds $\sup_{q_i\neq p^{\mathrm{m}}} \pi^g_i(q_i, p^{\mathrm{m}}_{-i}) \le \sup_{q_i\neq p^{\mathrm{m}}} \hat{\pi}^J_i(q) + \eta$. By construction, $\eta \le (\frac{1}{1-\hat{\delta}^J_{\mathrm{crit}}-\epsilon} - \frac{1}{1-\hat{\delta}^J_{\mathrm{crit}}})\kappa_i\pi^{\mathrm{m}}$. This in turn implies $\sup_{q_i\neq p^{\mathrm{m}}} \pi^g_i(q_i, p^{\mathrm{m}}_{-i}) \le \frac{1}{1-\hat{\delta}^J_i-\epsilon}\kappa_i\pi^{\mathrm{m}}$ so that $\delta^g_i \le \hat{\delta}^J_i + \epsilon$. As retailer i has been arbitrarily selected, we obtain $\delta^g_{\mathrm{crit}} \le \hat{\delta}^J_{\mathrm{crit}} + \epsilon$.

To sum up, we have shown in the above two steps that (i) $\delta^g_{\mathrm{crit}} \ge \hat{\delta}^J_{\mathrm{crit}}$ for any $g \in G$ satisfying $C(g) = J$ and (ii) for any $\epsilon > 0$, there exists some $g \in G$ so that $C(g) = J$ and $\delta^g_{\mathrm{crit}} \le \hat{\delta}^J_{\mathrm{crit}} + \epsilon$. These two properties establish the claim $\hat{\delta}^J_{\mathrm{crit}} = \inf\{\delta^g_{\mathrm{crit}} : g \in G \text{ satisfying } C(g) = J\}$.

Our next objective is to demonstrate that if σ is upshifting and non-constant on J, but $\hat{\delta}^{\sigma(J)}_{\mathrm{crit}} = \hat{\delta}^J_{\mathrm{crit}}$, then there exists some clause profile $g \in G$ so that $C(g) = \sigma(J)$ and $\delta^g_{\mathrm{crit}} \le \hat{\delta}^{\sigma(J)}_{\mathrm{crit}}$. That means, $\hat{\delta}^{\sigma(J)}_{\mathrm{crit}}$ then becomes the minimum of the critical discount factors resulting from the clause profiles in which $\sigma(J)$ constitutes the coalition of all CC-adopting retailers. To prove this claim, suppose from now on that σ is upshifting and non-constant on J and $\hat{\delta}^{\sigma(J)}_{\mathrm{crit}} = \hat{\delta}^J_{\mathrm{crit}}$. Obviously, it holds $K_J < K_{\sigma(J)}$. Moreover, we define $\tilde{b} := b \diamond \sigma$, i.e., $\tilde{b} = (b^{€,p^{\mathrm{m}}-c}_{\sigma(J)}, w_{-\sigma(J)})$.

Let us pick some arbitrary retailer i. We define $j := \sigma^{-1}(i)$ if $i \in \sigma(J)$ and $j := i$ otherwise. Obviously, $j \le i$. As σ is uplifting on J, it holds $k_j \le k_i$ and $K_{J\setminus\{j\}} \le K_{\sigma(J)\setminus\{i\}}$ where at least one of the two inequalities is strict. Suppose profile $q := (q_i, p^{\mathrm{m}}_{-i})$ of advertised prices satisfying $c + z < q_i < p^{\mathrm{m}}$ has been advertised by the retailers. By Assumptions (D1) and (D2), market demand mapping D is continuous and decreasing on $]c + z, p^{\mathrm{m}}[$. In consequence, there exists some $c < \tilde{q}_i(q_i) < q_i - z$ so that

$$D(q_i)-K_{\sigma(J)\setminus\{i\}}<D(\tilde{q}_i(q_i)+z)-K_{\sigma(J)\setminus\{i\}}<\frac{k_i}{k_j}\left(D(q_i)-K_{J\setminus\{j\}}\right)\ .$$

Note the latter inequality implies

$$\min\left\{\max\{D(\tilde{q}_i(q_i)+z)-K_{\sigma(J)\setminus\{i\}},0\},D(q_i)\right\}\le\frac{k_i}{k_j}\max\{D(q_i)-K_{J\setminus\{j\}},0\}\ .$$

Now, we construct clause profile $g := (g_k)_{k\in I}$. If $k \notin \sigma(J)$, then $g_k := w$. If $k \in \sigma(J)$, then

$$g_k(q) := \begin{cases} \tilde{q}_i(q_i) & \text{if } q = (q_i, p^{\mathrm{m}}_{-i}) \text{ and } c+z < q_i < p^{\mathrm{m}} \text{ for some } i \in I \setminus \{k\}, \\ \tilde{b}_k(q) & \text{otherwise.} \end{cases}$$

Obviously, clause profile g has the desired characteristic $C(g) = \sigma(J)$. Apart from that, we will demonstrate that it also satisfies the condition $\delta^g_{\mathrm{crit}} \leq \hat{\delta}^J_{\mathrm{crit}}$.

To verify this condition, pick some arbitrary retailer $i \in I$. As above, we define $j := \sigma^{-1}(i)$ if $i \in \sigma(J)$ and $j := i$ otherwise. In the following, we aim to compare the profits $\pi^g_i(q_i, p^{\mathrm{m}}_{-i})$ and $\frac{k_i}{k_j}\hat{\pi}^J_j(q_j, p^{\mathrm{m}}_{-j})$ for any $q_i = q_j$. It holds:

- If $q_i = q_j > p^{\mathrm{m}}$, then the constructions of g and b along with Assumption (D4) entail that $\pi^g_i(q_i, p^{\mathrm{m}}_{-i}) = 0 = \frac{k_i}{k_j}\hat{\pi}^J_j(q_j, p^{\mathrm{m}}_{-j})$ regardless of whether $i \in \sigma(J)$ or $i \notin \sigma(J)$.
- If $c + z < q_i = q_j < p^{\mathrm{m}}$, then the construction of g along with Assumption (R1) entails

$$\begin{aligned} R(q_i|g^{\mathrm{p}}(q_i, p^{\mathrm{m}}_{-i})) &\leq R_{\mathrm{l}}(q_i|g^{\mathrm{p}}(q_i, p^{\mathrm{m}}_{-i})) \\ &= \min\left\{\max\left\{D(\tilde{q}_i(q_i) + z) - K_{\sigma(J)\setminus\{i\}}, 0\right\}, D(q_i)\right\} \\ &\leq \frac{k_i}{k_j}\max\left\{D(q_j) - K_{J\setminus\{j\}}, 0\right\} . \\ &= \frac{k_i}{k_j} R_{\mathrm{e}}(q_j|b^{\mathrm{p}}(q_j, p^{\mathrm{m}}_{-j})) \end{aligned}$$

  As can be easily checked, it holds $K_{[g^{\mathrm{p}}(q_i,p^{\mathrm{m}}_{-i})=q_i]} = k_i$ and $K_{[b^{\mathrm{p}}(q_j,p^{\mathrm{m}}_{-j})]=q_j} = k_j$. Hence,

$$\begin{aligned} \pi^g_i(q_i, p^{\mathrm{m}}_{-i}) &= (q_i - c)\,\min\left\{k_i, \frac{k_i}{K_{[g^{\mathrm{p}}(q_i,p^{\mathrm{m}}_{-i})=q_i]}} R(q_i|g^{\mathrm{p}}(q_i, p^{\mathrm{m}}_{-i}))\right\} \\ &\leq (q_j - c)\,\min\left\{k_i, \frac{k_j}{K_{[b^{\mathrm{p}}(q_j,p^{\mathrm{m}}_{-j})=q_j]}}\frac{k_i}{k_j} R_{\mathrm{e}}(q_j|b^{\mathrm{p}}(q_j, p^{\mathrm{m}}_{-j}))\right\} \\ &= \frac{k_i}{k_j}\,(q_j - c)\,\min\left\{k_j, \frac{k_j}{K_{[b^{\mathrm{p}}(q_j,p^{\mathrm{m}}_{-j})=q_j]}} R_{\mathrm{e}}(q_j|b^{\mathrm{p}}(q_j, p^{\mathrm{m}}_{-j}))\right\} \\ &= \frac{k_i}{k_j}\hat{\pi}^J_j(q_j, p^{\mathrm{m}}_{-j}) . \end{aligned}$$

- If $q_i = q_j = c + z$, then $R(q_i|g^{\mathrm{p}}(q_i, p^{\mathrm{m}}_{-i})) = D(q_i) = D(q_j) = R_{\mathrm{e}}(q_j|b^{\mathrm{p}}(q_j, p^{\mathrm{m}}_{-j}))$. Moreover, it holds $K_{[g^{\mathrm{p}}(q_i,p^{\mathrm{m}}_{-i})=q_i]} = K_{\sigma(J)} > K_J = K_{[b^{\mathrm{p}}(q_j,p^{\mathrm{m}}_{-j})=q_j]}$ due to the constructions of g and b. We conclude from these findings that

$$\begin{aligned} \pi^g_i(q_i, p^{\mathrm{m}}_{-i}) &= (q_i - c)\,\min\left\{k_i, \frac{k_i}{k_j}\frac{k_j}{K_{\sigma(J)}} R(q_i|g^{\mathrm{p}}(q_i, p^{\mathrm{m}}_{-i}))\right\} \\ &\leq \frac{k_i}{k_j}\,(q_j - c)\,\min\left\{k_j, \frac{k_j}{K_J} R_{\mathrm{e}}(q_j|b^{\mathrm{p}}(q_j, p^{\mathrm{m}}_{-j}))\right\} \\ &= \frac{k_i}{k_j}\hat{\pi}^J_j(q_j, p^{\mathrm{m}}_{-j}) . \end{aligned}$$

- If $q_i = q_j < c + z$, then $R(q_i|g^{\mathrm{p}}(q_i, p^{\mathrm{m}}_{-i})) = D(q_i) = D(q_j) = R_{\mathrm{e}}(q_j|b^{\mathrm{p}}(q_j, p^{\mathrm{m}}_{-j}))$. Moreover, we observe $K_{[g^{\mathrm{p}}(q_i,p^{\mathrm{m}}_{-i})=q_i]} = k_i \geq k_j = K_{[b^{\mathrm{p}}(q_j,p^{\mathrm{m}}_{-j})=q_j]}$. These results ensure

$$\begin{aligned} \pi^g_i(q_i, p^{\mathrm{m}}_{-i}) &= (q_i - c)\,\min\{k_i, D(q_i)\} \\ &\leq (q_i - c)\,\min\{k_i, \frac{k_i}{k_j}D(q_i)\} \\ &= \frac{k_i}{k_j}\,(q_j - c)\,\min\{k_j, D(q_j)\} \\ &= \frac{k_i}{k_j}\hat{\pi}^J_j(q_j, p^{\mathrm{m}}_{-j}) . \end{aligned}$$

According to the above results, we have $\pi_i^g(q_i, p_{-i}^{\mathrm{m}}) \leq \frac{k_i}{k_j}\hat{\pi}_j^J(q_j, p_{-j}^{\mathrm{m}})$ for any advertised prices $q_i = q_j$. Hence, it holds $\pi_i^{\ddagger}(g) = \sup_{q_i \neq p^{\mathrm{m}}} \pi_i^g(q_i, p_{-i}^{\mathrm{m}}) \leq \frac{k_i}{k_j} \sup_{q_j \neq p^{\mathrm{m}}} \hat{\pi}_j^J(q_j, p_{-j}^{\mathrm{m}})$. We define $\hat{\pi}_j^{\ddagger}(J) := \sup_{q_j \neq p^{\mathrm{m}}} \hat{\pi}_j^J(q_j, p_{-j}^{\mathrm{m}})$. As just shown, $\pi_i^{\ddagger}(g) \leq \frac{k_i}{k_j}\hat{\pi}_j^{\ddagger}(J)$. This in turn implies

$$\frac{1}{1-\hat{\delta}_{j,\mathrm{crit}}^J}\kappa_i\pi^{\mathrm{m}} = \frac{k_i}{k_j}\frac{1}{1-\hat{\delta}_{j,\mathrm{crit}}^J}\kappa_j\pi^{\mathrm{m}} \geq \frac{k_i}{k_j}\hat{\pi}_j^{\ddagger}(J) \geq \hat{\pi}_i^{\ddagger}(g)$$

so that $\delta_{i,\mathrm{crit}}^g \leq \hat{\delta}_{j,\mathrm{crit}}^J$. As the latter holds for any retailer $i \in I$, we obtain $\delta_{\mathrm{crit}}^g \leq \hat{\delta}_{\mathrm{crit}}^J$. Recall our assumption $\hat{\delta}_{\mathrm{crit}}^J = \hat{\delta}_{\mathrm{crit}}^{\sigma(J)}$. On the other side, we have shown above that $\hat{\delta}_{\mathrm{crit}}^{\sigma(J)} \leq \delta_{\mathrm{crit}}^g$. Due these results, our claim $\delta_{\mathrm{crit}}^g = \hat{\delta}_{\mathrm{crit}}^{\sigma(J)}$ is verified. □

Proof of Lemma 4.3. Consider a competition game $\Gamma_{\mathrm{r}}(\delta, f, n, z)$. Pick some symmetric clause profile $g \in G^{\mathrm{s}}$ and some permutation $\sigma \in \Sigma_n$ upshifting on $C(g)$. To simplify the subsequent expositions, we define $\tilde{g} := g \circ \sigma$. Let us pick some arbitrary retailer $i \in I$.

