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Essays on Demographic Change and R&D-based Economic Growth

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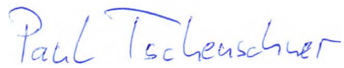

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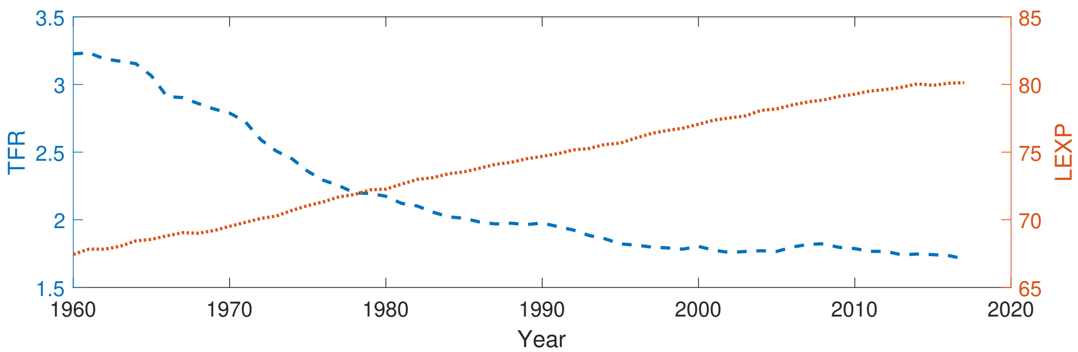
CHAPTER 1

General Introduction

Ageing is widely seen as one of the most significant risks to global prosperity in the decades ahead because of its potentially profound economic, social and political implications.
(Klaus Schwaab, World Economic Forum, 2012)

This statement of Klaus Schwaab, the founder and president of the World Economic Forum, provides an idea about the importance of population aging for today's society. Population aging is the change in the age structure, often referred to as demographic change, which has especially been observed in western countries and which is associated with great challenges that need to be addressed. The two main drivers of population aging are the increase in longevity and the decrease in fertility. Both effects combined result in an increase of a population's average age (United Nations, 2019). Figure 1.1 shows the aggregate developments of the total fertility rate (TFR) and life expectancy at birth (LEXP) over all OECD countries since 1960.

Figure 1.1: Average TFR and LEXP over all OECD countries, 1960–2017



Source: World Bank (2019)

It is apparent that the two described developments have taken place simultaneously. Fertility (dashed blue line) has decreased from 3.2 in 1960 to 1.7 in 2017, while life expectancy (dotted red line) has increased from 67 to 80 years over the same time horizon, resulting in a significant change in the populations' age structures.

Among others, potential macroeconomic concerns are higher tax rates to maintain the current pay-as-you-go pension systems, less innovation and, in the long run, a negative effect on economic growth and economic prosperity (Bloom et al., 2015). While some of these concerns are undoubtedly justified, there exist channels through which demographic change can exert a positive effect on economic growth. Most importantly, for an increasing lifetime horizon, savings increase in order to be able to afford old-age consumption (Futagami and Nakajima, 2001; Bloom et al., 2003, 2010). Furthermore, longer expected working lives raise the incentives to invest in education, which promotes economic growth (Cervellati and Sunde, 2013; Strulik and Werner, 2016). The consequent reduction in fertility (Becker and Lewis, 1973; Boucekkine et al., 2002) can exhibit an additional positive growth impulse through higher labor force participation (Bloom et al., 2009).

Plenty of research has been conducted on how demography affects economic growth in the medium run. Just to mention a few contributions, Bloom et al. (2010) as well as Lee and Mason (2010) investigate the effects of falling support ratios on the production side of an economy. Heijdra (2009) and İmrohoroğlu and Kitao (2012) analyze how social security contributions and the pension system are affected by changes in the propensity to save. In general, the changing savings behavior over the life-cycle is a big issue. Among others, Bloom et al. (2007), d'Albis (2007) and Krueger and Ludwig (2007) cover this issue and find mixed results. Many other important contributions about this heavily discussed debate could be added. All of them consider medium-run effects and are based on neoclassical models of economic growth such as the Solow (1956) model, the Cass (1965) and Koopmans (1965) model and the Diamond (1965) model. While these contributions provide important economic insights, they are missing a crucial point: long-run economic growth in industrialized countries is not determined by capital accumulation but by endogenously driven research and development. The mentioned frameworks might, therefore, not be able to accurately explain the underlying mechanisms of how demography affects long-run economic growth and the well-being of individuals or, at best, miss important channels.

Economic research has picked up on that issue by relating to the workhorse models of endogenous growth with horizontal innovations (Romer, 1990) and vertical innovations (Grossman and Helpman, 1991; Aghion and Howitt, 1992) and semi-endogenous frameworks with horizontal innovations (Jones, 1995) and vertical innovations (Segerström, 1998). While these growth models imply a positive relationship between the population size, respectively the population growth rate, and economic growth, the empirical literature finds a negative relationship for most industrialized economies (Brander and Dowrick, 1994; Ahituv, 2001; Li and Zhang, 2007). Therefore, a substantial part of this literature has focused on integrating longevity effects as in Blanchard (1985), Heijdra and Romp (2009) and Dalgaard and Strulik (2011) and a quantity-quality tradeoff in the spirit of Becker and Lewis (1973) and Galor and Weil (2000). To pick out two contributions, Strulik et al. (2013) model educational decisions of parents and their effects on fertility and human capital accumulation in an endogenous growth framework in the spirit of Romer (1990) in the very long run by relying on the unified growth theory kicked-off by Galor and Weil (2000). Prettner (2013) introduces lifetime uncertainty and fertility by combining an overlapping generations model with a model of endogenous growth. He shows that, while a decrease in fertility can have a negative effect on long-run economic growth, the effects of increasing longevity can even be large enough to outweigh this negative effect.

Understanding better the effects of population aging, separating them into different channels and being able to identify potential policy measures to, at least, counteract the negative effects of demographic change is one of the most crucial economic and political challenges ahead. This dissertation contributes to the debate by extending the existing literature on general equilibrium models of economic growth by including more realistic demographic structures to enhance the general understanding of how population aging affects individual behavior and long-run economic growth. More specifically, the main questions under consideration are:

- How do changes in longevity impact on economic growth through changes in the saving behavior?
- What are the growth effects of endogenous fertility decisions and how does fertility interact with increasing longevity?
- What are the underlying effects of demographic change on economic growth in the very long run?
- What are the welfare implications of longer lifetime horizons?

This dissertation contains three papers that evolve around the aforementioned research questions. In Chapter 2, the first paper titled "Longevity-induced vertical innovation and the tradeoff between life and growth"[1] introduces an exogenous survival probability into a three period overlapping generations framework. Economic growth is driven by quality improving innovations in the vein of Aghion and Howitt (1992). In the existing literature on endogenous vertical innovations, infinitely lived representative agents are assumed. The chosen approach is novel as it introduces a finite lifetime horizon and allows to analyze the growth effects of changing survival probabilities. The channel through which life expectancy exerts growth effects is the saving decision of households. A higher probability to survive until retirement increases aggregate savings which places downward pressure on the market interest rate. As a consequence, the net present value of holding a patent increases, which leads to higher R&D employment, a higher probability to innovate and faster economic progress. The relationship between the survival probability and long-run economic growth is shown to be strictly concave and decreasing in the elasticity of intertemporal substitution, i.e., in the willingness of individuals to substitute consumption over time. Additionally, the welfare effects of increasing longevity are analyzed. As a consequence of a higher life expectancy, productivity growth and, with it, consumption increase, which positively affects individual utility. At the same time, the direct effect of living longer also raises individual utility. The two channels are disentangled and it is shown that the direct utility gain due to higher life expectancy increases with the development of an economy and is, usually, larger than the indirect effect of longevity-induced economic growth. This result strongly emphasizes to not only consider the growth effects of health care.

The discussion surrounding the growth and welfare effects of rising longevity is extended in Chapter 3 by the second paper titled "Endogenous Life Expectancy and R&D-based Economic Growth". Baldanzi et al. (2019) focuses on balanced growth path effects with an exogenous and constant survival probability and a constant population size. In Chapter 3, these restrictions are relaxed by introducing endogenous survival similar to

[1]This paper is joint work with Annarita Baldanzi and Klaus Prettner and has been published in the *Journal of Population Economics* as Baldanzi et al. (2019).

Chakraborty (2004) as well as a quantity-quality tradeoff as in Prettner and Werner (2016) into a growth framework of horizontal innovations á la Romer (1990). Life expectancy increases in the public resources devoted toward health and still entails a positive relationship with individual and aggregate savings. Additionally, individuals need to decide on the number of children to have and on the children's level of education. Accordingly, higher life expectancy induces parents to have less children and to work more to be able to afford higher savings for retirement. It is shown that, on the one hand, higher savings increase monopoly profits and, thus, employment and productivity in the R&D sector. On the other hand, the longevity-induced growth effect through the workforce is mostly negative, as the increase in labor force participation is hardly able to compensate for the decrease in fertility. In a comparative static exercise, the aggregate growth effects of endogenously increasing life expectancy are analyzed by calibrating the model economy to U.S. data over the time period 1960–2017. The findings suggest that 11.9% of U.S. output growth over the mentioned timeframe is due to increases in life expectancy. Another important finding is that the optimal size of the health care sector might lie beyond what is observed in industrialized countries, nowadays. Also, the size of the health care sector that would maximize life expectancy is found to be much larger than its growth maximizing size. These results shed a relatively positive light on the growth effects of rising longevity and are in line with the finding in Chapter 2 that, from a welfare perspective, also the direct gains in life expectancy need to be considered.

Chapter 4 takes a closer look at the determinants of economic growth over the very long run. More specifically, in the paper titled "The Scientific Revolution and Its Role in the Transition to Sustained Economic Growth"[2], basic scientific knowledge is embedded into a Unified Growth framework similar to Strulik et al. (2013). Thus, knowledge about the laws of nature, about the scientific method etc. is a necessary input in applied R&D and increases in the overall number of thinkers in the economy and in their education. Households face a quantity-quality tradeoff. Fertility decreases and education increases in income. However, educational investments are zero for low levels of development and

[2]This paper is joint work with Klaus Prettner.

only start to increase once income reaches a certain threshold. This is when the economy experiences the takeoff to sustained economic growth with decreasing fertility, increasing education, faster productivity growth and faster economic progress. It is shown that the timing of the takeoff crucially hinges on the rate at which scientific knowledge accumulates as well as on its availability in applied research. For a faster accumulation and a higher availability of basic scientific knowledge, applied scientists become more productive, which, eventually, results in a steeper economic takeoff. For low growth rates or poor availability of basic scientific knowledge, the takeoff occurs up to one generation later since it takes longer for R&D to become profitable. In the extreme case of zero basic scientific knowledge, e.g., if scientific achievements are suppressed due to religious believes, no takeoff might occur at all. The presented framework provides one possible explanation why some regions might have experienced the Industrial Revolution later than others and what the consequences for today's economic performances are.

Last, Chapter 5 summarizes the key findings of Chapters 2–4 and draws conclusions with respect to the research questions posed.