Suppose $i \in \sigma(C(g))$ and choose $j := \sigma^{-1}(i)$. Obviously, it holds $j \leq i$ and $j \in C(g)$. Let us consider some arbitrary profile $q := (q_i, p_{-i}^{\mathrm{m}})$ of advertised prices. We define $\tilde{q} := (\tilde{q}_j, p_{-j}^{\mathrm{m}})$ where $\tilde{q}_j = q_i$. As will be shown next, we obtain $\pi_i^{\tilde{g}}(q) \leq \frac{\kappa_i}{\kappa_j}\pi_j^g(\tilde{q})$.

- If $q_i > p^{\mathrm{m}}$, then we observe $\tilde{g}_i(q) = g_{\sigma^{-1}(i)}(q \circ \sigma) = g_j(\tilde{q})$. Moreover, it follows from Assumption (G1) that $\tilde{g}_k(q) = p^{\mathrm{m}}$ for any $k \neq i$ and $g_k(\tilde{q}) = p^{\mathrm{m}}$ for any $k \neq j$. Summing up, we have $\tilde{g}^{\mathrm{p}}(q) = g^{\mathrm{p}}(\tilde{q}) \circ \tau_{i,j}$. Due to Assumptions (D4) and (R1), it holds

$$X_i(\tilde{g}^{\mathrm{p}}(q)) = \begin{cases} 0 & \text{if } \tilde{g}_i^{\mathrm{p}}(q) > p^{\mathrm{m}} \\ \frac{k_i}{K}D(p^{\mathrm{m}}) & \text{if } \tilde{g}_i^{\mathrm{p}}(q) = p^{\mathrm{m}} \\ \min\{k_i, D(\tilde{g}_i^{\mathrm{p}}(q))\} & \text{if } \tilde{g}_i^{\mathrm{p}}(q) < p^{\mathrm{m}} \end{cases}$$

  and

$$X_j(g^{\mathrm{p}}(\tilde{q})) = \begin{cases} 0 & \text{if } g_j^{\mathrm{p}}(\tilde{q}) > p^{\mathrm{m}} \\ \frac{k_j}{K}D(p^{\mathrm{m}}) & \text{if } g_j^{\mathrm{p}}(\tilde{q}) = p^{\mathrm{m}} \\ \min\{k_j, D(g_j^{\mathrm{p}}(\tilde{q}))\} & \text{if } g_j^{\mathrm{p}}(\tilde{q}) < p^{\mathrm{m}} \end{cases}.$$

  As $c \leq \tilde{g}_i^{\mathrm{s}}(q) = g_j^{\mathrm{s}}(\tilde{q})$ follows from Assumption (G2), we obtain $\pi_i^{\tilde{g}}(q) \leq \frac{\kappa_i}{\kappa_j}\pi_j^g(\tilde{q})$.

- If $c < q_i < p^{\mathrm{m}}$, then we observe $\tilde{g}_i(q) = q_i = \tilde{q}_j = g_j(\tilde{q})$ due to Assumption (G1). Due to the symmetry, it holds $\tilde{g}_k(q) = g_{\sigma^{-1}(k)}(q \circ \sigma) = g_{\sigma^{-1}(k)}(\tilde{q})$ for any $k \in \sigma(C(g)) \setminus \{i\}$. Moreover, it holds $\tilde{g}_k(q) = p^{\mathrm{m}} = g_{\sigma^{-1}(k)}(\tilde{q})$ for any $k \notin \sigma(C(g))$. Summing up, we have $\tilde{g}^{\mathrm{p}}(q) = g^{\mathrm{p}}(\tilde{q}) \circ \sigma^{-1}$. We note that σ is upshifting on $[g^{\mathrm{p}}(\tilde{q}) < g_j^{\mathrm{p}}(\tilde{q})] \subseteq C(g)$. As the rationing rule is regular, Remark 2.1 is applicable and, thus, $R(\tilde{g}_i^{\mathrm{p}}(q)|\tilde{g}^{\mathrm{p}}(q)) = R(g_j^{\mathrm{p}}(\tilde{q})|g^{\mathrm{p}}(\tilde{q}) \circ \sigma^{-1}) \leq R(g_i^{\mathrm{p}}(\tilde{q})|g^{\mathrm{p}}(\tilde{q}))$. As σ is upshifting on $[g^{\mathrm{p}}(\tilde{q}) = g_j^{\mathrm{p}}(\tilde{q})] \subseteq C(g)$ and $\sigma([g^{\mathrm{p}}(\tilde{q}) = g_j^{\mathrm{p}}(\tilde{q})]) \subseteq [\tilde{g}^{\mathrm{p}}(q) = \tilde{g}_j^{\mathrm{p}}(q)]$, we also obtain $K_{[\tilde{g}^{\mathrm{p}}(q) = \tilde{g}_i^{\mathrm{p}}(q)]} \geq K_{[g^{\mathrm{p}}(\tilde{q}) = g_j^{\mathrm{p}}(\tilde{q})]}$. Hence, it holds

$$\begin{aligned} X_i(\tilde{g}^{\mathrm{p}}(q)) &= k_i \min\left\{1, \frac{R(\tilde{g}_i^{\mathrm{p}}(q)|\tilde{g}^{\mathrm{p}}(q))}{K_{[\tilde{g}^{\mathrm{p}}(q) = \tilde{g}_i^{\mathrm{p}}(q)]}}\right\} \\ &\leq k_i \min\left\{1, \frac{R(g_j^{\mathrm{p}}(\tilde{q})|g^{\mathrm{p}}(\tilde{q}))}{K_{[g^{\mathrm{p}}(\tilde{q}) = g_j^{\mathrm{p}}(\tilde{q})]}}\right\} \\ &= \frac{\kappa_i}{\kappa_j} k_j \min\left\{1, \frac{R(g_j^{\mathrm{p}}(\tilde{q})|g^{\mathrm{p}}(\tilde{q}))}{K_{[g^{\mathrm{p}}(\tilde{q}) = g_j^{\mathrm{p}}(\tilde{q})]}}\right\} \\ &= \frac{\kappa_i}{\kappa_j} X_j(g^{\mathrm{p}}(\tilde{q})) \end{aligned}$$

  As $c \leq \tilde{g}_i^{\mathrm{s}}(q) = g_j^{\mathrm{s}}(\tilde{q})$ follows from Assumption (G2), we obtain $\pi_i^{\tilde{g}}(q) \leq \frac{\kappa_i}{\kappa_j}\pi_j^g(\tilde{q})$.

Suppose $i \notin \sigma(C(g))$. We define $j := \sigma^{-1}(i)$ and $l := \sigma(i)$. Let us consider some arbitrary profile $q := (q_i, p_{-i}^{\mathrm{m}})$ of advertised prices. As will be shown next, we obtain $\pi_i^{\tilde{g}}(q) \leq \pi_i^g(q)$.

- If $q_i > p^{\mathrm{m}}$, then we observe $\tilde{g}_i(q) = q_i \leq g_i(q)$. Moreover, it follows from Assumption (G1) that $\tilde{g}_k(q) = p^{\mathrm{m}} = g_k(q)$ for any $k \neq i$. Due to Assumption (D4), it holds $X_i(\tilde{g}^{\mathrm{p}}(q)) = 0 \leq X_i(g^{\mathrm{p}}(q))$ and, thus, $\pi_i^{\tilde{g}}(q) = 0 \leq \pi_i^g(q)$.
- If $c < q_i < p^{\mathrm{m}}$, then we observe $\tilde{g}_i(q) = q_i = g_i(q)$ due to Assumption (G1). Symmetry implies $\tilde{g}_k(q) = g_{\sigma^{-1}(k)}(q \circ \sigma) = g_{\sigma^{-1}(k)}(q)$ for any $k \in \sigma(C(g)) \setminus \{l\}$. We also have $\tilde{g}_k(q) = p^{\mathrm{m}} = g_{\sigma^{-1}(k)}(q)$ for any $k \notin C(g) \cup \{j\}$. Moreover, note that if $\sigma(i) \neq i$, then $\tilde{g}_l(q) \leq p^{\mathrm{m}} = g_j(q)$ as $j \notin C(g)$, but $l \in C(g)$ or $l \notin C(g)$. Let $\tau \in \Sigma_n$ be the transposition $\tau_{i,l}$ if $l \neq i$ and the identity mapping otherwise. We define $\tilde{\sigma} = \tau \circ \sigma$. As can be easily checked, it holds $\tilde{\sigma}(i) = i$, $\tilde{\sigma}(j) = l$, and $\tilde{\sigma}(k) = \sigma(k)$ for any $k \neq i, j$.

  We note that $\tilde{g}_k^{\mathrm{p}}(q) = g^{\mathrm{p}}_{\tilde{\sigma}^{-1}(k)}(q)$ if k satisfies $g^{\mathrm{p}}_{\tilde{\sigma}^{-1}(k)}(q) < g_i^{\mathrm{p}}(q)$. Due to Assumption (R2), we obtain $R(\tilde{g}_i^{\mathrm{p}}(q)|\tilde{g}^{\mathrm{p}}(q)) \leq R(g_i^{\mathrm{p}}(q)|g^{\mathrm{p}}(q) \circ \tilde{\sigma}^{-1})$. Moreover, $\tilde{\sigma}$ is upshifting on $[g^{\mathrm{p}}(q) < g_i^{\mathrm{p}}(q)]$ so that Remark 2.1 implies $R(g_i^{\mathrm{p}}(q)|g^{\mathrm{p}}(q) \circ \tilde{\sigma}^{-1}) \leq R(g_i^{\mathrm{p}}(q)|g^{\mathrm{p}}(q))$. We conclude that $R(\tilde{g}_i^{\mathrm{p}}(q)|\tilde{g}^{\mathrm{p}}(q)) \leq R(g_i^{\mathrm{p}}(q)|g^{\mathrm{p}}(q))$.