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CHAPTER 2

Longevity-induced Vertical Innovation and the Tradeoff Between Life and Growth*

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Abstract

We analyze the economic growth effects of rising longevity in a framework of endogenous growth driven by quality-improving innovations. A rise in longevity increases savings and thereby places downward pressure on the market interest rate. Since the monopoly profits generated by a successful innovation are discounted by the endogenous market interest rate, this raises the net present value of innovations, which, in turn, fosters R&D investments. The associated increase in the employment of scientists leads to faster technological progress and a higher long-run economic growth rate. From a welfare perspective, the direct effect of an increase in life expectancy tends to be larger than the indirect effect of the induced higher consumption due to faster economic growth. Consequently, the debate on rising health care expenditures should not be predominantly based on the growth effects of health care.

2.1 Introduction

At the global level, the total fertility rate has fallen from almost 5 children per woman in 1960 to 2.5 children per woman in 2015. Over the same period, average life expectancy has increased from 53 years to almost 72 years (World Bank, 2016). Falling fertility and rising life expectancy are the two main causes of the unprecedented population aging that industrialized countries started experiencing over the last decades (Bloom et al., 2010). Of these two forces, declining fertility has contributed more to population aging than rising life expectancy (Weil, 1997; Bloom and Luca, 2016).

While there are deep-rooted concerns on the negative economic consequences of population aging in the public debate (see, for example, The Economist, 2011a,b), a negative effect on economic growth is not yet visible in the data (Acemoglu and Restrepo, 2017). One of the reasons might be that increasing longevity has positive side effects that tend to raise economic growth. For example, if individuals expect a longer life, they save more in order to sustain their living standards in a prolonged retirement period (Futagami and Nakajima, 2001; Bloom et al., 2003, 2007, 2010). In addition, the longer expected working life that comes with rising longevity implies stronger incentives to invest in education, which raises labor productivity and thereby economic growth (Cervellati and Sunde, 2013; Ang and Madsen, 2015; Strulik and Werner, 2016).

From an empirical perspective, Acemoglu and Johnson (2007) and Hansen and Lønstrop (2015) find a negative effect of increasing life expectancy on economic growth. However, most of the empirical evidence points to the contrary. For example, Lorentzen et al. (2008), Aghion et al. (2011), and Bloom et al. (2014) show that higher life expectancy has been a driver of economic growth in the past. Cervellati and Sunde (2011) argue that the stage of development of a country is crucial in assessing the effects of increasing life expectancy on economic growth. In less developed countries, which did not yet experience the demographic transition, increasing life expectancy leads to faster population growth. This raises capital dilution and thereby tends to reduce economic growth. By contrast, in developed countries, which already experienced the demographic transition in the past, a rise in life expectancy reduces population growth and capital dilution and thereby exerts a positive effect on economic growth. Hansen and Lønstrop (2015) and Gehringer and Prettner (2019) corroborate this finding when focusing exclusively on developed economies.

We aim at contributing to this debate by proposing a theoretical mechanism by which increasing longevity affects economic growth in modern knowledge-based economies. In these economies, growth is driven by endogenous technological progress (Romer, 1990; Jones, 1995; Strulik et al., 2013) such that the analysis needs to be based on a model that takes into account the effects of increasing longevity on the incentives to innovate.[1] While there has been some progress in the analysis of the effects of increasing longevity on horizontal innovation, i.e., the introduction of new intermediate product varieties (see, for example, Prettner, 2013; Hashimoto and Tabata, 2016; Prettner and Trimborn, 2017; Futagami and Konishi, 2019), we are not aware of a comparable study that is based on vertical innovations, i.e., the quality-improvements of existing intermediate products. Closing this gap in the literature is important for two reasons: i) due to the different mechanism that generates growth in the vertical innovation framework as compared to the horizontal innovation framework, the pathway by which increasing longevity affects economic growth might be different; ii) the welfare implications of both frameworks could differ because there is a Schumpeterian creative destruction effect in the vertical innovation framework, where each new innovation drives the old incumbent out of business. This implies a potentially negative welfare effect of innovation that is not present in the framework of horizontal innovation. Thus, the long-run growth rate might be too high from a social point of view within the vertical innovation framework, a result that cannot occur within the horizontal innovation setting.

Endogenous and semi-endogenous growth models with vertical innovations (Grossman and Helpman, 1991; Aghion and Howitt, 1992; Segerström, 1998) are suited to analyze the economic growth effects of changing population size and changing population growth. The

[1] For the growth effects of demographic change in economies in which medium- and long-run growth are driven by physical capital accumulation – either according to the neoclassical growth model (Solow, 1956; Cass, 1965; Diamond, 1965) or according to an AK type of perpetual growth model (Romer, 1986) – see, for example, Chakraborty (2004), Heijdra and Romp (2006), Heijdra and Ligthart (2006), Heijdra and Mierau (2011), Sánchez-Romero (2013), Mierau and Turnovsky (2014a,b), and Sánchez-Romero et al. (2018).

basic mechanism in this framework is that research activities randomly lead to quality-improving innovations. The more researchers an economy employs and the more productive these researchers are, the higher is the probability that a new innovation occurs in a given time period. If a quality-improving innovation occurs, it raises the productivity of intermediate goods in producing the final consumption good. Thus, a higher probability of innovation raises long-run economic growth. In a recent extension of this model class, Chu et al. (2013) endogenize fertility and human capital accumulation and analyze the effects of changing patent policies. They find that strengthening patent protection tends to have a positive effect on long-run growth although it raises fertility and thereby reduces human capital accumulation.[2]

When it comes to the analysis of changing longevity, however, the described models of vertical innovation cannot be used directly because they rely on the notion of a representative agent. By definition, the representative agent lives forever, such that changes in life expectancy cannot be conceptualized. To be able to analyze the effects of increasing longevity in this setting, we introduce an overlapping generations structure into the model proposed by Aghion and Howitt (1992), which has been simplified subsequently by Aghion and Howitt (1999, 2005, 2009). In so doing, we assume a demographic structure with three phases in the individual life cycle: childhood, adulthood, and retirement. Individuals face an exogenously given survival probability from adulthood to retirement. Varying this parameter allows us to analyze the long-run growth effects of changing longevity and the mechanisms by which they unfold.

We show that, if longevity increases, the economy saves more at the aggregate level, which places downward pressure on the market interest rate. Since innovators discount the future expected profits of an innovation by the market interest rate, the net present value of an innovation and, thus, the incentive to come up with a quality-improving new idea rise. To increase the probability of a successful quality-improvement, R&D firms hire additional researchers, which raises the innovation rate and productivity growth.

[2]For other contributions that focus on various aspects of the protection of intellectual property, see, for example, Helpman (1993), Chu (2009), Iwaisako and Futagami (2013), Cozzi and Galli (2014), and Pan et al. (2018).

The presence of this channel in our extension is based on an important difference to the underlying model of Aghion and Howitt (1992). For simplicity, they assume a constant and exogenous interest rate. Since this assumption switches off the crucial mechanism by which increasing life expectancy could affect the incentives to innovate and thereby exert an influence on long-run economic growth, our extension of the model allows for gaining important additional insights with respect to the growth effects of demographic changes.

We apply our overlapping generations model of vertical innovation to analyze the welfare effects of increasing life expectancy by decomposing them into two separate effects. The direct effect is that higher life expectancy allows individuals to enjoy consumption over a longer expected time period. The indirect effect is the induced increase in the growth rate of output and consumption as described above. We show that the direct welfare effect tends to be higher than the indirect welfare effect. This result is consistent with the literature showing that an increase in investments in the health sector of an economy beyond the growth-maximizing point – such that additional resources channeled toward the health sector *reduce* economic growth – can be Pareto improving (Kuhn and Prettner, 2016). Furthermore, we show that the relative importance of the direct welfare effect increases with economic development. This is consistent with Hall and Jones (2007), who show that, as the economy develops, it is optimal to invest an ever larger share of aggregate income in better health.

The paper is organized as follows. In Section 2.2, we introduce the overlapping generations structure that we implement into a standard model of vertical innovation. In Section 2.3, we derive our main analytical results with respect to the growth effects of increasing life expectancy and present the numerical results with respect to the welfare decomposition into the direct effect and into the indirect effect. In Section 2.4, we summarize, draw an important lesson from a policy perspective, and discuss interesting questions for further research.

2.2 The model

2.2.1 Consumption side

Consider an overlapping generations economy populated by single-sex individuals living for three time periods: childhood, adulthood, and retirement. Childhood lasts for 20 years, adulthood for 40 years, and the phase of retirement can last for 40 years after which an individual dies for sure (cf. Ludwig and Vogel, 2010). As a consequence, the maximum achievable life span is 100 years. However, there is a survival probability from adulthood to retirement, which determines actual life expectancy. Children face no economic decisions and fulfill their consumption needs via parental expenditures. Adults consume, save for retirement, work for the wage rate w_t, and give birth to one child such that the cohort size of adults stays constant over time. We assume that parents give birth to children in the middle of the adulthood period, implying that children enter adulthood at the time when adults enter retirement. Finally, retirees consume out of the savings accumulated as adults (see Samuelson, 1958; Diamond, 1965, for the corresponding consumption-savings decision).

We conceptualize an adult's remaining lifetime utility (u_t) by a logarithmic utility function that ensures analytical tractability:

$$u_t = \ln(c_{1,t}) + \phi\beta \ln(c_{2,t+1}), \tag{2.1}$$

where $c_{1,t}$ is the consumption level of an adult at time t, $c_{2,t+1}$ refers to the consumption level in retirement at time $t+1$, $\beta \in (0,1)$ is the discount factor, and $\phi \in (0,1)$ is the survival probability between adulthood and retirement.[3]

Following Yaari (1965) and Blanchard (1985), there are perfect and fair annuity markets at which individuals insure themselves against the risk of dying with positive assets. The importance of annuity markets in industrialized countries is mainly due to public pension systems with mandatory annuitization (cf. Sheshinski, 2008; Heijdra and Mierau,

[3]For similar treatments of the survival probability in the overlapping generations literature, see, for example, Blackburn and Cipriani (2002), Chakraborty (2004), and Zhang and Zhang (2005).

2012; Schneider and Winkler, 2016; Kuhn and Prettner, 2018). Hosseini (2015) shows, for example, that approximately half of retirement wealth in the United States is held in the form of annuities. At the end of the first period, every individual deposits her savings with a mutual fund. This fund buys assets in the form of shares of firms yielding a gross rate of return of $(1 + r_{t+1})/\phi$, where r_{t+1} is the real rate of return on savings that is equivalent to the dividend yield plus the valuation gain of the investment and where the denominator accounts for the redistribution of assets by the life insurance company from those who died among those who survived.