  Obviously, $\tilde{\sigma}$ is upshifting on $[g^{\mathrm{p}}(\tilde{q}) = g_j^{\mathrm{p}}(\tilde{q})]$ and $\tilde{\sigma}([g^{\mathrm{p}}(q) < g_i^{\mathrm{p}}(q)]) \subseteq [\tilde{g}^{\mathrm{p}}(q) < \tilde{g}_i^{\mathrm{p}}(q)]$. These findings entail $K_{[\tilde{g}^{\mathrm{p}}(q) = \tilde{g}_i^{\mathrm{p}}(q)]} \geq K_{[g^{\mathrm{p}}(q) = g_i^{\mathrm{p}}(q)]}$. Hence, we obtain

$$\begin{aligned} X_i(\tilde{g}^{\mathrm{p}}(q)) &= k_i \min\left\{1, \frac{R(\tilde{g}_i^{\mathrm{p}}(q)|\tilde{g}^{\mathrm{p}}(q))}{K_{[\tilde{g}^{\mathrm{p}}(q) = \tilde{g}_i^{\mathrm{p}}(q)]}}\right\} \\ &\leq k_i \min\left\{1, \frac{R(g_i^{\mathrm{p}}(q)|g^{\mathrm{p}}(q))}{K_{[g^{\mathrm{p}}(q) = g_i^{\mathrm{p}}(q)]}}\right\} \\ &= X_i(g^{\mathrm{p}}(q)) \end{aligned}$$

  As $\tilde{g}_i^{\mathrm{s}}(q) = g_i^{\mathrm{s}}(q)$ follows from Assumption (G2), we obtain $\pi_i^{\tilde{g}}(q) \leq \pi_i^g(q)$.

The above results along with Remark 3.1(a) ensure that for any $i \in I$, there exists some $j \in I$ so that $\pi_i^{\ddagger}(\tilde{g}) \leq \frac{\kappa_i}{\kappa_j} \pi_j^{\ddagger}(g)$. This in turn entails

$$\pi_i^{\ddagger}(\tilde{g}) \leq \frac{\kappa_i}{\kappa_j} \pi_j^{\ddagger}(g) \leq \frac{\kappa_i}{\kappa_j} \frac{1}{1 - \delta^g_{j,\mathrm{crit}}} \kappa_j \pi^{\mathrm{m}} \leq \frac{1}{1 - \delta^g_{j,\mathrm{crit}}} \kappa_i \pi^{\mathrm{m}}$$

and, thus, $\delta^{\tilde{g}}_{i,\mathrm{crit}} \leq \delta^g_{j,\mathrm{crit}}$. As this is satisfied for any $i \in I$, we obtain $\delta^{\tilde{g}}_{\mathrm{crit}} \leq \delta^g_{\mathrm{crit}}$. Finally, we note that due to Remark 3.3, the weak inequality turns into an equality if $\hat{\delta}^{\tilde{g}}_{\mathrm{crit}} = \delta_{\mathrm{crit}}$. Obviously, the equality also appears if $\hat{\delta}^g_{\mathrm{crit}} = 0$. ☐

Proof of Lemma 4.4. To simplify our formal expositions, we define $J := C(g)$ and $\tilde{J} := \sigma(J)$. Applying Remark 4.2, we obtain $\hat{\delta}^J_{\mathrm{crit}} \leq \delta^g_{\mathrm{crit}}$. Moreover, $\hat{\delta}^{\tilde{J}}_{\mathrm{crit}} \leq \hat{\delta}^J_{\mathrm{crit}}$ due to Lemma 4.3.

- Suppose $\hat{\delta}^{\tilde{J}}_{\mathrm{crit}} = \hat{\delta}^J_{\mathrm{crit}}$. If σ is constant on J, then $\tilde{g} = g$ and the claim follows immediately. Let us now suppose σ is non-constant on J. According to Remark 4.2, there exists some $\tilde{g} \in G$ so that $C(\tilde{g}) = \tilde{J}$ and $\delta^{\tilde{g}}_{\mathrm{crit}} = \hat{\delta}^{\tilde{J}}_{\mathrm{crit}}$. The latter entails $\delta^{\tilde{g}}_{\mathrm{crit}} \leq \delta^g_{\mathrm{crit}}$.
- Suppose $\hat{\delta}^{\tilde{J}}_{\mathrm{crit}} < \hat{\delta}^J_{\mathrm{crit}}$. According to Remark 4.2, there exists some $\tilde{g} \in G$ so that $C(\tilde{g}) = \tilde{J}$ and $\hat{\delta}^{\tilde{J}}_{\mathrm{crit}} \leq \delta^{\tilde{g}}_{\mathrm{crit}} < \hat{\delta}^J_{\mathrm{crit}}$. The latter inequality entails $\delta^{\tilde{g}}_{\mathrm{crit}} < \delta^g_{\mathrm{crit}}$.

We conclude from the above results that there exists some $\tilde{g} \in G$ satisfying $C(\tilde{g}) = \tilde{J}$ and $\delta^{\tilde{g}}_{\mathrm{crit}} \leq \delta^g_{\mathrm{crit}}$. Moreover, if $\delta^{\tilde{J}}_{\mathrm{crit}} < \delta^J_{\mathrm{crit}}$, then even $\delta^{\tilde{g}}_{\mathrm{crit}} < \delta^g_{\mathrm{crit}}$. ☐

Proof of Proposition 4.5. As $\delta < \delta_{\mathrm{crit}}$, we conclude from Theorem 4.1 that the perfectly collusive clause profile g has the property $0 < \ell := |C(g)| \leq n$. The claim of the proposition will be established by induction on ℓ. For this purpose, we define $j := \min C(g) < n + 1 - |C(g)|$. Moreover, let υ be the extreme upshift permutation on $C(g)$.

Suppose $\ell = 1$. In this case, it holds $C(g) = \{j\}$ where $1 \leq j < n$. We know from LEMMA 4.4 that there exists some clause profile $\tilde{g}$ so that $C(\tilde{g}) = \upsilon(C(g))$ and $\delta^{\tilde{g}}_{\text{crit}} \leq \delta^{g}_{\text{crit}}$. Note that $C(\tilde{g}) = \{n\}$ (i.e., $\min C(\tilde{g}) = n + 1 - |C(\tilde{g})|$) and, thus, $f^{\tilde{g},\delta}_{\text{crit}} > f^{g,\delta}_{\text{crit}}$. Applying PROPOSITION 3.10, we conclude that g as a perfectly collusive clause profile satisfies PROPERTIES (M1) and (M3). This implies that $\tilde{g}$ fulfills both properties, too. Moreover, as $\delta < \delta_{\text{crit}}$, it also fulfills PROPERTY (M2). We then infer from PROPOSITION 3.10 that $\tilde{g}$ is perfectly collusive. Summing up, we have shown that $|C(\tilde{g})| = |C(g)|$ as well as $f^{\tilde{g},\delta}_{\text{crit}} > f^{g,\delta}_{\text{crit}}$ and $\delta^{\tilde{g}}_{\text{crit}} \leq \delta^{g}_{\text{crit}}$. This means nothing but $\tilde{g}$ is more collusive than g.

Let us turn to the case $1 < \ell < n$ and suppose that our claim holds for any perfectly collusive clause profile in which at least one, but less than ℓ retailers adopt CCs. We already know from LEMMA 4.4 that there exists some clause profile g' so that $C(g') = \upsilon(C(g))$ and $\delta^{g'}_{\text{crit}} \leq \delta^{g}_{\text{crit}}$. Note that $\min C(g') = n + 1 - |C(g')| > j$ and, thus, $f^{g',\delta}_{\text{crit}} > f^{g,\delta}_{\text{crit}}$.

It follows from PROPOSITION 3.10 that g as a perfectly collusive clause profile satisfies PROPERTIES (M1) and (M3). For this reason, g' fulfills both properties, too. If g' also satisfies PROPERTY (M2), the proof is completed. Otherwise, we make use of PROPOSITION 3.11 and conclude that there exists some perfectly collusive clause profile $\tilde{g} := (g'_J, w_{-J})$ satisfying $\emptyset \neq J \subset C(g')$. As can be easily checked, it holds $|C(\tilde{g})| < |C(g)|$. That is, $\tilde{g}$ proves to be more collusive than g. If $\min C(\tilde{g}) = n + 1 - |C(\tilde{g})|$, our claim follows immediately. Otherwise, we obtain $1 \leq |C(\tilde{g})| < \ell$ so that we apply the induction premise in order to establish our claim. □

Proof of Corollary 4.6. Let us consider a perfectly collusive clause profile g. As $\delta < \delta_{\text{crit}}$ is assumed, THEOREM 4.1 implies $C(g) \neq \emptyset$. We prove the claim indirectly. Suppose there exists some $i \notin C(g)$ so that $i > j := \min C(g)$. We already know from PROPOSITION 4.5 that there exists some perfectly collusive clause profile $\tilde{g}$ being more collusive than g and having the property $\min C(\tilde{g}) = n + 1 - |C(\tilde{g})|$. That means, g is not the most collusive among the perfectly collusive clause profiles. Hence, any clause profile $\hat{g}$ which proves to be robustly collusive has the property that there exists some $k \in I$ so that $C(\hat{g}) = \{k, k+1, \ldots, n\}$. Due to the cost-efficiency criterion, k has to be identical for any of those clause profiles. □

Proof of Theorem 5.1.

(a) Let $\hat{g} := (w_{-n}, b_n^{€, p^{\text{m}} - c})$ be the clause profile in which the largest retailer adopts the BCC with lump sum $p^{\text{m}} - c$ and the other retailers abstain from offering a CC. To prove this claim, our first step is to demonstrate that $1 - \kappa_n \leq \delta^{\hat{g}}_{\text{crit}} \leq \delta$ and $\delta^{\hat{g}}_{\text{crit}} \leq \delta^{g}_{\text{crit}}$ for any clause profile g where $|C(g)| = 1$.

Let us first suppose that clause profile $\hat{g}$ has been realized in the clause implementation phase. Consider the largest retailer, i.e., retailer n, and suppose $q := (q_n, p^{\text{m}}_{-n})$ is the profile of the advertised prices. Apparently, if $q_n > p^{\text{m}}$, we observe $\hat{g}^{\text{s}}_n(q) = c$ so that $\pi^{\hat{g}}_n(q) = 0 = \pi_n(q)$ due to ASSUMPTIONS (D2) and (D4). If $q_n \leq p^{\text{m}}$, we observe $\hat{g}^{\text{s}}_n(q) = \hat{g}^{\text{p}}_n(q) = q_n$ and $\hat{g}^{\text{p}}_k(q) = p^{\text{m}} = q_k$ for any $k \neq n$ so that $\pi^{\hat{g}}_n(q) = \pi_n(q)$. Bringing together these findings, we obtain $\pi^{\hat{g}}_n(q) = \pi_n(q)$ for any $q := (q_n, p^{\text{m}}_{-n}) \in \mathbb{R}^I_+$. It follows $\pi^{\ddagger}_n(\hat{g}) = \pi^{\ddagger}_n(w)$. Note our assumption $k_n \geq D(c)$ along with ASSUMPTIONS (D1) and (D2) entail that $\pi^{\ddagger}_n(w) = (p^{\text{m}} - c)D(p^{\text{m}})$ and, thus, $\delta^{\hat{g}}_{n,\text{crit}} = \delta_{n,\text{crit}} = 1 - \kappa_n \leq \delta$.

Pick now some retailer $i \neq n$ and let $q := (q_i, p^{\text{m}}_{-i})$ be the profile of the advertised prices. We obtain the following results:

- If $q_i > p^{\text{m}}$, then $\hat{g}^{\text{p}}_i(q) = q_i$ and $\hat{g}^{\text{p}}_k(q) = p^{\text{m}}$ for any $k \neq i$. We conclude from ASSUMPTIONS (D2) and (D4) that $R(q_i|\hat{g}^{\text{p}}(q)) = 0$. This in turn implies $X_i(\hat{g}^{\text{p}}(q)) = 0$ and, thus, $\pi^{\hat{g}}_i(q) = 0$.
- If $c + z < q_i < p^{\text{m}}$, then $\hat{g}^{\text{s}}_i(q) = \hat{g}^{\text{p}}_i(q) = q_i$ and $\hat{g}^{\text{p}}_n(q) = c + z$. Moreover, we observe $\hat{g}^{\text{p}}_k(q) = p^{\text{m}}$ for any $k \neq i, n$. It follows from $k_n \geq D(c)$ as well as ASSUMPTIONS (R1) and (D2) that $0 \leq R(q_i|\hat{g}^{\text{p}}(q)) \leq R_{\text{l}}(q_i|\hat{g}^{\text{p}}(q)) = \max\{D(c+z) - k_n, 0\} = 0$. This entails $X_i(\hat{g}^{\text{p}}(q)) = 0$ and, thus, $\pi^{\hat{g}}_i(q) = 0$.

- If $q_i = c + z$, then $\hat{g}_i^{\mathrm{s}}(q) = \hat{g}_i^{\mathrm{p}}(q) = c + z = \hat{g}_n^{\mathrm{p}}(q)$. Moreover, we observe $\hat{g}_k^{\mathrm{p}}(q) = p^{\mathrm{m}}$ for any $k \neq i, n$. It follows from $k_n \geq D(c)$ and Assumptions (D2) that $X_i(\hat{g}^{\mathrm{p}}(q)) = \frac{k_i}{k_i+k_n} D(c+z) < \min\{k_i, D(c+z)\}$ and, thus, $\pi_i^{\hat{g}}(q) < \bar{\pi}_i(c+z)$. Note that as $z \leq \bar{z}_1^{\delta}$, it holds $z \leq \bar{z}_i^{\delta}$ due to Remark 3.8. By Assumption (D3), we obtain $\bar{\pi}_i(c+z) \leq \bar{\pi}_i(c+\bar{z}_i^{\delta}) = \frac{1}{1-\delta}\kappa_i \pi^{\mathrm{m}}$. We conclude from these findings that $\pi_i^{\hat{g}}(q) < \frac{1}{1-\delta}\kappa_i\pi^{\mathrm{m}}$.
- If $c < q_i < c + z$, then $\hat{g}_i^{\mathrm{p}}(q) = q_i < c + z = \hat{g}_n^{\mathrm{p}}(q)$. Moreover, we observe $\hat{g}_k^{\mathrm{p}}(q) = p^{\mathrm{m}}$ for any $k \neq i, n$. It follows $X_i(\hat{g}^{\mathrm{p}}(q)) = \min\{k_i, D(q_i)\}$ and, thus, $\pi_i^{\hat{g}}(q) = \bar{\pi}_i(q_i)$. Moreover, as $z \leq \bar{z}_1^{\delta}$, we have $z \leq \bar{z}_i^{\delta}$ due to Remark 3.8. It follows from Assumption (D3) that $\bar{\pi}_i(q_i) < \bar{\pi}_i(c+\bar{z}_i^{\delta}) = \frac{1}{1-\delta}\kappa_i\pi^{\mathrm{m}}$. We conclude from these findings that $\pi_i^{\hat{g}}(q) < \frac{1}{1-\delta}\kappa_i\pi^{\mathrm{m}}$.

The above calculations and Remark 3.1(a) ensure $\pi_i^{\ddagger}(\hat{g}) \leq \frac{1}{1-\delta}\kappa_i\pi^{\mathrm{m}}$. This in turn entails $\delta_{i,\mathrm{crit}}^{\hat{g}} \leq \delta$ for any $i \neq n$. Summing up, we have shown that clause profile $\hat{g}$ has the desired property $1 - \kappa_n \leq \delta_{\mathrm{crit}}^{\hat{g}} \leq \delta$ and, thus, the first part of the above claim has been verified.

Our next objective is to prove the second part of the above claim. That is, we aim to establish that $\delta_{\mathrm{crit}}^{\hat{g}} \leq \delta_{\mathrm{crit}}^{g}$ for any clause profile g satisfying $|C(g)| = 1$. For this purpose, let us pick such clause profile g and henceforth denote the only CC-adopting retailer in g by j, i.e., $\{j\} = C(g)$. As will be shown next, it turns that for any $i \in I$, there is some $k \in I$ so that $\delta_{i,\mathrm{crit}}^{\hat{g}} \leq \delta_{k,\mathrm{crit}}^{g}$. Apparently, this finding would confirm the second part of the claim.

Let us first consider retailer n. We know from Remark 3.5 that $\delta_{j,\mathrm{crit}}^{g} \geq 1 - \max\{\kappa_j, \frac{D(p^{\mathrm{m}})}{K}\}$. Recall that $\kappa_n \geq \kappa_j$. Moreover, our assumption $k_n \geq D(c)$ along with Assumption (D2) entails $\kappa_n > \frac{D(p^{\mathrm{m}})}{K}$. It follows $\delta_{j,\mathrm{crit}}^{g} \geq 1 - \kappa_n$. As $\delta_{n,\mathrm{crit}}^{\hat{g}} = 1 - \kappa_n$, it holds $\delta_{n,\mathrm{crit}}^{\hat{g}} \leq \delta_{j,\mathrm{crit}}^{g}$.

Let us turn to retailer j. As retailer j is the only CC-adopting retailer, we obtain $\pi_j^{\ddagger}(g) = (p^{\mathrm{m}} - c)\min\{k_j, D(p^{\mathrm{m}})\}$ due to Assumption (D1). Moreover, we know from Remark 3.1(b) that $\pi_j^{\ddagger}(\hat{g}) \leq (p^{\mathrm{m}} - c)\min\{k_j, D(p^{\mathrm{m}})\}$ and, thus, $\pi_j^{\ddagger}(\hat{g}) \leq \pi_j^{\ddagger}(g)$. This in turn implies $\delta_{j,\mathrm{crit}}^{\hat{g}} \leq \delta_{j,\mathrm{crit}}^{g}$.

Let us finally consider some retailer $i \neq j, n$ and let $q := (q_i, p_{-i}^{\mathrm{m}})$ be the profile of the advertised prices. We obtain the following results:

- If $q_i > p^{\mathrm{m}}$ or $c + z < q_i < p^{\mathrm{m}}$, then $g_i^{\mathrm{s}}(q) = g_i^{\mathrm{p}}(q) = q_i > c + z$ so that $\pi_i^{g}(q) \geq 0$. Recall that $\pi_i^{\hat{g}}(q) = 0$ in any of these two cases and, thus, $\pi_i^{g}(q) \geq \pi_i^{\hat{g}}(q)$.
- If $q_i = c + z$, then $g_i^{\mathrm{s}}(q) = g_i^{\mathrm{p}}(q) = c + z = g_j^{\mathrm{p}}(q)$. Moreover, we observe $g_k^{\mathrm{p}}(q) = p^{\mathrm{m}}$ for any $k \neq i, n$. It follows $X_i(g^{\mathrm{p}}(q)) = \frac{k_i}{k_i+k_j}\min\{k_i + k_j, D(q_i)\}$. Recall that $X_i(\hat{g}^{\mathrm{p}}(q)) = \frac{k_i}{k_i+k_n} D(q_i)$. As $k_j \leq k_n$ and $k_i + k_n > D(q_i)$ due to Assumption (D2), we obtain $X_i(g^{\mathrm{p}}(q)) \geq X_i(\hat{g}^{\mathrm{p}}(q))$. Note that $g_i^{\mathrm{s}}(q) = \hat{g}_i^{\mathrm{s}}(q)$. Hence, $\pi_i^{g}(q) \geq \pi_i^{\hat{g}}(q)$.
- If $c < q_i < c + z$, then $g_i^{\mathrm{s}}(q) = g_i^{\mathrm{p}}(q) = q_i < c + z = g_j^{\mathrm{p}}(q)$. Moreover, we observe $g_k^{\mathrm{p}}(q) = p^{\mathrm{m}}$ for any $k \neq i, j$. It follows $X_i(g^{\mathrm{p}}(q)) = \min\{k_i, D(q_i)\} = X_i(\hat{g}^{\mathrm{p}}(q))$. As $g_i^{\mathrm{s}}(q) = \hat{g}_i^{\mathrm{s}}(q)$, we obtain $\pi_i^{g}(q) \geq \pi_i^{\hat{g}}(q)$.

The above calculations confirm that $\pi_i^{\hat{g}}(q) \leq \pi_i^{g}(q)$ for any profile $q := (q_i, p_{-i}^{\mathrm{m}})$ of advertised prices satisfying $c < p \neq p^{\mathrm{m}}$. This result and Remark 3.1(a) entail $\pi_i^{\ddagger}(\hat{g}) \leq \pi_i^{\ddagger}(g)$. Hence, $\delta_{i,\mathrm{crit}}^{\hat{g}} \leq \delta_{i,\mathrm{crit}}^{g}$ for any $i \neq j, n$. Summing up, we have established that for any $i \in I$, there exists some $k \in I$ so that $\delta_{i,\mathrm{crit}}^{\hat{g}} \leq \delta_{k,\mathrm{crit}}^{g}$. This in turn implies $\delta_{\mathrm{crit}}^{\hat{g}} \leq \delta_{\mathrm{crit}}^{g}$, i.e., the second part of our claim.

As just shown, it holds $1 - \kappa_n \leq \delta_{\mathrm{crit}}^{\hat{g}} \leq \delta$ and $\delta_{\mathrm{crit}}^{\hat{g}} \leq \delta_{\mathrm{crit}}^{g}$ for any clause profile g where $|C(g)| = 1$. With these findings at our disposal, we prove the "if"- and "only if"-part of the theorem.

(*"if"*) Suppose $1 - \kappa_n \leq \delta < \delta_{\mathrm{crit}}$ and $f \leq \frac{1}{1-\delta}\kappa_n\pi^{\mathrm{m}}$ are satisfied. It follows from $\delta_{\mathrm{crit}}^{\hat{g}} \leq \delta$ that $\hat{g}$ satisfies Property (M1). Moreover, it holds $(w_i, \hat{g}_{-i}) = w$ for any $i \in C(g)$. Therefore, Property (M2) results from assumption $\delta < \delta_{\mathrm{crit}}$. Finally, we remark that $f_{\mathrm{crit}}^{\hat{g},\delta} = \frac{1}{1-\delta}\kappa_n\pi^{\mathrm{m}}$ and, thus, Property (M3) is also satisfied. We conclude from Proposition 3.10 that $\hat{g}$ is perfectly collusive.

It remains to demonstrate that $\hat{g}$ is robustly collusive. For this purpose, consider some arbitrary clause profile g being perfectly collusive at common discount factor $\delta < \delta^{g}_{\text{crit}}$. Note that THEOREM 4.1 implies $|C(g)| \geq |C(\hat{g})| = 1$. Therefore, our task is to show that if $|C(g)| = |C(\hat{g})|$, then $\delta^{g}_{\text{crit}} \geq \delta^{\hat{g}}_{\text{crit}}$ and $f^{g,\delta}_{\text{crit}} \leq f^{\hat{g},\delta}_{\text{crit}}$.

The former weak inequality has already been proved above. The latter inequality follows immediately from the fact that $\kappa_n \geq \kappa_j$ where j denotes the retailer contained in singleton $C(g)$. To sum up, $\hat{g}$ proves to be robustly collusive and, in consequence, any robustly collusive clause profile has the property that the largest retailer is the only CC-adopting retailer.

(*"only if"*) Suppose $\hat{g}$ is robustly collusive at common discount factor δ. Obviously, any robustly collusive clause profile at common discount factor δ then has the property that the largest retailer is the only CC-adopting retailer.

We remark that $\hat{g}$ is perfectly collusive at common discount factor δ by assumption. According to the characterization in PROPOSITION 3.10, it satisfies PROPERTIES (M1) - (M3). We already know from the above calculations that $1-\kappa_n \leq \delta^{\hat{g}}_{\text{crit}}$. Hence, PROPERTY (M1) requires $1-\kappa_n \leq \delta$. Moreover, we conclude from PROPERTY (M2) and THEOREM 4.1 that $\delta < \delta_{\text{crit}}$. Finally, PROPERTY (M3) implies $f \leq f^{\hat{g},\delta}_{\text{crit}} = \frac{1}{1-\delta}\kappa_n \pi^{\text{m}}$.

(b) Let $\hat{g} := (w_{J_{n-2}}, b^{€,p^{\text{m}}-c}_{n-1}, b^{€,p^{\text{m}}-c}_{n})$ be the clause profile in which the two largest retailers adopt the BCC with lump sum $p^{\text{m}} - c$ while the other retailers abstain from offering a CC. To prove the claim, our first step is to demonstrate that $\delta^{\hat{g}}_{\text{crit}} \leq \delta$ and $\delta^{\hat{g}}_{\text{crit}} \leq \delta^{g}_{\text{crit}}$ for any clause profile g satisfying $|C(g)| = 2$.