As a consequence of this setting, the budget constraints of adults and retirees are given by

$$c_{1,t} + s_t = w_t, \tag{2.2}$$

$$c_{2,t+1} = \frac{1 + r_{t+1}}{\phi} \cdot s_t, \tag{2.3}$$

where consumption as adult plus savings for retirement (s_t) cannot exceed wage income in the first period of life [Equation (2.2)] and consumption of retirees who survived from adulthood to retirement is given by the savings carried over from adulthood plus the return earned by investing in the annuity market [Equation (2.3)]. Combining these two equations yields the lifetime budget constraint

$$c_{1,t} + \phi \frac{c_{2,t+1}}{1 + r_{t+1}} = w_t. \tag{2.4}$$

From the first-order conditions of the associated optimization problem, the individual Euler equation follows as

$$\frac{c_{2,t+1}}{c_{1,t}} = (1 + r_{t+1})\,\beta. \tag{2.5}$$

Notice that the survival probability drops out of the individual Euler equation because of the fully insured mortality risk. In the absence of full annuitization, the mechanisms of our model would even be strengthened because the probability of surviving showed up on the right hand side of the individual Euler equation with a positive influence on consumption growth. Thus, as far as the growth effects of increasing longevity are concerned, we are conservative in assuming a perfect annuity market.

Since the birth rate is equal to the replacement rate, the relative cohort size between adults and retirees is only influenced by the survival probability to the extent that $N_{2,t+1} = \phi N_{1,t}$. Defining consumption of adults and of retirees by $C_{1,t} = c_{1,t}N_{1,t}$ and $C_{2,t+1} = c_{2,t+1}N_{2,t+1}$, respectively, aggregation yields $c_{2,t+1}N_{2,t+1} = (1 + r_{t+1})\beta c_{1,t}\phi N_{1,t}$. From this expression, the "cohort" Euler equation follows immediately as

$$\frac{C_{2,t+1}}{C_{1,t}} = (1 + r_{t+1})\,\beta\phi. \tag{2.6}$$

To our best knowledge, we are the first to distinguish between the individual and the cohort Euler equation in discrete time with lifetime uncertainty. It is important to note that the cohort Euler equation differs from the "aggregate" Euler equation that is well-known from the literature on lifetime uncertainty in continuous time (cf. Blanchard, 1985; Buiter, 1988; Heijdra, 2009). The cohort Euler equation only takes into account the growth of consumption of the adult cohort in period t to the next period $t+1$, when the members of the cohort are retired. It does not consider the change in consumption that is due to different levels of consumption between those who are retired in period t and the adults in period $t + 1$. The aggregate Euler equation would also take this part of the consumption change into account and it can be shown that, if the economy is on its long-run balanced growth path, the cohort-specific consumption growth rate and the aggregate consumption growth rate are equal.

2.2.2 Production side

The production side of the economy is a simplified version of the production side of the Schumpeterian growth model developed by Aghion and Howitt (1992) and further elaborated upon in Aghion and Howitt (1999, 2005). There are three sectors, the final goods sector, the intermediate goods sector, and the R&D sector. The aggregate final good is produced under perfect competition using the intermediate good as input. The intermediate good in turn is produced by a monopoly that converts one unit of labor into one unit of the intermediate good (i.e, intermediate goods producers utilize a one-to-one technology). The monopolist owns the patent of the most recent quality-improving innovation developed in the R&D sector. When the next quality-improving innovation occurs, the incumbent is driven out of business and loses the monopoly rents. Since monopolies in the intermediate goods sector earn a positive profit stream as long as they are the incumbent, the firms have positive value and their shares are sold via the mutual funds to the households in the economy. We abstract from physical capital such that these shares represent the only savings vehicle. Introducing physical capital would complicate the model substantially without changing the main mechanisms.[4]

The final good is denoted by Y_t and produced according to the production function

$$Y_t = A_t x_t^{\alpha}, \tag{2.7}$$

where A_t refers to the productivity of the intermediate good in producing the final good, x_t is the intermediate good produced by the monopolist, and $\alpha \in (0,1)$ is the elasticity of output with respect to intermediate inputs. The intermediate good is produced by using the amount x_t of the production factor labor, which is fixed due to our demographic assumptions and denoted by L. Labor is also used in the research sector, with the amount being denoted by n_t. Consequently, the labor market clearing condition is

$$L = x_t + n_t. \tag{2.8}$$

Given employment n_t in the research sector, the arrival of innovations follows a random Poisson arrival rate $\lambda \cdot n_t$, where $\lambda > 0$ denotes the productivity of researchers. This means that more researchers and a higher productivity of these researchers both increase the probability of a successful innovation in a given period. The firm that succeeds to obtain the newest innovation from the R&D sector monopolizes the intermediate goods sector until it is replaced by the next innovator.

[4]For a framework of horizontal innovation with increasing longevity in which both types of assets, physical capital and shares of intermediate goods producers, are available, see Prettner (2013).

Innovations are tantamount to the introduction of a new variety of the single intermediate good with a higher quality that replaces the previous one. The new intermediate good increases the productivity level A_t by a constant factor $\gamma > 1$. Rewriting the index of productivity such that it refers to the number of innovations that already happened in the past (as denoted by i) instead of time, we therefore have

$$A_{i+1} = \gamma \cdot A_i. \tag{2.9}$$

It is important to keep the distinction between the time period, t, and the number of innovations, i, because, in an overlapping generations model in which a time period lasts for 40 years, many innovations will occur within one single period. This is also the reason why there are no jumps in the growth rate over time along a balanced growth path. As compared to the continuous time setting of Aghion and Howitt (1992), which assumes a continuum of intermediate goods producing firms to avoid discontinuities in the growth rate, the law of large numbers applies over time in our case and ensures a smooth growth rate. For the robustness of our results in a model variant with a continuum of intermediate good sectors, see the appendix.

The employment level of researchers can be calculated with the help of the no-arbitrage condition

$$w_i = \lambda V_{i+1}, \tag{2.10}$$

where w_i refers to the wage of the researchers and V_{i+1} to the discounted expected payoff of innovation $i + 1$. The no-arbitrage condition states that the investment of the research firm in terms of the wage bill for scientists has to be equal to the discounted payoff of the expected monopoly rents over the period in which the innovator will be the new incumbent. The value of V_{i+1} is in turn determined by the no-arbitrage equation for the investments of households given by

$$r_{i+1}V_{i+1} = \pi_{i+1} - \lambda n_{i+1}V_{i+1}. \tag{2.11}$$

The left-hand side is the income earned on an investment of the amount V_{i+1} at the risk free interest rate r_{i+1}, while the right-hand side consists of the monopoly profits due to owning the incumbent firm minus the expected loss that occurs when the incumbent is driven out of business by a new quality-improving innovation (the Schumpeterian creative destruction effect). While the interest rate is exogenously given in Aghion and Howitt (1999), the aggregate Euler equation that coincides with the cohort Euler equation along the balanced growth path determines the real interest rate in our overlapping generations framework. Hence, we endogenize the interest rate, which is particularly important when analyzing the growth effects of increasing life expectancy.

From Equation (2.11), we obtain

$$V_{i+1} = \frac{\pi_{i+1}}{r_{i+1} + \lambda n_{i+1}}, \tag{2.12}$$

stating that the value of the firm is equal to the discounted expected profit with the effective discount rate being the interest rate (r_{i+1}) plus the probability of a new innovation that drives the incumbent out of business (λn_{i+1}). It is straightforward that $\partial V_{i+1}/\partial n_{i+1} < 0$, i.e., if more researchers are employed in the R&D sector, the probability of the next innovation is larger, which implies lower expected monopoly profits because an incumbent tends to be replaced earlier. The incumbent innovator determines optimal output x_i by maximizing

$$\pi_i = p_i(x_i)x_i - w_i x_i \tag{2.13}$$

with respect to the choice of x_i. This choice in turn determines the profits π_i. Given perfect competition in the final good sector, $p_i(x_i) = A_i \alpha x_i^{\alpha-1}$ is the inverse demand function for intermediates. The maximization of profits then yields

$$x_i = \left(\frac{\alpha^2 A_i}{w_i}\right)^{\frac{1}{1-\alpha}}. \tag{2.14}$$

Substituting $p_i(x_i)$ in Equation (2.13) by the inverse demand function for intermediates, we obtain

$$\pi_i = \left(\frac{1}{\alpha} - 1\right) w_i x_i = A_i \frac{1-\alpha}{\alpha} \omega_i x_i = A_i \tilde{\pi}, \tag{2.15}$$

where $\omega_i = w_i/A_i$ is the productivity-adjusted wage rate and $\tilde{\pi} = (1-\alpha)\omega x/\alpha$. Exploiting the new definition of profits in Equation (2.15) and dividing both sides of Equation (2.10) by A_i, acknowledging that $A_{i+1}/A_i = \gamma$ from Equation (2.9), we can rewrite the no-arbitrage condition as

$$\omega_i = \lambda \frac{\gamma \tilde{\pi}\left(\omega_{i+1}\right)}{r_{i+1} + \lambda n_{i+1}}. \tag{2.16}$$

To be consistent, we also rewrite the labor market clearing condition in terms of the productivity-adjusted wage rate

$$L = n_i + \left(\frac{\alpha^2}{\omega_i}\right)^{\frac{1}{1-\alpha}}. \tag{2.17}$$

The production function, Equation (2.7), can be reformulated as $Y_i = A_i \left(L - n_i\right)^{\alpha}$ by exploiting the labor market clearing condition. This implies that

$$Y_{i+1} = \gamma Y_i, \tag{2.18}$$

meaning that output grows at the rate $\gamma - 1$ with each innovation that occurs. Since the time between two innovations is random, we compute the average growth rate of the economy over a generation by relying on the relation

$$\ln Y_{t+1} = \ln Y_t + \ln(\gamma) \cdot \epsilon_t, \tag{2.19}$$

where ϵ_t is the number of innovations between time t and time $t+1$. The number of innovations is Poisson distributed with the parameter $\lambda \cdot n_t$ referring to the average number of innovations over a generation. Computing the expectation of Equation (2.19), we get the average growth rate of the economy, g_t, as

$$g_t = E\left(\ln Y_{t+1} - \ln Y_t\right) = \lambda \cdot n_t \cdot \ln(\gamma). \tag{2.20}$$

This is also the growth rate of per capita GDP at the steady state because we assume that fertility is at the replacement rate. Notice that there are no jumps in the growth rate because many innovations occur within a generation of 40 years such that the law of large numbers applies over time. Thus, it is not necessary in our framework to assume a continuum of intermediate firms to avoid discontinuities.

2.2.3 The balanced growth path

Along the balanced growth path, all markets clear and the common long-run growth rate of technology, output, and consumption is constant. The endogenous consumption-savings decision of individuals affects the real interest rate and, as obvious from Equation (2.16), exerts an influence on the demand for research, which affects the number of scientists and thereby the frequency of quality-improving innovations. This, in turn, determines the growth rate of the economy. Along the balanced growth path, the time dimension is not relevant because the growth rate is constant such that we suppress the time index and the innovation index from now on. The equilibrium dynamics along the balanced growth path are summarized by the following four-dimensional system of equations that brings together the demand-side and the supply-side of the economy and allows for determining the endogenous variables:

$$g_C = (1+r)\beta\phi - 1, \tag{2.21}$$

$$g_Y = \lambda n \cdot \ln(\gamma), \tag{2.22}$$

$$1 = \lambda \frac{\gamma \left(\frac{1-\alpha}{\alpha}\right)(L-n)}{r+\lambda n}, \tag{2.23}$$

$$L = n + \left(\frac{\alpha^2}{\omega}\right)^{\frac{1}{1-\alpha}}. \tag{2.24}$$

Along the balanced growth path, aggregate consumption, cohort consumption, and aggregate output all grow at the same rate such that $g_C = g_Y \equiv g$ in Equations (2.21) and (2.22).

Using Equations (2.21)–(2.24), we solve for the four unknowns g, r, n, and ω. The associated long-run growth rate of the economy boils down to

$$g = \max\left\{\frac{\ln(\gamma)\{\alpha + \beta\phi[(\alpha-1)\gamma\lambda L - \alpha]\}}{\beta\phi[\alpha(\gamma-1) - \gamma] - \alpha\ln(\gamma)}, 0\right\}. \tag{2.25}$$

The first expression within the curly brackets is positive as long as the product of β, λ, L, ϕ, and γ is high enough. However, there is the theoretical possibility that research incentives are too low to sustain a positive growth rate, for example, if the productivity of researchers, λ, is very low for a given population size, L. Since employment of researchers and therefore technological progress cannot become negative, we have the lower bound of zero on the long-run growth rate, as reflected in our formulation of Equation (2.25). This behavior of the model in the corner solution is in line with the literature that usually employs similar non-negativity assumptions, either implicitly or explicitly (see, for example, Romer, 1990; Kuhn and Prettner, 2016). In case of the corner solution, there would not be any technological progress. However, in actually existing highly developed countries, this is an empirically irrelevant case because technological progress has been positive in any 40 year period since the Industrial Revolution.

2.3 Results

2.3.1 The growth effects of increasing longevity

Since we are interested in the consequences of rising life expectancy for long-run economic growth, we analyze the effect of the survival probability, ϕ. A higher survival probability implies a higher life expectancy and also a higher average age in the economy, such that an increase in ϕ is tantamount to population aging. We can state the following proposition.

Proposition 2.1. *The long-run growth rate (g) increases unambiguously in response to a higher survival probability (ϕ).*

Proof. The partial derivative of the growth rate with respect to the survival probability is given by

$$\frac{\partial g}{\partial \phi} = \frac{\alpha\beta\ln(\gamma)\{\alpha + \gamma - \alpha\gamma + \ln(\gamma)[\alpha + (1-\alpha)\gamma\lambda L]\}}{[\beta\phi(\alpha + \gamma - \alpha\gamma) + \alpha\ln(\gamma)]^2}. \tag{2.26}$$

The denominator of this expression is always positive. Taking into account that $\alpha \in (0,1)$, the numerator is also always positive, such that the survival probability has a strictly positive effect on the long-run growth rate of the economy.

□

The economic intuition for this finding is that an increase in life expectancy reduces the generational turnover and therefore raises aggregate savings. Given the higher savings, the mutual funds can invest more into the shares of intermediate goods companies, which raises the demand for new innovations. This, in turn, leads to a higher employment level in the research sector such that the probability of successful innovations within each period increases. As a consequence, the common long-run growth rate of aggregate consumption and aggregate output rises.

Interestingly, since higher savings imply a decrease in the interest rate, individual consumption growth, at first, decreases. This is due to the perfectly insured risk of death: a change in the survival probability does not affect individual consumption growth directly but only indirectly via the reduction in the interest rate. A lower interest rate implies that individual consumption growth decreases. At the cohort level, however, there are two opposing effects. An increase in ϕ has a direct positive effect on cohort consumption growth because of the increase in the number of individuals who survive, whereas the decrease in the interest rate has an indirect negative effect on cohort consumption growth via the individual Euler equation. The negative indirect effect of a lower interest rate, r, only partially offsets the positive direct effect of a higher survival rate, ϕ. Otherwise, aggregate savings would not rise and the interest rate could not have decreased in the first place. That aggregate savings rise and the interest rate decreases are unambiguous general equilibrium effects. Altogether, therefore, cohort-specific consumption growth increases with the survival probability. Thus, a rising survival probability reduces the wedge between individual consumption growth and cohort consumption growth. In the limit of $\phi = 1$, the individual and the cohort Euler equation are the same. In this case, we would be back in the standard formulation of an overlapping generations model without longevity risk in which all individuals die at the age of 100.

Inspecting the long-run growth rate g in Equation (2.25) shows that the effects of β and λ are positive, while the effect of α is negative. These results are in line with the standard literature. The effect of γ is ambiguous, but for reasonably high values of γ, it is positive because the monopolist enjoys higher rents after an innovation, which eventually increases the incentives to invest in research.

If mortality risks were only imperfectly insured, our qualitative results would not change. From a quantitative point of view, the magnitude of the effects would be even larger. This is due to the following mechanism. Saving becomes less attractive for individuals if their lifetime risk is only partially insured. As a result, the individual saving rate and the aggregate saving rate both decrease. Altogether, a rise in life expectancy would then have a larger positive effect on saving than in the case of fully insured lifetime risks such that longevity's impact on economic growth would be stronger.

Our result holds true when individuals save for their own retirement or the social security system is based on a fully-funded system in which the contributions of individuals are invested and later paid out as pensions when they retire. The presence of a pay-as-you-go pension system would lead to a crowding out of individual savings and thereby reduce the growth effect that is due to our mechanism. However, as long as individual savings are not fully crowded out by the pension system, the channel that we describe is still operative and increasing longevity would raise economic growth as stated in Proposition 2.1.

In Figure 2.1, we show the relationship between the growth rate, g, and the survival probability, ϕ, for standard parameter values. We normalize the size of the workforce such that $L \equiv 1$ to eliminate considerations with respect to the scale of the economy. The choice of the discount factor $\beta = 0.14$ corresponds to a yearly discount rate of approximately 4% and the size of $\gamma = 1.04$ implies that every successful quality improvement increases productivity by 4%. The survival probability is set such that life expectancy corresponds to the value in the United States of 78.7 years (World Bank, 2016). Finally, we set the productivity of scientists to $\lambda = 42.5$ to fit the implied growth rate of the model to the

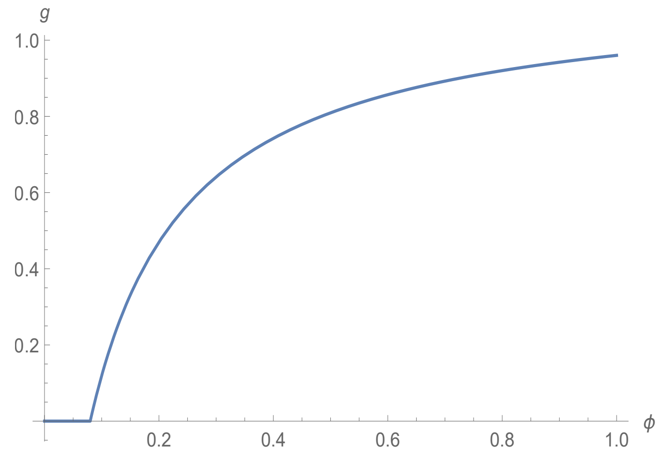

Figure 2.1: Survival probability and the long-run growth rate

actual aggregate growth rate of the United States over 40 years.[5]

In line with the analytical result stated in Proposition 2.1, we observe a strictly positive effect of the survival probability on the long-run economic growth rate. Notice that the effect of the survival probability on economic growth is concave such that the positive effect of an increase in the survival probability is high for low levels of survival probability, and low for high levels of survival probability. For very low values of ϕ, the economy would be trapped in the corner solution of zero employment in the research sector and a stagnation of technology.[6]

To keep our analytical results as simple as possible, we used the special case of logarithmic utility that conforms to an elasticity of intertemporal substitution (EIS) of one. However, since our general equilibrium effects crucially hinge on the interest rate and on the individual consumption-savings decision, we examine numerically how changes in the EIS affect the long-run economic growth rate. According to Chetty (2006), values for the EIS can vary between 0.5 and 1, while others find the EIS to be even closer to zero (e.g. Guvenen, 2006). An EIS of one, which corresponds to the logarithmic utility function, is usually found to be still within the range of plausible values. Table 2.1 shows the implied growth rates for different values of the EIS and the effect that an increase in

[5]The large value of λ is due to the normalization of the population size to unity. Alternatively, we could have assumed a realistic population size and then λ would be much smaller to generate a realistic economic growth rate. However, both procedures would have led to similar growth rates.

[6]While one could, in principle, include a depreciation rate of technology to account for technological regress and negative growth, this would not change any of our arguments above.

life expectancy has on economic growth for the given EIS. We also include values of the EIS that are larger than one to assess the sensitivity of our results with respect to this parameter.

Table 2.1: Sensitivity of the growth rate to changes in the elasticity of intertemporal substitution (EIS)

EIS	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
Growth rate (g)	0	0.19	0.5	0.69	0.79	0.85	0.89	0.92	0.94	0.95
Growth effect of increasing life expectancy	0	0.041	0.032	0.022	0.015	0.011	0.008	0.005	0.005	0.004

As shown in Table 2.1, economic growth varies positively with the willingness of individuals to substitute consumption over time. This result is as expected and conforms to the standard economic growth literature (cf. Romer, 1990). A higher willingness to substitute consumption over time increases the effect of the interest rate on the saving rate. Higher savings increase investment in R&D, which eventually raises long-run economic growth. For very low values of the EIS, the economy is again at the corner solution with zero growth. When considering different values of the EIS, an increase in life expectancy of one year always raises economic growth. This confirms that our underlying mechanism not only holds for the special case of a logarithmic utility function.

2.3.2 The welfare effects of increasing longevity

In our overlapping generations economy in which individual utility rises with consumption, faster economic growth implies higher welfare at the individual as well as at the aggregate level. Keeping that in mind, the total welfare gains that are due to increasing longevity can be decomposed into two separate effects:

(1) As shown in Proposition 2.1, increases in longevity positively impact on productivity growth and, thus, on the growth rate of the economy. In the long run, this increases consumption and therefore welfare.

(2) Individuals do not only derive utility from the higher consumption induced by the effect of higher life expectancy on economic growth but also from the direct effect of living longer.

Considering these two channels, it is interesting to analyze their relative importance depending on the health status of the inhabitants of an economy (measured by ϕ) and depending on how developed an economy already is (measured by the level of technology, A_t, or by the corresponding wage rate, w_t). We do so by comparing how an increase in longevity of one year, i.e., an increase of ϕ by 0.025, impacts on the relative importance of productivity gains for the gains in overall welfare. To this end, we isolate the welfare effect of productivity gains by calculating the welfare derived from higher consumption due to higher productivity, given a counterfactually unchanged survival probability. Then we calculate the relative importance of the welfare effect that is due to productivity gains as the share of the total increase in welfare that is due to an increase in longevity. Denoting the share of the welfare increase that is due to rising productivity as $WS_{productivity}$, we have

$$WS_{productivity} = \frac{U_{productivity} - U_{initial}}{U_{new} - U_{initial}},$$

where $U_{initial}$ and U_{new} are the utility levels before and after the increase in life expectancy of one year, while $U_{productivity}$ is the increase in utility that is induced by increases in productivity, assuming an unchanged life expectancy. We isolate this effect by allowing ϕ to change on the production side of the economy and holding it constant in the utility function when calculating utility levels.