Suppose for the time being that clause profile $\hat{g}$ has been realized in the clause implementation phase. Moreover, we pick some arbitrary retailer $i \in I$ and let $q := (q_i, p^{\text{m}}_{-i})$ be the profile of the advertised prices. It holds:

- If $q_i > p^{\text{m}}$, then $\hat{g}^{\text{s}}_i(q) = \hat{g}^{\text{p}}_i(q) = q_i$ if $i \in J_{n-2}$ and $\hat{g}^{\text{s}}_i(q) = c$ otherwise. Regardless of which of the two cases applies, we observe $\hat{g}^{\text{p}}_k(q) = p^{\text{m}}$ for any $k \neq i$. If $i \in J_{n-2}$, we conclude from ASSUMPTIONS (D2) and (D4) that $D(p^{\text{m}}) < K_{-i}$. This in turn implies $R(q^{\text{p}}_i|\hat{g}^{\text{p}}(q)) = 0$ and, thus, $X_i(\hat{g}^{\text{p}}(q)) = 0$. Hence, we obtain $\pi^{\hat{g}}_i(q) = 0$. If $i \notin J_{n-2}$, it follows immediately from the sales price $\hat{g}^{\text{s}}_i(q) = c$ that $\pi^{\hat{g}}_i(q) = 0$.
- If $c + z < q_i < p^{\text{m}}$, then $\hat{g}^{\text{s}}_i(q) = \hat{g}^{\text{p}}_i(q) = q_i$ and $\hat{g}^{\text{p}}_j(q) = c + z$ for any $j \in C(g) \setminus \{i\}$. Moreover, we observe $\hat{g}^{\text{p}}_k(q) = p^{\text{m}}$ for any $k \notin C(g) \cup \{i\}$. Pick some $j \in C(g) \setminus \{i\}$. It follows from $k_j \geq k_{n-1} \geq D(c)$ and ASSUMPTIONS (D2) that $K_{C(g)\setminus\{i\}} > D(c + z)$. This in turn entails $R(\hat{g}^{\text{p}}_i(q)|\hat{g}^{\text{p}}(q)) = 0$ and, thus, $X_i(\hat{g}^{\text{p}}(q)) = 0$. Hence, we obtain $\pi^{\hat{g}}_i(q) = 0$.
- If $q_i = c + z$, then $\hat{g}^{\text{s}}_i(q) = \hat{g}^{\text{p}}_i(q) = c + z$ and $\hat{g}^{\text{p}}_j(q) = c + z$ for any $j \notin C(g) \setminus \{i\}$. Moreover, we observe $\hat{g}^{\text{p}}_k(q) = p^{\text{m}}$ for any $k \notin C(g) \cup \{i\}$. We note that $K_{[\hat{g}^{\text{p}}(q)=\hat{g}^{\text{p}}_i(q)]} = K_{C(\hat{g})\setminus\{i\}} + k_i$. Pick some $j \notin C(\hat{g}) \setminus \{i\}$. It follows from $K_{C(\hat{g}))\setminus\{i\}} \geq k_j \geq D(c)$ and ASSUMPTION (D2) that $K_{C(\hat{g}\setminus\{i\}} + k_i > D(q_i)$. Hence, it holds $X_i(\hat{g}^{\text{p}}(q)) = \frac{k_i}{K_{C(\hat{g})\setminus\{i\}}+k_i} D(c + z) < \min\{k_i, D(c + z)\}$ and, thus, $\pi^{\hat{g}}_i(q) = z\frac{k_i}{K_{C(\hat{g})\setminus\{i\}}+k_i} D(c + z) < \bar{\pi}_i(c + z)$. Moreover, as $z \leq \bar{z}^{\delta}_1$, we obtain $z \leq \bar{z}^{\delta}_i$ due to REMARK 3.8. It follows from ASSUMPTION (D3) that $\bar{\pi}_i(c+z) \leq \bar{\pi}_i(c+\bar{z}^{\delta}_i) = \frac{1}{1-\delta}\kappa_i\pi^{\text{m}}$. We conclude from these results that $\pi^{\hat{g}}_i(q) < \frac{1}{1-\delta}\kappa_i\pi^{\text{m}}$.
- If $c < q_i < c + z$, then $\hat{g}^{\text{s}}_i(q) = \hat{g}^{\text{p}}_i(q) = q_i < c + z$ and $c + z \leq \hat{g}^{\text{p}}_k(q)$ for any $k \neq i$. Obviously, this entails $X_i(\hat{g}^{\text{p}}(q)) = \min\{k_i, D(q_i)\}$. Hence, we have $\pi^{\hat{g}}_i(q) = \bar{\pi}_i(q_i)$. Note that as $z \leq \bar{z}^{\delta}_1$, it holds $z \leq \bar{z}^{\delta}_i$ due to REMARK 3.8. It follows from ASSUMPTION (D3) that $\bar{\pi}_i(q_i) < \bar{\pi}_i(c + \bar{z}^{\delta}_i) = \frac{1}{1-\delta}\kappa_i\pi^{\text{m}}$. We conclude from these results that $\pi^{\hat{g}}_i(q) < \frac{1}{1-\delta}\kappa_i\pi^{\text{m}}$.

The above calculations and REMARK 3.1(a) imply $\pi^{\updownarrow}_i(\hat{g}) < \frac{1}{1-\delta}\kappa_i\pi^{\text{m}}$. This in turn entails $\delta^{\hat{g}}_{i,\text{crit}} \leq \delta$ for any $i \in I$. Summing up, we have established that clause profile $\hat{g}$ has the property $\delta^{\hat{g}}_{\text{crit}} \leq \delta$.

Our next task is to prove that $\delta^{\hat{g}}_{\mathrm{crit}} \leq \delta^{g}_{\mathrm{crit}}$ for any clause profile g satisfying $|C(g)| = 2$ (i.e., there are exactly two CC-adopting retailers in clause profile g). For this purpose, let us suppose for the time being that such clause profile g has been realized in the clause implementation phase. In the following, we argue that for any $i \in I$, there is some $j \in I$ so that $\delta^{\hat{g}}_{i,\mathrm{crit}} \leq \delta^{g}_{j,\mathrm{crit}}$. Apparently, this result would verify our claim $\delta^{\hat{g}}_{\mathrm{crit}} \leq \delta^{g}_{\mathrm{crit}}$.

Let us first pick some retailer $i \in C(\hat{g}) = \{n-1, n\}$. Moreover, we consider retailer $j := \min C(g)$. Obviously, it holds $i \geq j$ and, thus, $\frac{k_i}{k_j} \geq 1$ where the strict inequality results whenever $i > j$. As will be argued next, it turns out that $\frac{k_i}{k_j}\pi^{\ddagger}_j(g) \geq \pi^{\ddagger}_i(\hat{g})$.

- If $q_i = q_j > p^{\mathrm{m}}$ or $c+z < q_i = q_j < p^{\mathrm{m}}$, then Assumption (G2) ensures that $g^{\mathrm{p}}_j(q_j, p^{\mathrm{m}}_{-j}) \geq c+z$ and, thus, $g^{\mathrm{s}}_j(q_j, p^{\mathrm{m}}_{-j}) \geq c$. It immediately follows $\pi^{g}_j(q_j, p^{\mathrm{m}}_{-j}) \geq 0$. Recall that $\pi^{\hat{g}}_i(q_i, p^{\mathrm{m}}_{-i}) = 0$. Hence, it holds $\frac{k_i}{k_j}\pi^{g}_j(q_j, p^{\mathrm{m}}_{-j}) \geq 0 = \pi^{\hat{g}}_i(q_i, p^{\mathrm{m}}_{-i})$.
- If $q_i = q_j = c+z$, then $g^{\mathrm{s}}_j(q_j, p^{\mathrm{m}}_{-j}) = g^{\mathrm{p}}_j(q_j, p^{\mathrm{m}}_{-j}) = c+z$. Moreover, Assumption (G2) ensures that $g^{\mathrm{p}}_k(q_j, p^{\mathrm{m}}_{-j}) \geq c+z$ for the other CC-adopting retailer $k \in C(g) \setminus \{j\}$ in clause profile g. Apart from that, we observe $g^{\mathrm{p}}_k(q_j, p^{\mathrm{m}}_{-j}) = p^{\mathrm{m}}$ for any non CC-adopting retailer $k \notin C(g)$ in clause profile g. Define $J := [g^{\mathrm{p}}(q_j, p^{\mathrm{m}}_{-j}) = q_j]$. As $J \subseteq C(g)$, we obtain $K_J \leq K_{C(g)}$. Hence, it holds $X_j(g^{\mathrm{p}}(q_j, p^{\mathrm{m}}_{-j})) = \frac{k_j}{K_J}\min\{K_J, D(c+z)\} \geq \frac{k_j}{K_{C(g)}}\min\{K_{C(g)}, D(c+z)\}$ and, thus, $\pi^{g}_j(q_j, p^{\mathrm{m}}_{-j}) \geq z\frac{k_j}{K_{C(g)}}\min\{K_{C(g)}, D(c+z)\}$. Recall that $\pi^{\hat{g}}_i(q_i, p^{\mathrm{m}}_{-i}) = z\frac{k_i}{K_{C(\hat{g})}}D(c+z)$ and $D(c+z) \leq K_{C(\hat{g})}$ due to $k_{n-1} \geq D(c)$ and Assumption (D2). As $K_{C(g)} \leq K_{C(\hat{g})}$, we finally obtain $\frac{k_i}{k_j}\pi^{g}_j(q_j, p^{\mathrm{m}}_{-j}) \geq \pi^{\hat{g}}_i(q_i, p^{\mathrm{m}}_{-i})$.
- If $c < q_i = q_j < c+z$, then $g^{\mathrm{s}}_i(q_j, p^{\mathrm{m}}_{-j}) = g^{\mathrm{p}}_i(q_j, p^{\mathrm{m}}_{-j}) = q_i < c+z$. Moreover, we observe $g^{\mathrm{p}}_k(q_j, p^{\mathrm{m}}_{-j}) \geq c+z$ for any $k \neq i$ due to Assumption (G2). It follows $X_j(g^{\mathrm{p}}(q)) = \min\{k_j, D(q_j)\}$ and, thus, $\pi^{g}_j(q_j, p^{\mathrm{m}}_{-j}) = \bar{\pi}_j(q_j)$. Recall that $\pi^{\hat{g}}_i(q_i, p^{\mathrm{m}}_{-i}) = \bar{\pi}_i(q_i)$. As can be easily checked, it holds $\frac{k_i}{k_j}\bar{\pi}_j(q_j) \geq \bar{\pi}_i(q_i)$. Hence, we obtain $\frac{k_i}{k_j}\pi^{g}_j(q_j, p^{\mathrm{m}}_{-j}) \geq \pi^{\hat{g}}_i(q_i, p^{\mathrm{m}}_{-i})$.

We have shown by the above calculations that $\frac{k_i}{k_j}\pi^{g}_j(q_j, p^{\mathrm{m}}_{-j}) \geq \pi^{\hat{g}}_i(q_i, p^{\mathrm{m}}_{-i})$ for any advertised prices $c < q_i = q_j \neq p^{\mathrm{m}}$. This result and Remark 3.1(a) entail $\frac{k_i}{k_j}\pi^{\ddagger}_j(g) \geq \pi^{\ddagger}_i(\hat{g})$. In consequence, we obtain

$$\pi^{\ddagger}_i(\hat{g}) \leq \frac{k_i}{k_j}\pi^{\ddagger}_j(g) \leq \frac{k_i}{k_j}\frac{1}{1-\delta^{g}_{j,\mathrm{crit}}}\kappa_j\pi^{\mathrm{m}} \leq \frac{1}{1-\delta^{g}_{j,\mathrm{crit}}}\kappa_i\pi^{\mathrm{m}}$$

and, thus, $\delta^{\hat{g}}_{i,\mathrm{crit}} \leq \delta^{g}_{j,\mathrm{crit}}$.