The three different utility levels required to calculate the welfare shares are obtained by the following expressions:

$$
\begin{aligned}
U_{initial} &= \ln(c_{1,initial}) + \phi\beta\ln(c_{2,initial}) \\
&\quad + \ln[(1+g_{initial})\cdot c_{1,initial}] + \phi\beta\ln[(1+g_{initial})\cdot c_{2,initial}] \\
&\quad + [(1+g_{initial})^2\cdot c_{1,initial}] + \phi\beta\ln[(1+g_{initial})^2\cdot c_{2,initial}],
\end{aligned}
$$

$$
\begin{aligned}
U_{new} &= \ln(c_{1,new}) + (\phi+0.025)\beta\ln(c_{2,new}) \\
&\quad + \ln[(1+g_{new})\cdot c_{1,new}] + (\phi+0.025)\beta\ln[(1+g_{new})\cdot c_{2,new}] \\
&\quad + \ln[(1+g_{new})^2\cdot c_{1,new}] + (\phi+0.025)\beta\ln[(1+g_{new})^2\cdot c_{2,new}],
\end{aligned}
$$

$$
\begin{aligned}
U_{productivity} &= \ln(c_{1,new}) + \phi\beta\ln(c_{2,new}) \\
&\quad + \ln[(1+g_{new})\cdot c_{1,new}] + \phi\beta\ln[(1+g_{new})\cdot c_{2,new}] \\
&\quad + \ln[(1+g_{new})^2\cdot c_{1,new}] + \phi\beta\ln[(1+g_{new})^2\cdot c_{2,new}].
\end{aligned}
$$

Subscripts 1 and 2 refer to adulthood and retirement, while “initial” and “new” indicate the consumption level and the growth rate of the economy before and after the increase in life expectancy by one year. In the expression for U_{new} we have a survival probability of $\phi + 0.025$ because an increase in life expectancy by one year corresponds to an increase in ϕ of 0.025. To derive the welfare shares, we take into account the lifetime utility of three generations, which corresponds to 160 years excluding childhood (80 years of the first generation and 40 years for each additional generation).

In Table 2.2 we apply this calculation to two economies that resemble parameter values that lead to income levels and life expectancies as in the United States and as in Russia, respectively. We adjust initial life expectancy, ϕ, wage income, w, and the productivity of researchers, λ, such that our model matches the growth rates of GDP over a period of 40 years in both countries. Afterwards, we derive the corresponding welfare shares and report the welfare share that is due to changes in productivity.

The importance of longevity gains and productivity gains differ substantially between

Table 2.2: Decomposition of additional utility for an increase in life expectancy of 1 year

	Productivity share	
	United States	Russia
Actual parameter values	24.81%	46.54%
Initial life expectancy reduced by 5 years	37.03%	63.73%
Yearly wage rate reduced by $5,000	24.95%	47.24%

both economies. In the United States, the productivity share of a rise in life expectancy by one year is 25%, while the direct welfare gains of living longer amount to 75%. For Russia, the relative importance changes considerably. Almost half of the longevity-induced welfare gains are due to improvements in productivity growth. Nevertheless, the direct welfare gains of longevity improvements still exceed the indirect welfare gains due to rising productivity in the baseline specification. We summarize this result in the following remark.

Remark 2.1. *In highly developed countries, the direct utility gain due to increasing life expectancy is higher than the indirect utility gain via induced productivity growth.*

In a second step, we conduct a comparative statics analysis by separately decreasing initial life expectancy by five years and wage income by $5000 ceteris paribus. This allows us to understand how different parameters impact on the relative importance of both welfare shares. It is obvious that initial life expectancy is most important in that respect. For both economies, the welfare share of productivity increases considerably when decreasing initial life expectancy. For Russia, the effect is even more pronounced than for the United States. Although being smaller in magnitude, the same relationship holds when considering the wage rate of an economy. Lower initial wages increase the productivity share and, again, the effect is stronger for Russia than for the United States. We summarize this finding in the following remark.

Remark 2.2. *Improvements in productivity become less important for welfare the better the health status of the inhabitants of an economy and the more developed an economy already is.*

This is consistent with the results of Kuhn and Prettner (2016) who show that an increase of the size of the health sector beyond its growth-maximizing size is actually a Pareto improvement. In other words, choosing the size of the health care sector solely by considering its economic growth effect misses the point because it disregards the large direct welfare effects of higher life expectancy.

Furthermore, the relative welfare gain that is due to living longer increases with the level of income. This is in line with the result of Hall and Jones (2007) that a continuous increase in the share of resources devoted to health care is an optimal outcome in the course of economic development.

2.4 Conclusions

We introduce a demographic structure with three overlapping generations, childhood, adulthood, and retirement into an endogenous growth model based on vertical innovations. Furthermore, individuals face an exogenously given survival probability from adulthood to retirement, which determines life expectancy. We show that increasing longevity leads to higher aggregate savings, which raises the demand for innovation. This, in turn, leads to higher employment in the R&D sector, faster technological progress, and a higher long-run economic growth rate. Our mechanism yields an alternative theoretical explanation for the positive effects of increasing life expectancy on economic growth in developed countries as found by Cervellati and Sunde (2011) in their empirical analysis.

We use the resulting framework to assess the welfare effects of rising longevity and decompose them into the direct welfare effect of living longer, and the indirect welfare effect of higher consumption due to the induced faster economic growth. We show that the direct welfare effect tends to be larger than the indirect welfare effect. Furthermore, the relative importance of the direct welfare effect increases with economic development. Both of these results are consistent with the recent literature on the welfare maximizing size of healthcare sectors and on the evolution of optimal expenditures on health care in the course of economic development.

From a policy perspective, our results show that investments in health care have positive direct welfare effects as well as positive side effects on economic growth that both need to be taken into account when reforming health care systems. Focusing solely on the growth effects of increasing life expectancy misses a large part of the welfare gains. These welfare gains become even more important with economic development. Thus, in assessing the optimal resources to be invested in health care, it is important to adopt a comprehensive view that considers both the indirect economic growth effects as well as the direct welfare effects of living longer.

As far as the scope for future research is concerned, we identify the following important dimensions. First, while our results would not change from a qualitative perspective if we allowed for imperfect annuitization and for a pay-as-you-go pension system (as long as there is not a full crowding out of private savings), it could be interesting to assess the quantitative implications of these aspects in a more thorough numerical analysis. Second, following the analysis of Kuhn and Prettner (2016) in the horizontal innovation framework, it might be interesting to introduce an explicit health care sector and to assess its growth-maximizing and its welfare-maximizing size. Doing so would allow to assess whether there are differences in the size of growth-maximizing and welfare-maximizing health care sectors between the horizontal innovation framework and the vertical innovation framework. Finally, to allow for richer demographic dynamics, it could be worthwhile to introduce endogenous population growth and endogenous education investments along the lines of Strulik et al. (2013). However, all of these extensions would raise the complexity of the model considerably such that only numerical results by means of a detailed calibration exercise could be obtained. Of course, this is beyond the scope of our present paper, which is mainly concerned with the analytical characterization of the main channels.

Appendix A: Model solution for a continuum of intermediate goods firms

The model solution in the main text refers to a setting in which output is produced by using only one intermediate good as input. In this section, we show that the central results remain valid when assuming a continuum of intermediate goods sectors instead (Aghion and Howitt, 1999). To be able to derive analytical solutions for this case, however, we have to use continuous time approximations in the derivation of the long-run economic growth rate.

Assume, in contrast to the baseline model, that there is one R&D sector for each intermediate good, with the firms in each research sector competing to discover the next generation of that particular good. One necessary additional assumption is that, although the arrival rates in different sectors are independent of each other, innovations are drawn from the same pool of knowledge, i.e., each new innovation increases the technological frontier available to all research firms. This ensures that innovations during one period arrive gradually. Using a continuous time approximation to be able to derive the analytical solution of the long-run economic growth rate, the labor market clearing condition and the research arbitrage condition are given by

$$L = n + \frac{(1-\alpha)\left(\frac{\alpha^2}{\omega}\right)^{\frac{1}{1-\alpha}}}{1+\ln(\gamma)-\alpha}, \tag{2.27}$$

$$\left(\frac{\alpha^2}{\omega}\right)^{\frac{1}{\alpha-1}} = \frac{\lambda\frac{1-\alpha}{\alpha}}{r+\lambda n+\frac{\alpha}{1-\alpha}\lambda n\ln(\gamma)}. \tag{2.28}$$

The main difference to the baseline model is the crowding out effect, represented by λn in the denominator of Equation (2.28). This is a consequence of the competition between the sectors because lower monopoly profits reduce the incentives to invest in R&D. The number of researchers across all sectors is the same because the expected payoffs in all research sectors are identical. The flow of innovations can, therefore, still be expressed as

$$g_t = \lambda \cdot n_t \cdot \ln\gamma. \tag{2.29}$$

Since the new production structure does not affect the consumption-savings behavior of the households, the long-run growth rate of the economy is given by

$$g = \max \left\{ \frac{(1-\alpha)\log(\gamma)\{\beta\phi[(1-\alpha)\lambda L + \alpha] + \beta\lambda L\phi\log(\gamma) - \alpha\}}{\log(\gamma)[(1-\alpha)\alpha(1-\beta\phi) + \beta\phi] + (1-\alpha)\beta\phi}, 0 \right\}. \tag{2.30}$$

The effect of population aging, as represented by an increase in the survival probability (ϕ) is still unambiguously positive. This is formulated in the following proposition.

Proposition 2.2. *If there is a continuum of intermediate goods sectors instead of a single sector in our vertical innovation economic growth model with overlapping generations, the long-run growth rate (g) still increases in response to a higher survival probability (ϕ).*

Proof. The partial derivative of the growth rate with respect to the survival probability is given by

$$\frac{\partial g}{\partial \phi} = \frac{(1-\alpha)\alpha\beta\log(\gamma)[1+\log(\gamma)-\alpha][(1-\alpha)\lambda L\log(\gamma)+1]}{\{(\alpha-1)\beta\phi - \log(\gamma)[(1-\alpha)\alpha(1-\beta\phi)+\beta\phi]\}^2}. \tag{2.31}$$

The denominator of this expression is always positive. Since $0 < \alpha < 1$, the numerator is also always positive such that the survival probability has a strictly positive effect on the long-run growth rate of the economy. □

The economic intuition is identical to the one in the baseline case with only one intermediate good. An increase in the probability to survive raises aggregate savings. Higher savings induce higher investments into shares of intermediate goods companies. This, in turn, raises the demand for innovation and, thus, for scientists. Having a larger number of scientists in the economy raises the frequency at which new innovations occur, increasing the long-run growth rate of aggregate consumption and of output. As apparent from Table 2.3, the growth effect of increasing life expectancy is still positive for different values of the EIS.

The welfare effects of increasing longevity are reported in Table 2.4. The slightly larger productivity shares, as compared to the one-sector model, are a result of the crowding

Table 2.3: Sensitivity of the growth rate to changes in the elasticity of intertemporal substitution (EIS)

EIS	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
Growth rate (g)	0	0.2	0.51	0.69	0.79	0.86	0.89	0.92	0.94	0.95
Growth effect of increasing life expectancy	0	0.042	0.032	0.022	0.015	0.011	0.008	0.007	0.005	0.005

out effect mentioned above, which reduces the incentives to invest in research. This, in turn, increases the productivity share in the welfare analysis.