Let us now consider some retailer $i \notin C(\hat{g})$. Moreover, we suppose that profile $q := (q_i, p^{\mathrm{m}}_{-i})$ has been advertised by the retailers. It holds:

- If $q_i > p^{\mathrm{m}}$ or $c+z < q_i < p^{\mathrm{m}}$, then Assumption (G2) ensures that $g^{\mathrm{p}}_i(q) \geq c+z$ and, thus, $g^{\mathrm{s}}_i(q) \geq c$. It immediately follows $\pi^{g}_i(q) \geq 0$. Recall that $\pi^{\hat{g}}_i(q) = 0$. Hence, we obtain $\pi^{g}_i(q) \geq 0 = \pi^{\hat{g}}_i(q)$.
- If $q_i = c+z$, then $g^{\mathrm{s}}_i(q) = g^{\mathrm{p}}_i(q) = c+z$. By Assumption (G2), it holds $g^{\mathrm{p}}_j(q) \geq c+z$ for any $j \in C(g) \setminus \{i\}$. Moreover, we observe $g^{\mathrm{p}}_k(q) = p^{\mathrm{m}}$ for any $k \notin C(g) \cup \{i\}$. Let us define $J := [g^{\mathrm{p}}(q) = q_i]$. Obviously, $J \subseteq C(g) \cup \{i\}$. It follows $K_J \leq K_{C(g)\cup\{i\}}$ and, thus, $X_i(g^{\mathrm{p}}(q)) = \frac{k_i}{K_J}\min\{K_J, D(c+z)\} \geq \frac{k_i}{K_{C(g)\cup\{i\}}}\min\{K_{C(g)\cup\{i\}}, D(c+z)\}$. This in turn implies $\pi^{g}_j(q) \geq z\frac{k_i}{K_{C(g)\cup\{i\}}}\min\{K_{C(g)\cup\{i\}}, D(c+z)\}$. Recall that $\pi^{\hat{g}}_i(q) = z\frac{k_i}{K_{C(\hat{g})\cup\{i\}}}D(c+z)$ and $D(c+z) \leq K_{C(\hat{g})\cup\{i\}}$ due to $k_{n-1} \geq D(c)$ and Assumption (D2). As $K_{C(g)\cup\{i\}} \leq K_{C(\hat{g})\cup\{i\}}$, we finally obtain $\pi^{g}_i(q) \geq \pi^{\hat{g}}_i(q)$.
- If $c < q_i < c+z$, then $g^{\mathrm{s}}_i(q) = g^{\mathrm{p}}_i(q) = q_i < c+z$. Moreover, we observe $g^{\mathrm{p}}_k(q) \geq c+z$ for any $k \neq i$ due to Assumption (G2). It follows $X_i(g^{\mathrm{p}}(q)) = \min\{k_i, D(q_i)\}$ and, thus, $\pi^{g}_i(q) = \bar{\pi}_i(q_i)$. Recall that $\pi^{\hat{g}}_i(q) = \bar{\pi}_i(q_i)$. Hence, we obtain $\pi^{g}_i(q) = \pi^{\hat{g}}_i(q)$.

We have established by the above calculations that $\pi_i^g(q) \geq \pi_i^{\hat{g}}(q)$ for any profile $q := (q_i, p_{-i}^{\mathrm{m}})$ of advertised prices where $c < q_i \neq p^{\mathrm{m}}$. This result and REMARK 3.1(a) entail $\pi_i^{\ddagger}(g) \geq \pi_i^{\ddagger}(\hat{g})$. In consequence, we obtain

$$\pi_i^{\ddagger}(\hat{g}) \leq \pi_i^{\ddagger}(g) \leq \frac{1}{1-\delta_{i,\mathrm{crit}}^g} \kappa_i \pi^{\mathrm{m}}$$

and, thus, $\delta_{i,\mathrm{crit}}^{\hat{g}} \leq \delta_{j,\mathrm{crit}}^g$. Summing up, we have established that for any $i \in I$, there exists a $j \in I$ so that $\delta_{i,\mathrm{crit}}^{\hat{g}} \leq \delta_{j,\mathrm{crit}}^g$. Apparently, this result entails the above claim $\delta_{\mathrm{crit}}^{\hat{g}} \leq \delta_{\mathrm{crit}}^g$.

As just argued, clause profile $\hat{g}$ fulfills the properties $\delta_{\mathrm{crit}}^{\hat{g}} \leq \delta$ and $\delta_{\mathrm{crit}}^{\hat{g}} \leq \delta_{\mathrm{crit}}^g$ for any clause profile g satisfying $|C(g)| = 2$. With this result at our hand, we are now able to prove the "if"- and "only if"-part of the theorem.

(*"if"*) Suppose $\delta < 1 - \kappa_n$ and $f \leq \frac{1}{1-\delta}\kappa_{n-1}\pi^{\mathrm{m}}$. We already have shown that $\delta_{\mathrm{crit}}^{\hat{g}} \leq \delta$ and, thus, $\hat{g}$ satisfies PROPERTY (M1). Moreover, as we already know from part (a), any clause profile $g := (w_k, \hat{g}_{-k})$ where $k \in C(\hat{g})$ fulfills $\delta_{\mathrm{crit}}^g \geq 1 - \kappa_n$. For this reason, PROPERTY (M2) also holds. Finally, we remark that $f_{\mathrm{crit}}^{\hat{g},\delta} = \frac{1}{1-\delta}\kappa_{n-1}\pi^{\mathrm{m}}$ and, thus, PROPERTY (M3) is also satisfied. Hence, according to PROPOSITION 3.10, clause profile $\hat{g}$ is perfectly collusive.

Our next step is to demonstrate that $\hat{g}$ is robustly collusive. For this purpose, consider some arbitrary clause profile g being perfectly collusive at common discount factor δ. We infer from THEOREM 4.1 and above part (a) that $|C(g)| \geq |C(\hat{g})| = 2$. Therefore, it remains to show that if $|C(g)| = |C(\hat{g})|$ then $\delta_{\mathrm{crit}}^g \geq \delta_{\mathrm{crit}}^{\hat{g}}$ and $f_{\mathrm{crit}}^{g,\delta} \leq f_{\mathrm{crit}}^{\hat{g},\delta}$.

The former weak inequality has already been confirmed above. The latter inequality follows immediately from the fact that $\kappa_{n-1} \geq \kappa_j$ where j denotes the smallest retailer contained in doubleton $C(g)$. Hence, $\hat{g}$ proves to be robustly collusive and, in consequence, any robustly collusive clause profile has the characteristic that the two largest retailer are the only CC-adopting retailers.

(*"only if"*) Suppose $\hat{g}$ is robustly collusive at common discount factor δ. Obviously, any robustly collusive clause profile at common discount factor δ then has the property that the two largest retailer are the only CC-adopting retailers. We already know from THEOREM 4.1 and above part (a) that whenever $\delta \geq 1 - \kappa_n$, then $|C(g)| < 2$ for any robustly collusive clause profile g. As clause profile $\hat{g}$ is assumed to be robustly collusive, it holds $\delta < 1 - \kappa_n$. Note, by assumption, $\hat{g}$ is perfectly collusive at common discount factor δ. According to PROPOSITION 3.10, it then satisfies PROPERTY (M3). This in turn implies $f \leq f_{\mathrm{crit}}^{\hat{g},\delta} = \frac{1}{1-\delta}\kappa_{n-1}\pi^{\mathrm{m}}$. □

Proof of Corollary 5.2. This corollary results from THEOREMS 4.1 and 5.1. Let us consider competition games $\Gamma(\delta, f, n, z)$ where $f \leq \kappa_{n-1}p^{\mathrm{m}}$ and $z \leq \bar{z}_1^0$. Moreover, we assume that $k_{n-1} \geq D(c)$, i.e., at least the two largest retailer are dominant. If $\delta \geq \delta_{\mathrm{crit}}$, then THEOREM 4.1 ensures that the trivial clause profile w is the only element of solution set $\mathcal{S}^{\mathrm{r}}(\delta, f, n, z)$. If $1 - \kappa_n \leq \delta < \delta_{\mathrm{crit}}$, then THEOREM 5.1(a) guarantees that clause profile $(w_{-n}, b_n^{€,p^{\mathrm{m}}-c})$ belongs to $\mathcal{S}^{\mathrm{r}}(\delta, f, n, z)$. If $\delta < 1 - \kappa_n$, then we know from THEOREM 5.1(b) that clause profile $(w_{J_{n-2}}, b_{n-1}^{€,p^{\mathrm{m}}-c}, b_{n-1}^{€,p^{\mathrm{m}}-c})$ belongs to $\mathcal{S}^{\mathrm{r}}(\delta, f, n, z)$. To sum up, regardless of the value of the common discount factor, solution set $\mathcal{S}^{\mathrm{r}}(\delta, f, n, z)$ always contains a conventional clause profile in which at most the two largest retailers adopt CCs. □

Proof of Proposition 6.1. Let g be some conventional and perfectly collusive clause profile in $\Gamma_{\mathrm{r}}(\delta, f, n, z)$. We denote the extreme upshift permutation on $C(g)$ by υ. As $\delta < \delta_{\mathrm{crit}}$, we conclude from THEOREM 4.1 that $0 < \ell := |C(g)| < n$. The proposition will be proved by induction on ℓ.

Suppose $\ell = 1$. In this case, it holds $C(g) = \{j\}$ for some $1 \leq j < n$. Define $\tilde{g} := g \diamond \upsilon$. We obtain $C(\tilde{g}) = \{n\}$ so that $\min C(\tilde{g}) = n + 1 - |C(\hat{g})|$. Moreover, it holds $f_{\mathrm{crit}}^{\tilde{g},\delta} > f_{\mathrm{crit}}^{g,\delta}$. As υ is upshifting on $C(g)$, we conclude from LEMMA 4.3 that $\delta_{\mathrm{crit}}^{\hat{g}} \leq \delta_{\mathrm{crit}}^g$.

Note that g as a perfectly clause profile satisfies PROPERTIES (M1) and (M3) due to PROPOSITION 3.10. We also know that $f^{\tilde{g},\delta}_{\text{crit}} > f^{g,\delta}_{\text{crit}}$ and $\delta^{\tilde{g}}_{\text{crit}} \leq \delta^{g}_{\text{crit}}$. These results ensure that $\tilde{g}$ satisfies PROPERTIES (M1) and (M3). As $|C(\tilde{g})| = 1$ and $\delta < \delta_{\text{crit}}$, it also satisfies PROPERTY (M2). We infer from PROPOSITION 3.10 that $\tilde{g}$ is perfectly collusive. Indeed, as $|C(\tilde{g})| = |C(g)|$ as well as $f^{\tilde{g},\delta}_{\text{crit}} > f^{g,\delta}_{\text{crit}}$ and $\delta^{\tilde{g}}_{\text{crit}} \leq \delta^{g}_{\text{crit}}$, clause profile $\tilde{g}$ proves to be more collusive than clause profile g.

Let us turn to the case $1 < \ell < n$ and suppose that our claim holds for any conventional and perfectly collusive clause profile in which at least one, but less than ℓ retailers adopt CCs. Define $\tilde{g} := g \diamond v$. By construction, we observe $|C(\tilde{g})| = |C(g)|$, but $\min C(\tilde{g}) = n+1-|C(\tilde{g})| > \min C(g)$. It follows $f^{\tilde{g},\delta}_{\text{crit}} > f^{g,\delta}_{\text{crit}}$. Moreover, LEMMA 4.3 ensures $\delta^{\tilde{g}}_{\text{crit}} \leq \delta^{g}_{\text{crit}}$. Note that g satisfies PROPERTIES (M1) and (M3) due to PROPOSITION 3.10. Hence, $\tilde{g}$ fulfills both properties, too. If $\tilde{g}$ is perfectly collusive, it is more collusive than g and the proof is completed. Otherwise, PROPOSITION 3.11 guarantees that there is some conventional and perfectly collusive clause profile $\hat{g}$ satisfying $C(\hat{g}) \subset C(\tilde{g})$. As $|C(\hat{g})| < |C(g)|$, it is more collusive than g. If $\min C(\hat{g}) = n+1-|C(\hat{g})|$, our claim follows immediately. Otherwise, as $1 \leq |C(\hat{g})| < \ell$, the induction premise ensures the claim. □

Proof of Corollary 6.2. Let us consider some conventional and perfectly collusive clause profile g in $\Gamma_{\text{r}}(\delta, f, n, z)$. As $\delta < \delta_{\text{crit}}$ is assumed, THEOREM 4.1 implies $C(g) \neq \emptyset$. We prove the claim of this theorem indirectly. Suppose there exists some $j \notin C(g)$ so that $j > i := \min C(g)$. We already know from PROPOSITION 6.1 that there exists a conventional and perfectly collusive clause profile $\hat{g}$ being more collusive than g and satisfying $\min C(\hat{g}) = n+1-|C(\hat{g})|$. That means, g is not the most collusive among the conventional and perfectly collusive clause profiles. Hence, any clause profile $\hat{g}$ belonging to the most collusive among those clause profiles has the characteristic that there exists a $k \in I$ so that $C(\hat{g}) = \{k, k+1, \ldots, n\}$. Due to the cost-efficiency criterion, k has to be identical for any of them. □

Proof of Remark 6.3.

(a) Consider some competition game $\Gamma_{\text{r}}(\delta, f, n, z)$ and some coalitions $J \subseteq \tilde{J} \subseteq I$. If $z \geq p^{\text{m}} - c$, then $\hat{\delta}^{J}_{\text{crit}} = \hat{\delta}^{\tilde{J}}_{\text{crit}} = \delta_{\text{crit}}$ due to REMARK 4.2. Therefore, our claim is immediately satisfied in this case. From now on, we take for granted that $z < p^{\text{m}} - c$. Let us define $g := (b^{€,p^{\text{m}}-c}_{J}, w_{-J})$ and $\tilde{g} := (b^{€,p^{\text{m}}-c}_{\tilde{J}}, w_{-\tilde{J}})$. Moreover, we specify $\ell := |\tilde{J} \setminus J|$. Our claim $\hat{\delta}^{\tilde{J}}_{\text{crit}} \leq \hat{\delta}^{J}_{\text{crit}}$ is proved by induction on ℓ.

Obviously if $\ell = 0$, i.e., $\tilde{J} = J$, then the claim follows immediately. Next, consider the case $\ell = 1$ and denote the retailer belonging to $\tilde{J} \setminus J$ by i. Obviously, we then have $\tilde{g} := (b^{€,p^{\text{m}}-c}_{i}, g_{-i})$. In the following, it will be demonstrated that $\delta^{\tilde{g}}_{k,\text{crit}} \leq \delta^{g}_{k,\text{crit}}$ for any $k \in I$. Apparently, this result would verify the claim $\delta^{\tilde{g}}_{\text{crit}} \leq \delta^{g}_{\text{crit}}$.

Let us begin by considering some retailer $j \neq i$ and let $q := (q_j, p^{\text{m}}_{-j})$ be the profile of the advertised prices. We obtain the following results:

- If $q_j > p^{\text{m}}$, we observe $\tilde{g}^{\text{s}}_j(q) = g^{\text{s}}_j(q) \leq q_j$ and $\tilde{g}^{\text{p}}_j(q) = g^{\text{p}}_j(q) \leq q_j$ due to the constructions of $\tilde{g}$ and g. Besides, they imply $\tilde{g}^{\text{p}}_k(q) = g^{\text{p}}_k(q) = p^{\text{m}}$ for any retailer $k \neq j$. Hence, it holds $\pi^{\tilde{g}}_j(q) = \pi^{g}_j(q)$.
- If $c < q_j < p^{\text{m}}$, we observe $\tilde{g}^{\text{s}}_j(q) = g^{\text{s}}_j(q) = q_j$ and $\tilde{g}^{\text{p}}_j(q) = g^{\text{p}}_j(q) = q_j$. Besides, the constructions of $\tilde{g}$ and g entail $c + z \leq \tilde{g}^{\text{p}}_k(q) = g^{\text{p}}_k(q)$ for any retailer $k \neq i, j$. We also observe $c = \tilde{g}_i(q) < g_i(q) = p^{\text{m}}$ so that $c + z = \tilde{g}^{\text{p}}_i(q) \leq g^{\text{p}}_i(q) = p^{\text{m}}$. These results imply $K_{[\tilde{g}^{\text{p}}(q)<q_j]} \geq K_{[g^{\text{p}}(q)<q_j]}$ and $K_{[\tilde{g}^{\text{p}}(q)=q_j]} + K_{[\tilde{g}^{\text{p}}(q)<q_j]} - K_{[g^{\text{p}}(q)<q_j]} \geq K_{[g^{\text{p}}(q)=q_j]}$. It will be shown next that $X_j(\tilde{g}^{\text{p}}(q)) \leq X_j(g^{\text{p}}(q))$. As this claim is trivially satisfied for $X_j(\tilde{g}^{\text{p}}(q)) = 0$, we take $X_j(\tilde{g}^{\text{p}}(q)) > 0$ for

granted from now on. Due to efficient rationing, it holds

$$\begin{aligned}
X_j(\tilde{g}^{\mathrm{p}}(q)) &= k_j \min\left\{1, \frac{D(q_j) - K_{[\tilde{g}^{\mathrm{p}}(q)<q_j]}}{K_{[\tilde{g}^{\mathrm{p}}(q)=q_j]}}\right\} \\
&\le k_j \min\left\{1, \frac{D(q_j) - K_{[\tilde{g}^{\mathrm{p}}(q)<q_j]} + K_{[\tilde{g}^{\mathrm{p}}(q)<q_j]} - K_{[g^{\mathrm{p}}(q)<q_j]}}{K_{[\tilde{g}^{\mathrm{p}}(q)=q_j]} + K_{[\tilde{g}^{\mathrm{p}}(q)<q_j]} - K_{[g^{\mathrm{p}}(q)<q_j]}}\right\} \\
&\le k_j \min\left\{1, \frac{D(q_j) - K_{[g^{\mathrm{p}}(q)<q_j]}}{K_{[g^{\mathrm{p}}(q)=q_j]}}\right\} \\
&= X_j(g^{\mathrm{p}}(q)).
\end{aligned}$$

It follows from $\tilde{g}^{\mathrm{s}}_j(q) = g^{\mathrm{s}}_j(q)$ and $X_j(\tilde{g}^{\mathrm{p}}(q)) \le X_j(g^{\mathrm{p}}(q))$ that $\pi^{\tilde{g}}_j(q) \le \pi^g_j(q)$.

Summing up, it has been established that $\pi^{\tilde{g}}_j(q) \le \pi^g_j(q)$ for any $q := (q_j, p^{\mathrm{m}}_{-j})$ where $c < q_j \ne p^{\mathrm{m}}$. This result along with Remark 3.1(a) ensures $\pi^{\updownarrow}_j(\tilde{g}) \le \pi^{\updownarrow}_j(g)$. Hence, we obtain $\delta^{\tilde{g}}_{j,\mathrm{crit}} \le \delta^g_{j,\mathrm{crit}}$.

Let us turn to retailer i and let $q := (q_i, p^{\mathrm{m}}_{-i})$ be the profile of the advertised prices. We obtain the following results:

- If $q_i > p^{\mathrm{m}}$, then $c = \tilde{g}_i(q) < g_i(q) = q_i$ by construction. Hence, it holds $\tilde{g}^{\mathrm{s}}_i(q) = c$ or $\tilde{g}^{\mathrm{p}}_i(q) = q_i$. Moreover, we observe $\tilde{g}^{\mathrm{p}}_k(q) = g^{\mathrm{p}}_k(q) = p^{\mathrm{m}}$ for any $k \ne i$. It follows from Assumptions (D2) and (D4) that $0 = \pi^{\tilde{g}}_i(q) = \pi^g_i(q)$.
- If $c < q_i < p^{\mathrm{m}}$, then we observe $\tilde{g}^{\mathrm{s}}_i(q) = g^{\mathrm{s}}_i(q) = q_i$ and $\tilde{g}^{\mathrm{p}}_i(q) = g^{\mathrm{p}}_i(q) = q_i$ as well as $\tilde{g}^{\mathrm{p}}_k(q) = g^{\mathrm{p}}_k(q)$ for any retailer $k \ne i$. It follows immediately from these equalities that $\pi^{\tilde{g}}_i(q) = \pi^g_i(q)$.

The above calculations and Remark 3.1(a) imply $\pi^{\updownarrow}_i(\tilde{g}) = \pi^{\updownarrow}_i(g)$. This in turn entails $\delta^{\tilde{g}}_{i,\mathrm{crit}} = \delta^g_{i,\mathrm{crit}}$. Summing up, we have shown that $\delta^{\tilde{g}}_{k,\mathrm{crit}} \le \delta^g_{k,\mathrm{crit}}$ for any $k \in I$ and, thus, $\delta^{\tilde{g}}_{\mathrm{crit}} \le \delta^g_{\mathrm{crit}}$.

To complete the proof, let us now examine the case $\ell > 1$ and suppose that our claim is satisfied for any coalition J' of retailers where $J \subseteq J'$ and $0 \le |J' \setminus J| < \ell$. Let us pick some $i \in J$ and define $J' := \tilde{J} \setminus \{i\}$ as well as $g' := (b^{€,p^{\mathrm{m}}-c}_{J'}, w_{-J'})$. Obviously, it holds $|J' \setminus J| = \ell - 1$. Our induction premise ensures $\hat{\delta}^{J'}_{\mathrm{crit}} \le \hat{\delta}^{J}_{\mathrm{crit}}$. As $\tilde{g} := (b^{€,p^{\mathrm{m}}-c}_i, g'_{-i})$ and $|\tilde{J} \setminus J'| = 1$, our induction premise also implies $\delta^{\tilde{g}}_{\mathrm{crit}} \le \delta^{\hat{g}}_{\mathrm{crit}}$ and, thus, $\delta^{\tilde{g}}_{\mathrm{crit}} \le \delta^g_{\mathrm{crit}}$.

(b) Consider some non-trivial clause profile $g := (b^{€,p^{\mathrm{m}}-c}_J, w_{-J})$ where $\delta^g_{j,\mathrm{crit}} \le \delta$ is satisfied for $j := \max J$. Our aim is to prove $\delta^g_{\mathrm{crit}} \le \delta$. For this purpose, we pick some arbitrary retailer $i \ne j$ and suppose that $q := (q_i, p^{\mathrm{m}}_{-i})$ is the profile of the advertised prices. We note that as $z \le \bar{z}^{\delta}_1$ and $\delta < \delta_{\mathrm{crit}}$, it holds $z < p^{\mathrm{m}} - c$. Hence, we obtain:

- If $q_i > p^{\mathrm{m}}$, we observe $q^{\mathrm{s}}_i = c$ due to our construction of g. It follows from Assumptions (D2) and (D4) that $\pi^g_i(q) = 0 \le \frac{1}{1-\delta}\kappa_i p^{\mathrm{m}}$.
- If $c + z < q_i < p^{\mathrm{m}}$, we observe $q^{\mathrm{s}}_i = q^{\mathrm{p}}_i = q_i$. Besides, the construction of g entails $q^{\mathrm{p}}_k = p^{\mathrm{m}}$ for any $k \notin J$ and $q^{\mathrm{p}}_k = c + z$ for any $k \in J \setminus \{i\}$. Note that $\pi^g_i(q) = 0 \le \frac{1}{1-\delta}\kappa_i p^{\mathrm{m}}$ results immediately from $X_i(g^{\mathrm{p}}(q)) = 0$. For this reason, we take $X_i(g^{\mathrm{p}}(q)) > 0$ for granted from now on. It then holds $0 < R_{\mathrm{e}}(q^{\mathrm{p}}_i | q^{\mathrm{p}}) = D(q_i) - K_{J\setminus\{i\}} \le D(q_i) - K_{J\setminus\{j\}} - k_j + k_i$.