Table 2.4: Decomposition of additional utility for an increase in life expectancy of 1 year, multi-sector approach

	Productivity share	
	United States	Russia
Actual parameter values	29.21%	48.99%
Initial life expectancy reduced by 5 years	41.58%	68.15%
Yearly wage rate reduced by $5,000	29.37%	49.69%

To summarize, the introduction of a continuum of intermediate goods does not change our baseline results from a qualitative perspective and only slightly from a quantitative perspective. While the multi-sector case is arguably more realistic, it comes at the cost of using a continuous time approximation to be able to calculate analytical results.

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CHAPTER 3

Endogenous Life Expectancy and R&D-based Economic Growth*

*This chapter is my own work and is under revision at the Journal of Population Economics.

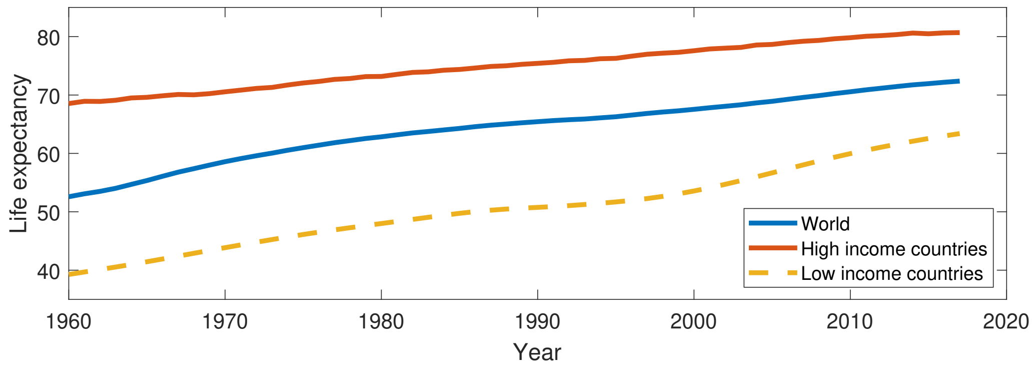
Abstract

We propose an overlapping generations framework in which life expectancy is determined endogenously by governmental health investments. As a novelty, we are able to examine the feedback effects between life expectancy and R&D-driven economic growth for the transitional dynamics. We find that i) higher survival induces economic growth through higher savings and higher labor force participation; ii) longevity-induced reductions in fertility hamper economic development; iii) the positive life expectancy effects of larger savings and higher labor force participation outweigh the negative effect of a reduction in fertility, and iv) there exists a growth-maximizing size of the health care sector that might lie beyond what is observed in most countries. Altogether, the results support a rather optimistic view on the relationship between life expectancy and economic growth and contribute to the debate surrounding rising health shares and economic development.

3.1 Introduction

Over the past decades, mortality has decreased at an historically unprecedented pace all over the world. As depicted in Figure 3.1, on a global level, life expectancy at birth has increased from 52.6 years in 1960 to 72.4 years in 2017. Nonetheless, substantial differences between different regions of the world remain. Over the same time period, the real GDP p.c. has risen roughly by a factor of 3.5 from $4,254 to $14,574 (Roser, 2019).

Figure 3.1: Life expectancy at birth, 1960–2017



Source: World Bank (2019)

In the literature, the following important reasons for increasing life expectancy have been proposed: medical progress, economic development, increased health shares and behavioral changes (see, for example, Christensen and Vaupel, 1996; Harper, 2014; Currie and Schwandt, 2016; Dwyer-Lindgren et al., 2017). However, the dynamic interrelations between life expectancy and economic development are less clear.

Economists have tried to shed light on the correlation and causality between health and income. The Preston curve (see Preston, 1975) provides interesting insights by highlighting the positive correlation between the development status and the health status of a country, which flattens out for high levels of income. Furthermore, as Bloom et al. (2019) show, the Preston curve has shifted upwards between 1960 and 2015 and still exhibits a strong positive correlation. All this points to a relationship between health and income that requires further investigation. In a prominent work, Acemoglu and Johnson (2007) find a negative relationship between health and economic growth by exploiting the epidemiological transition after 1940. However, this result got challenged by several contributions. Critiques by Aghion et al. (2011) and Bloom et al. (2014) mainly emphasize the role of initial health that is not considered in Acemoglu and Johnson (2007). In fact, Cervellati and Sunde (2011, 2015) show that the effect of health on economic growth hinges on a country's stage of the demographic transition. Economic growth in less developed countries with high mortality and fertility might even decrease in better health because longer lifetimes do not decrease fertility or raise educational investments. Therefore, higher survival primarily raises the population size, potentially affecting p.c. income negatively. Focusing on high income countries, the relationship between health and economic growth reverses since higher survival reduces population growth and spurs human capital investments. This negative relationship is also supported by various other works (for example Lorentzen et al., 2008; Cervellati and Sunde, 2013; Bloom et al., 2014; Hansen and Lønstrop, 2015; Gehringer and Prettner, 2019). Given the tremendous improvements in health care during large parts of the twentieth century, infant and child mortality is contributing less and less to gains in life expectancy in high income countries, leaving decreases in old-age mortality as the main engine for future increases in individual life spans (Breyer et al., 2010; Eggleston and Fuchs, 2012). Also, since health care sectors in high income countries are usually well-developed, additional gains in life expectancy come at ever higher cost, thereby raising the opportunity cost of health care, e.g. productive investments (Bhargava et al., 2001).

To analyze the complex relationship within models of economic growth, two dimensions can be distinguished – exogenous vs. endogenous life expectancy and exogenous vs. endogenous growth frameworks. There already exists quite a rich literature on exogenous life expectancy effects in both exogenous and endogenous growth models (see, for example, Blanchard, 1985; Reinhart, 1999; Heijdra and Romp, 2008; Prettner, 2013; Baldanzi et al., 2019). These models usually emphasize the saving effect, the role of human capital and the corresponding effects on productivity and provide an important basis for a better understanding of the consequences of changes in the lifetime horizon for economic growth. Nonetheless, no feedback effects on longevity can be considered, which limits the models' explanatory power. Life expectancy has been endogenized mostly in exogenous growth models as in Boucekkine et al. (2002), Chakraborty (2004) and Fanti and Gori (2014), where life expectancy increases in the public resources directed toward health or in the level of human capital. Given the endogenous nature of technological progress, these models are also limited in their explanatory power. Last, there is the category of models incorporating both endogenous survival and endogenous growth. Contributions from this category are rare. A recent paper by Kuhn and Prettner (2016) introduces individual survival into a Romer (1990) type of economy, where life expectancy increases in the health care personnel. The authors find a hump-shaped relationship between the size of the health care sector (which determines life expectancy) and economic growth. However, in their analysis they solely focus on balanced growth path effects with constant fertility and an exogenous health care sector.

To provide additional insights, we fully endogenize life expectancy similar to Chakraborty (2004) and combine it with a discrete time endogenous growth framework in the vein of Romer (1990) and Jones (1995). Old-age life expectancy increases in output per worker as well as in the health tax levied by the government. Additionally, there is a quantity-quality trade-off in the sense that adults have to decide on the number of children to have as well as on the children's education. We include three channels through which changes in the survival probability (and vice versa) can affect economic development: the saving rate, fertility and labor force participation. Given an increase in the lifetime horizon, individuals save a larger fraction of their income to account for the prolonged retirement period (Bloom et al., 2007, 2010). To finance the increase in savings, adult consumption and fertility decrease and labor force participation increases (for works on the fertility and labor force participation effects of health see Zhang and Zhang, 2005; Angeles, 2010; Cai, 2010; García-Gómez, 2011). We show that, in principle, longer individual lifespans foster economic growth, as the positive saving and labor force participation effects outweigh the negative fertility effect. For a constant survival probability as well as for an over-sized health care sector, economic growth slows down.

The paper is organized as follows. In Section 3.2, we set up the model structure by implementing endogenous life expectancy into a R&D-based growth framework. In Section 3.3, the balanced growth path is derived analytically. In Sections 3.4 and 3.5, we calibrate the model to U.S. data and use the results to further investigate the feedback effects between life expectancy and economic growth. Finally, in Section 3.6, we draw conclusions and discuss potentials for future research.

3.2 The model

3.2.1 Consumption side

In the vein of Samuelson (1958) and Diamond (1965), the economy is populated by three single-sex overlapping generations: children, young adults and retirees. Childhood lasts for 20 years, adulthood for 40 years and, depending on the probability to survive to old age, ϕ_t, retirement lasts for up to 40 years.[1] Consequently, the maximum possible lifespan an individual can reach is 100 years. This is as in Baldanzi et al. (2019) with the exception that life expectancy is determined endogenously according to

$$\phi_t = \frac{b_t}{1+b_t},$$

with b_t being the health status of an adult. We closely follow Chakraborty (2004), such that b_t increases in the public resources devoted to health (see also Preston, 1975; Wolfe, 1986; Lichtenberg, 2004). The survival probability has the following properties

- $\frac{\partial \phi_t(b_t)}{\partial b_t} > 0,$
- $\lim_{b_t \to 0} \phi_t(b_t) = 0,$
- $\lim_{b_t \to \infty} \phi_t(b_t) = 1.$

There is a strictly concave non-decreasing relationship between governmental health investments and life expectancy. In the limits, if $\phi = 0$, the corresponding life expectancy

[1] Focusing on well-developed countries only, we assume that all children survive to adulthood. For interrelations between economic development, fertility and child mortality see Cigno (1998).

equals 60 years and if $\phi = 1$, individuals reach the maximum life expectancy of 100 years with certainty.

As a simplification, all economically relevant decisions are made by the adult generation. We implicitly assume that children's consumption needs are covered for by parental consumption. Adults supply up to one unit of labor and, exactly after half of their adult period, give birth to n number of children who will become adults when their parents finish adulthood. Retirees consume out of the savings accumulated and die with certainty at the end of the period, leaving zero bequests. Maximum lifetime utility is then determined by a consumption-saving decision and by choosing the number of children, n_t, to have and the children's educational level, e_t. Consequently, individual utility is given by

$$u_t = \ln(c_{1,t}) + \phi_t \beta \ln(c_{2,t+1}) + \xi \ln(n_t) + \theta \ln(e_t), \tag{3.1}$$

where $c_{1,t}$ is adult consumption in period t, $c_{2,t+1}$ is old-age consumption in period $t+1$ and β, ξ, $\theta \in (0,1)$ are the utility weights of old-age consumption, fertility and children's education, respectively. Individuals need to satisfy two constraints

$$(1 - \psi n_t - \eta n_t e_t) w_t h_t = c_{1,t} + s_t, \tag{3.2}$$

$$\frac{s_t R_{t+1}}{\phi_t} = c_{2,t+1}. \tag{3.3}$$

Equation (3.2) describes the economic constraint of an adult. Labor income depends on the wage rate per effective labor, w_t, on the level of individual human capital, h_t, and on the labor force participation rate, $1 - \psi n_t - \eta n_t e_t$. As in Galor and Weil (2000), we assume that labor supply decreases in the fertility rate to account for the time spent raising children. Furthermore, as a simplification, we follow Prettner and Werner (2016) in assuming home education.[2] Consequently, labor force participation decreases in the time cost of raising children, ψ, in education, e_t, and in the time requirements for educating children, η. Labor income of an adult in period t can then be spent on consumption

[2]To slim down the model and to put the spotlight on life expectancy effects, we abstract from baseline education that children acquire without formal schooling as in Strulik et al. (2013).

during adulthood and on savings, s_t, for retirement.