  As usual, let $\tau_{i,j}$ be the transposition swapping i and j. We define $\tilde{q} := q \circ \tau_{i,j}$. It holds $\tilde{q}^{\mathrm{s}}_j = \tilde{q}^{\mathrm{p}}_j = \tilde{q}_j = q_i$ due to the construction of g. Moreover, we observe $\tilde{q}^{\mathrm{p}}_k = p^{\mathrm{m}}$ for any $k \notin J$ and $\tilde{q}^{\mathrm{p}}_k = c + z$ for any $k \in J \setminus \{j\}$. Hence, it holds $0 < R_{\mathrm{e}}(\tilde{q}^{\mathrm{p}}_j | \tilde{q}^{\mathrm{p}}) = D(q_i) - K_{J\setminus\{j\}}$ and, thus, $R_{\mathrm{e}}(q^{\mathrm{p}}_i | q^{\mathrm{p}}) \le R_{\mathrm{e}}(\tilde{q}^{\mathrm{p}}_j | \tilde{q}^{\mathrm{p}}_j) - k_j + k_i$. As $\frac{k_i}{K_{[q^{\mathrm{p}} = q^{\mathrm{p}}_i]}} = 1 = \frac{k_j}{K_{[\tilde{q}^{\mathrm{p}} = \tilde{q}^{\mathrm{p}}_j]}}$, we obtain,

$$\begin{aligned}
X_i(q^{\mathrm{p}}) &= \min\{k_i, R_{\mathrm{e}}\left(q^{\mathrm{p}}_i | q^{\mathrm{p}}\right)\} \\
&\le \min\{k_j, R_{\mathrm{e}}\left(\tilde{q}^{\mathrm{p}}_j | \tilde{q}^{\mathrm{p}}\right))\} - k_j + k_i \\
&= X_j(\tilde{q}^{\mathrm{p}}) - k_j + k_i.
\end{aligned}$$

Note that $X_j(\tilde{q}^{\mathrm{p}}) \leq k_j$. Therefore, it holds $X_j(\tilde{q}^{\mathrm{p}}) - k_j + k_i \leq X_j(\tilde{q}^{\mathrm{p}}) - \frac{\kappa_j - \kappa_i}{\kappa_j} X_j(\tilde{q}^{\mathrm{p}})$. This in turn implies $X_i(q^{\mathrm{p}}) \leq \frac{\kappa_i}{\kappa_j} X_j(\tilde{q}^{\mathrm{p}})$. As $q_i^{\mathrm{s}} = q_i = \tilde{q}_j = \tilde{q}_j^{\mathrm{s}}$, we obtain $\pi_i^g(q) \leq \frac{\kappa_i}{\kappa_j} \pi_j^g(\tilde{q})$. In consequence, it holds

$$\begin{aligned} \pi_i^g(q_i, p_{-i}^{\mathrm{m}}) &\leq \frac{\kappa_i}{\kappa_j} \sup_{c+z<q_j<p^{\mathrm{m}}} \pi_j^g(q_j, p_{-j}^{\mathrm{m}}) \\ &\leq \frac{\kappa_i}{\kappa_j} \frac{1}{1-\delta_{j,\mathrm{crit}}^g} \kappa_j \pi^{\mathrm{m}} \\ &= \frac{1}{1-\delta_{j,\mathrm{crit}}^g} \kappa_i \pi^{\mathrm{m}} \ . \end{aligned}$$

As $\delta_{j,\mathrm{crit}}^g \leq \delta$, we observe $\pi_i^g(q) \leq \frac{1}{1-\delta} \kappa_i \pi^{\mathrm{m}}$.

- If $c < q_i \leq c + z$, we observe $q_i^{\mathrm{s}} = q_i^{\mathrm{p}} = q_i$. Hence, it holds $X_i(q^{\mathrm{p}}) \leq \min\{k_i, D(q_i)\}$. We conclude from Assumption (D3) that $\pi_i^g(q) \leq \bar{\pi}_i(q_i) \leq \bar{\pi}_i(c+z)$. As $z \leq \bar{z}_1^\delta$, we obtain $z \leq \bar{z}_i^\delta$ due to Remark 3.8. Therefore, it holds $\bar{\pi}_i(c+z) \leq \frac{1}{1-\delta}\kappa_i \pi^{\mathrm{m}}$. We conclude from this that $\pi_i^g(q) \leq \frac{1}{1-\delta}\kappa_i \pi^{\mathrm{m}}$.

Resorting to the above results and Remark 3.1(a), we obtain $\pi_i^{\updownarrow}(g) \leq \frac{1}{1-\delta}\kappa_i \pi^{\mathrm{m}}$. This implies $\delta_{i,\mathrm{crit}}^g \leq \delta$ for any $i \neq j$. As $\delta_{j,\mathrm{crit}}^g \leq \delta$ is assumed, it holds $\delta_{\mathrm{crit}}^g \leq \delta$. □

Proof of Theorem 6.4.

(*"if"*) Suppose there is a $k \in I_0 \setminus \{n\}$ so that clause profile $\hat{g} := (w_{J_k}, b_{-J_k}^{€,p^{\mathrm{m}}-c})$ satisfies conditions (*i*) to (*iii*). We note that $f_{\mathrm{crit}}^{\hat{g},\delta} = \frac{1}{1-\delta}\kappa_{k+1}\pi^{\mathrm{m}}$. Therefore, condition (*iii*) is nothing but Property (M3). Moreover, Property (M2) follows immediately from condition (*ii*). Applying Remark 6.3(b) to condition (*i*), we obtain Property (M1). It follows from Proposition 3.10 that $\hat{g}$ is perfectly collusive.

To prove that $\hat{g}$ is even robustly collusive, we proceed as follows. First, we will show that any clause profile $g \in G$ satisfying $|C(g)| < |C(\hat{g})|$ is not perfectly collusive. Afterwards, it will be demonstrated that any perfectly collusive clause profile $g \in G$ satisfying $|C(g)| = |C(\hat{g})|$ has the properties $\delta_{\mathrm{crit}}^g \geq \delta_{\mathrm{crit}}^{\hat{g}}$ and $f_{\mathrm{crit}}^{g,\delta} \leq f_{\mathrm{crit}}^{\hat{g},\delta}$.

Consider some $g \in G$ satisfying $|C(g)| < |C(\hat{g})|$. If $|C(g)| = 0$, we resort to Theorem 4.1 and conclude that g is not perfectly collusive. Suppose $|C(g)| \geq 1$ from now on. Let $J' := C(g)$ and define $g' := (b_{J'}^{€,p^{\mathrm{m}}-c}, w_{-J'})$. We know from Remark 4.2 that $\delta_{\mathrm{crit}}^{g'} \leq \delta_{\mathrm{crit}}^g$. Let υ the extreme upshift permutation on J'. We specify $g'' := g' \diamond \upsilon$ and $J'' := \upsilon(J')$. It holds $\delta_{\mathrm{crit}}^{g''} \leq \delta_{\mathrm{crit}}^{g'}$ due to Lemma 4.3. Finally, we define $\tilde{g} := (w_{J_{k+1}}, b_{-J_{k+1}}^{€,p^{\mathrm{m}}-c})$ and $\tilde{J} := I \setminus (J_{k+1} \cup J'')$. Obviously, $\tilde{g} = (b_{\tilde{J}}^{€,p^{\mathrm{m}}-c}, g''_{-\tilde{J}})$. It follows from Remark 6.3(a) that $\delta_{\mathrm{crit}}^{\tilde{g}} \leq \delta_{\mathrm{crit}}^{g''}$. Moreover, condition (*ii*) implies $\delta < \delta_{\mathrm{crit}}^{\tilde{g}}$ and, thus, $\delta < \delta_{\mathrm{crit}}^g$. That means, g violates Property (M1). Hence, according to Proposition 3.10, it is not perfectly collusive.

Consider now some perfectly collusive clause profile $g \in G$ satisfying $|C(g)| = |C(\hat{g})|$. Obviously, it holds $f_{\mathrm{crit}}^{g,\delta} \leq f_{\mathrm{crit}}^{\hat{g},\delta}$. We define $g' := (b_{C(g)}^{€,p^{\mathrm{m}}-c}, w_{-C(g)})$. It follows $\delta_{\mathrm{crit}}^{g'} \leq \delta_{\mathrm{crit}}^g$ from Remark 4.2. Let υ the extreme upshift permutation on $C(g)$. Obviously, $\hat{g} = g' \diamond \upsilon$. Resorting to Lemma 4.3, we obtain $\delta_{\mathrm{crit}}^{\hat{g}} \leq \delta_{\mathrm{crit}}^{g'}$ and, thus, $\delta_{\mathrm{crit}}^{\hat{g}} \leq \delta_{\mathrm{crit}}^g$.

(*"only if"*) Suppose there exists some $k \in I_0 \setminus \{n\}$ so that clause profile $\hat{g} := (w_{J_k}, b_{-J_k}^{€,p^{\mathrm{m}}-c})$ is robustly collusive. As $\hat{g}$ is perfectly collusive, Properties (M1) - (M3) are satisfied due to Proposition 3.10. We note that $f_{\mathrm{crit}}^{\hat{g},\delta} = \frac{1}{1-\delta}\kappa_{k+1}\pi^{\mathrm{m}}$. Hence, condition (*iii*) is nothing but Property (M3). Due to Property (M2), there is some retailer $i \in C(\hat{g})$ so that $g := (w_i, \hat{g}_{-i})$ satisfies $\delta < \delta_{\mathrm{crit}}^g$. This inequality along with Remark 6.3(b) entails that condition (*ii*) is also satisfied. It remains to verify condition (*i*). This condition follows immediately from Property (M1). □

Proof of Corollary 6.5. Let us consider some competition game $\Gamma_{\mathrm{e}}(\delta, f, n, z)$ where $f \leq \kappa_1 \pi^{\mathrm{m}}$ and $z \leq \bar{z}_1^{\delta}$. Moreover, we define clause profile $g^{[k]} := (w_{J_k}, b_{-J_k}^{€, p^{\mathrm{m}}-c})$ for any $k \in I_0$. Note that $g^{[k]}$ represents the conventional clause profile in which the $n-k$ largest retailers as the only CC-adopting retailers offer the BCC with lump sum refund $p^{\mathrm{m}} - c$. Obviously, we have $g^{[0]} = b^{€, p^{\mathrm{m}}-c}$. As can be easily checked, it holds $\delta_{\mathrm{crit}}^{g^{[0]}} = 0$. This in turn ensures the non-emptiness of $\{k \in I_0 : \delta_{\mathrm{crit}}^{g^{[k]}} \leq \delta\}$ and, thus, $\hat{k} := \max\{k \in I_0 : \delta_{\mathrm{crit}}^{g^{[k]}} \leq \delta\}$ proves to be well-defined.

We specify $\hat{g} := g^{[\hat{k}]}$. If $\hat{k} = n$, then Theorem 4.1 immediately entails that clause profile $\hat{g} = w$ is robustly collusive. If $\hat{k} < n$, then condition (i) of Theorem 6.4 results from the construction of $\hat{g}$. We also note that $\delta_{\mathrm{crit}}^{g^{[k+1]}} > \delta$. By applying Remark 6.3(b) to this inequality, we obtain condition (ii). Moreover, condition (iii) is satisfied as $f \leq \kappa_1 \pi^{\mathrm{m}} \leq \frac{1}{1-\delta} \kappa_{\hat{k}+1} \pi^{\mathrm{m}}$. Due to these results, Theorem 6.4 is applicable. We conclude that clause profile $\hat{g}$ is robustly collusive. Summing up, we have shown that solution set $\mathcal{S}^r(\delta, f, n, z)$ contains a conventional business policy profile for any common discount factor as long as the implementation and hassle costs are sufficiently small. □

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University of Hohenheim
Dean’s Office of the Faculty of Business, Economics and Social Sciences
Palace Hohenheim 1 B
70593 Stuttgart | Germany
Fon +49 (0)711 459 22488
Fax +49 (0)711 459 22785
wiso@uni-hohenheim.de
wiso.uni-hohenheim.de