Equation (3.3) captures individual old-age consumption. As in standard overlapping generations frameworks, the only self-determined source of income during retirement are interest payments on the savings accumulated in the previous period. This is reflected in $s_t R_{t+1}$. To account for individuals' savings who die before reaching retirement, we follow Yaari (1965), Blanchard (1985), Chakraborty (2004) and Baldanzi et al. (2019) and implement perfect and fair annuity markets which insure individuals against the risk of dying with positive bequests before reaching the retirement stage. More specifically, at the end of adulthood, all adults invest their savings in a mutual life insurance fund. Individuals who survive to retirement receive two transfers, their deposited savings and transfers of individuals' savings who died. This is accounted for by dividing $s_t R_{t+1}$ by ϕ_t.[3] In so doing, the individual saving behavior reflects the exact anticipated life expectancy. Combining Equations (3.1), (3.2) and (3.3), utility maximization yields the following optimality conditions

$$\begin{aligned} c_{1,t} &= \frac{w_t h_t}{1+\xi+\beta\phi_t}, \qquad s_t = \frac{\beta\phi_t w_t h_t}{1+\xi+\beta\phi_t}, \\ n_t &= \frac{\xi-\theta}{\psi(1+\xi+\beta\phi_t)}, \qquad e_t = \frac{\theta\psi}{\eta(\xi-\theta)}. \end{aligned} \tag{3.4}$$

First and foremost, the wage rate per effective labor, w_t, and the level of individual human capital, h_t, increase adult consumption and savings but have no direct effect on the fertility and education decisions. This is due to fertility and education being modeled in terms of time-opportunity cost rather than in terms of absolute cost. There is an income-independent quantity-quality trade-off, which is reflected in the optimality conditions for n_t and e_t. While the preferences for children, ξ, raise fertility, they decrease educational efforts. The reverse holds true for the preferences for education, θ. For the reminder of the paper, $\xi > \theta$ is assumed to rule out negative fertility rates.

The effects of the survival probability, ϕ_t, are similar to the ones of the individual

[3] Although, we do not explicitly include a pension system, the annuity dynamics are in line with Cipriani (2014) who finds a negative relationship between population aging and financial transfers received during retirement.

discount factor, β. A change in life expectancy induces the same individual behavior – the higher the probability to survive to old age, the less individuals discount utility derived from future consumption. Therefore, both β and ϕ_t increase savings and decrease consumption of adults. Also, fertility decreases in β and ϕ_t since a higher preference for future consumption decreases the willingness of adults to free resources for children. These relationships are well-known from previous works (Blackburn and Cipriani, 2002; Zhang and Zhang, 2005; Chen, 2010). However, since our survival probability is endogenous, its marginal effects change as the economy is growing. Therefore, the incentives to save and to have children change over time, which, in turn, affects research and production. Given that capital accumulation, population growth and technological progress are fundamental drivers of economic development (Ramsey, 1928; Solow, 1956; Cass, 1965; Koopmans, 1965; Romer, 1990), implementing an endogenous life expectancy improves our understanding of economic growth processes.

Inspecting the fertility rate more closely, we observe that it is constant in the long run if the economy experiences sustained economic growth.

$$n_t = \begin{cases} \frac{\xi-\theta}{\psi(1+\xi+\beta)} & \text{for } \lim_{b_t\to\infty} \phi_t\left(b_t\right) = 1, \\ \frac{\xi-\theta}{\psi(1+\xi+\beta\phi_t)} & \text{otherwise.} \end{cases}$$

The same framework without lifetime uncertainty would yield a constant fertility rate from the beginning on. The effect of economic development on fertility would be abrogated which can be reasonable if focusing solely on advanced economies that have exhibited low and relatively constant fertility rates over the last few decades (Prettner and Werner, 2016). By introducing an endogenously determined life expectancy, it is possible to separate the longevity effect on fertility decisions from the income effect on fertility decisions, which leads us to Remark 3.1.

Remark 3.1. *Increases in life expectancy induce higher savings for retirement at the expense of reduced fertility.*

These relationships are in line with the literature surrounding longevity and economic growth (see Blackburn and Cipriani, 2002; Zhang and Zhang, 2005; Cervellati and Sunde, 2005). Given that children born in period t become adults in period $t+1$, the size of the labor force, L_t, evolves according to

$$L_{t+1} = \frac{\xi - \theta}{\psi(1+\xi+\beta\phi_t)} L_t.$$

Again, for sustained economic growth, life expectancy will be at its upper limit such that population growth is constant in the long run. Turning towards human capital creation, the law of motion for individual human capital is given by

$$h_{t+1} = A_E e_t h_t.$$

Since home education is assumed, parents' level of human capital, h_t, the educational effort, e_t, and parents' productivity in education, A_E, determine the level of individual human capital of the next generation. Economy-wide aggregate human capital is then given by

$$H_t = h_t L_t, \tag{3.5}$$

and the labor force participation rate can be expressed as

$$\Omega_t = \frac{1+\beta\phi_t}{1+\xi+\beta\phi_t}. \tag{3.6}$$

Combining Equations (3.5) and (3.6), the stock of aggregate human capital available for production and R&D, $\tilde{H}_t$, can be calculated as

$$\tilde{H}_t = \Omega_t H_t. \tag{3.7}$$

The available stock of aggregate human capital determines overall labor input. Since the labor force participation rate increases but fertility decreases in life expectancy, the long-run effect is ambiguous and requires further investigation. We go into more details in Sections 3.4 and 3.5.

3.2.2 Government

Similar to Barro (1990), the government finances its health spending by taxing aggregate income, Y_t^{agg}, with a health tax, τ. It needs to keep a balanced budget at all times, i.e., health investments can only be financed by the tax income received in the same period. The governmental constraint then reads

$$\tau Y_t^{agg} = B_t,$$

where the left-hand side are aggregate governmental revenues and the right-hand side is aggregate governmental health spending, B_t. Incorporating a medical technology, λ, and remembering that the government only aims at improving adults' health, the health status of an adult is given by

$$b_t = \lambda \frac{B_t}{L_t}.$$

Individuals' health and, with it, the probability to survive to retirement increases in the medical resources per adult invested and in the productivity of these investments.

3.2.3 Production side

The production side of the economy builds up on Romer (1990), Jones (1995) and Prettner and Werner (2016). There are three sectors: the final goods sector, the intermediate goods sector and the R&D sector. In short, employing scientists, new ideas/blueprints are developed in the perfectly competitive R&D sector and are sold to an intermediate goods producer who then becomes the monopolist in producing that specific product for one period. The intermediate good of variant i, x_t^i, is produced in a one-for-one technology using physical capital, k_t, as the only input. Due to Dixit and Stiglitz (1977) monopolistic competition, the intermediate good is sold with a mark-up over the market price to the final goods producer. The operating profits are necessary to compensate for

R&D expenses.[4] The final goods producer uses the intermediate good as well as final goods workers as inputs to produce the final good under perfect competition.

Life expectancy affects productivity and production through two channels: aggregate savings, $S_t = s_t L_t$, and the available aggregate human capital, $\tilde{H}_t$. Longevity-induced changes in the saving behavior affect the capital that can be invested in intermediate goods companies, which, in turn, affects the production of new blueprints and final goods production. The size of the labor force and with it also R&D and final goods production, is affected twofold: on the one hand, through fertility, increases in life expectancy reduce the future size of the workforce, on the other hand, fewer children increase labor force participation.

The final good, Y_t, is produced according to

$$Y_t = (1-\tau)\left(H_t^Y\right)^{1-\alpha} \sum_{i=1}^{A_t} \left(x_t^i\right)^{\alpha},$$

where H_t^Y denotes workers employed in final goods production, A_t is the number of differentiated blueprints developed until time t, x_t^i is the number of machines for each blueprint i used in final goods production and α is the elasticity of final output with respect to machines. We include $(1-\tau)$ to account for the governmental health tax.[5] Perfect competition ensures that all production factors are paid their marginal value products, such that the wage per effective worker, w_t^Y, and the price for machines of type i, p_t^i, are given by, respectively

$$w_t^Y = \frac{\partial Y_t}{\partial H_t^Y} = (1-\alpha)(1-\tau)\left(H_t^Y\right)^{-\alpha} \sum_{i=1}^{A_t} \left(x_t^i\right)^{\alpha}, \tag{3.8}$$

$$p_t^{Y,i} = \frac{\partial Y_t}{\partial x_t^i} = \alpha(1-\tau)\left(H_t^Y\right)^{1-\alpha} \left(x_t^i\right)^{\alpha-1}. \tag{3.9}$$

Turning to the monopolistic intermediate goods sector, the profit function of intermediate

[4]The monopolist only owns the patent for one period. For all future periods, we assume that the government sells the patent and invests the proceeds unproductively.

[5]This formulation corresponds to the government taxing capital and labor income at equal rates.

goods producer i, $\pi_t^{x,i}$, reads

$$\pi_t^{x,i} = p_t^{Y,i} x_t^i - R_t x_t^i. \tag{3.10}$$

Combining Equations (3.9) and (3.10) and using $k_t^i = x_t^i$, profit maximization results in

$$\frac{\partial \pi_t^{x,i}}{\partial k_t^i} = 0 \qquad \Leftrightarrow \qquad p_t^i = \frac{R_t}{\alpha}. \tag{3.11}$$

The more differentiated the intermediate goods are, i.e., the smaller α is, the more market power the monopolist has and the higher the mark-up over the market price is. Since all monopolists are similar, we can infer from Equation (3.11) that all monopolists charge the same price and produce the same amount, i.e., $p_t^i = p_t$ and $x_t^i = x_t$. Therefore, dropping the machine variety index i, the aggregate production function can be written as

$$Y_t = (1-\tau)\left(A_t H_t^Y\right)^{1-\alpha} K_t^{\alpha},$$

where $K_t = A_t k_t$ is aggregate capital. The production function has now the same Cobb-Douglas form as in standard neoclassical growth models (see, for example, Solow, 1956).

Moving on to the R&D sector, new ideas are developed according to the following knowledge production function

$$A_{t+1} - A_t = \delta A_t^{\chi} H_t^A. \tag{3.12}$$

H_t^A are scientists employed in R&D. Their productivity depends on the general productivity in research, δ, and on the number of already existing ideas (standing on shoulders effect). We allow for intertemporal knowledge spillovers $\chi \in (0,1)$ to partly nest the Jones (1995) framework. Given the price of a patent, p_t^A, and the wage rate per effective labor of scientists, w_t^A, the profit function in the R&D sector is given by

$$\pi_t^A = p_t^A \delta A_t^{\chi} H_t^A - w_t^A H_t^A.$$

Perfect competition then allows to derive an expression for w_t^A.

$$\frac{\partial \pi_t^A}{\partial H_t^A} = 0 \qquad \Leftrightarrow \qquad w_t^A = p_t^A \delta A_t^\chi. \tag{3.13}$$

The wage per effective labor of scientists depends on the price that can be charged for one blueprint and on scientists' overall productivity.

3.2.4 Market clearing

The first condition for an equilibrium is that operating profits of the intermediate goods producer, π_t^x, and the price of a patent, p_t^A, equal. Combining Equations (3.9), (3.10) and (3.11), we observe that this relationship holds for

$$p_t^A = \pi_t^x = \alpha(1-\alpha)\frac{Y_t}{A_t}. \tag{3.14}$$

The second condition for an equilibrium is that wages of workers and scientists equal, $w_t^Y = w_t^A = w_t$. Equating Equations (3.8) and (3.13) and using Equation (3.14), we derive an equilibrium condition for aggregate human capital employed in final goods production

$$H_t^Y = \frac{A_t^{1-\chi}}{\delta\alpha}. \tag{3.15}$$

Considering that human capital is solely employed in final goods production and in R&D, H_t^A is given as

$$H_t^A = \tilde{H}_t - H_t^Y. \tag{3.16}$$

Inserting Equations (3.7) and (3.15) into Equation (3.16), we are able to derive an expression for aggregate human capital employed in R&D

$$H_t^A = \Omega_t H_t - \frac{A_t^{1-\chi}}{\delta\alpha}. \tag{3.17}$$

Using Equation (3.17) in the knowledge production function, the equilibrium stock of ideas evolves according to

$$A_{t+1} = \delta A_t^{\chi}\Omega_t H_t - A_t\left(\frac{1-\alpha}{\alpha}\right). \tag{3.18}$$

Defining a growth rate as $g_{x,t} = (x_{t+1} - x_t)/x_t$, the growth rate of new ideas, $g_{A,t}$, is given by

$$g_{A,t} = \delta A_t^{\chi-1}\Omega_t H_t - \frac{1}{\alpha}. \tag{3.19}$$

Since we are interested in the growth effects of changes in life expectancy, we inspect Equation (3.19) with respect to ϕ_t. We observe that the survival probability enters through the labor force participation rate, Ω_t, which leads us to the following remark.

Remark 3.2. *In the short run, through higher labor force participation, higher life expectancy increases the rate at which new ideas are developed.*

Proof. The partial derivative of $g_{A,t}$ with respect to the survival probability, ϕ_t, is given by

$$\frac{\partial g_{A,t}}{\partial \phi_t} = \delta A_t^{\chi-1} H_t \frac{\beta\xi}{(1+\xi+\beta\phi_t)^2}.$$

The denominator of this expression is always positive. For $\delta, \beta, \xi \in (0,1)$ and for the reasonable case of $A_t > 0$ and $H_t > 0$, the numerator is also positive, such that the survival probability has a strictly positive effect on productivity growth in the short run. □

Last, capital accumulation needs to be derived. For full depreciation of physical capital and since savings of period t are invested in period $t+1$, the standard law of motion for capital can be applied, where aggregate savings determine the stock of aggregate physical capital in the next period

$$K_{t+1} = s_t L_t.$$

Combining Equations (3.8) and (3.15), the wage rate per effective labor can be written as

$$w_t = (1-\alpha)(1-\tau)A_t^{1-\alpha}K_t^{\alpha}\left(\frac{A_t^{1-\chi}}{\delta\alpha}\right)^{-\alpha}.$$

Together with the expression for individual savings, s_t, from Equation (3.4), the stock of physical capital then evolves according to

$$K_{t+1} = L_t h_t \frac{\beta\phi_t}{1+\xi+\beta\phi_t}(1-\alpha)(1-\tau)A_t^{1-\alpha}K_t^{\alpha}\left(\frac{A_t^{1-\chi}}{\delta\alpha}\right)^{-\alpha}.$$

The survival probability, ϕ_t, enters positively through the propensity to save, $\beta\phi_t/(1+\xi+\beta\phi_t)$. Economic progress that raises life expectancy entails a multiplying effect, since a longer life span increases capital accumulation and, with it, economic progress and life expectancy.

To sum up, for labor market and financial market clearing, we have the following system of equations that describes how our model economy evolves over time

$$\begin{aligned}
A_{t+1} &= \delta A_t^{\chi}\frac{1+\beta\phi_t}{1+\xi+\beta\phi_t}h_t L_t - A_t\frac{1-\alpha}{\alpha}, \\
K_{t+1} &= L_t h_t \frac{\beta\phi_t}{1+\xi+\beta\phi_t}(1-\alpha)(1-\tau)A_t^{1-\alpha}K_t^{\alpha}\left(\frac{A_t^{1-\chi}}{\delta\alpha}\right)^{-\alpha}, \\
Y_{t+1}^{Agg} &= \left(\frac{A_{t+1}^{2-\chi}}{\delta\alpha}\right)^{1-\alpha}K_{t+1}^{\alpha}, \\
L_{t+1} &= L_t\frac{\xi-\theta}{\psi(1+\xi+\beta\phi_t)}, \\
h_{t+1} &= \frac{A_E\theta\psi}{\eta(\xi-\theta)}h_t, \\
\phi_{t+1} &= \frac{\lambda\tau\frac{Y_{t+1}^{Agg}}{L_{t+1}}}{1+\lambda\tau\frac{Y_{t+1}^{Agg}}{L_{t+1}}}.
\end{aligned}$$

For a given set of parameters and by choosing initial values for the endogenous variables A_0, K_0, L_0 and h_0, this system fully describes the evolution of our model economy over time. Life expectancy, represented by the survival probability ϕ_{t+1}, is, of course, also endogenous but is completely determined inside the model.

3.3 Balanced growth path

The system above can be solved analytically by deriving its balanced growth path (BGP). Along the BGP, $g_{x,t} = g_{x,t+1} = g_x$ must hold. In the limit, the survival probability approaches

$$\lim_{y_t \to \infty} \phi_t(y_t) = 1,$$

such that we can drop it in the BGP analysis. Accordingly, fertility is constant and the growth rate of the labor force is given by

$$g_L = \frac{L_{t+1}}{L_t} - 1 = \frac{\xi - \theta}{\psi(1+\xi+\beta)} - 1.$$

The growth rate of individual human capital is also constant and given by

$$g_h = \frac{h_{t+1}}{h_t} - 1 = A_E \frac{\theta\psi}{\eta(\xi - \theta)} - 1.$$

For a constant labor force participation rate, the growth rate of new ideas can be derived as

$$g_{A,t} = \frac{A_{t+1}}{A_t} - 1 = \delta A_t^{\chi - 1} \frac{1+\beta}{1+\xi+\beta} h_t L_t - \frac{1}{\alpha}.$$

Setting $g_{A,t} = g_{A,t+1}$ and solving for g_A, the BGP expression reads

$$g_A = \left[\frac{A_E \theta}{\eta(1+\xi+\beta)}\right]^{\frac{1}{1-\chi}} - 1,$$

where the term in square brackets represents the growth factor of aggregate human capital. The BGP expressions for physical capital and for aggregate production can be derived in the same way and read

$$g_K = \left[\frac{A_E\theta}{\eta(1+\xi+\beta)}\right]^{\frac{2-\chi}{1-\chi}} - 1,$$
$$g_{Y^{agg}} = \left[\frac{A_E\theta}{\eta(1+\xi+\beta)}\right]^{\frac{2-\chi}{1-\chi}} - 1,$$

where we note that $g_K = g_{Y^{agg}}$. Since there are no life expectancy effects in the long run, which seems reasonable considering that life expectancy gains are slowing down (see Olshansky et al., 2005; Dong et al., 2016), economic progress is solely driven by growth in aggregate human capital and in the intertemporal knowledge spillovers. This result is similar to Prettner and Werner (2016). We summarize this is in the following proposition.

Proposition 3.1. *As the economy progresses, life expectancy reaches an upper limit, such that there are no more longevity-induced growth effects in the long run.*

This proposition has to be treated with some caution. From an analytical perspective, setting $\phi = 1$ in the long run is correct, however, it ignores level effects and, most importantly, it ignores that life expectancy is still projected to further increase over the next decades (Oeppen et al., 2002; Strulik and Vollmer, 2013; Kontis et al., 2017). To account for that, we examine the transitional dynamics in Sections 3.4 and 3.5.

3.4 Simulation

To analyze the transitional dynamics and to discuss the plausibility of our results, we compare the model dynamics to U.S. data over the period 1960–2017. Given the scope of the paper, we aim at resembling developments in p.c. GDP, life expectancy at birth, fertility and the population size. All data have been taken from the World Bank (2019).

The parameter and initial values chosen are summarized in Table 3.1. The health tax rate is set to 11.4%, which corresponds to the average U.S. national health expenditure as percentage of GDP over the chosen time horizon (CMS.gov, 2019). The discount factor of $\beta = 0.67$ implies an annual discount rate of 2%, which is in line with Auerbach and Kotlikoff (1987). The value of the elasticity of final output with respect to intermediate inputs, α, is set to 0.3, which is close to the standard value used in the literature (Jones, 1995; Acemoglu, 2009). Also, $\psi = 0.08$ is similar to Strulik et al. (2013). All other values are chosen for the model to fit the U.S. data as precisely as possible.

Table 3.1: Parameter and initial values for the simulation

Parameter	Value	Parameter	Value	Variable	Value
τ	0.114	α	0.3	A_0	17.5
β	0.67	δ	6.5	K_0	0.8
ψ	0.08	χ	0.076	L_0	10
ξ	0.327	A_E	1.65	h_0	1
θ	0.181	η	0.16		
λ	0.9				

To ensure sure that the model resembles the economic development of the U.S. reasonably well, Figures 3.2 and 3.3 show to the evolution of p.c. output and of the population size in the data (dashed red line) and in the model (solid blue line). Keeping in mind that the model economy consists of three overlapping generations, the model population size needs to be derived first. We do so by calculating the size of the entire population in period t as $Pop_t = \phi_{t-1}L_{t-1} + L_t + 0.5n_tL_t$, where the first term refers to retirees, the second term are adults and the third term represents children. The model p.c. GDP is then calculated by dividing aggregate output by the population size, i.e., $y_t = Y_t^{agg}/Pop_t$. We observe that p.c. GDP in the data and in the model increase over time by a factor of three, approximately. In the data, p.c. GDP growth accelerates in the 1980's and 1990's and partially slows down afterwards. In the model, a similar trend is visible. Economic progress is strong until around the year 2000 when the dot-com bubble burst and slows down afterwards. Also, the increase in the size of the population matches the one in the U.S.

Turning to the underlying driving forces of economic growth, changes in life expectancy are of the main interest. U.S. life expectancy has been rising by approximately nine years from 1960–2010 and is stagnant since then. In Figure 3.4, the model life expectancy resembles the data quite well, only underestimating the slowdown during the last decade.

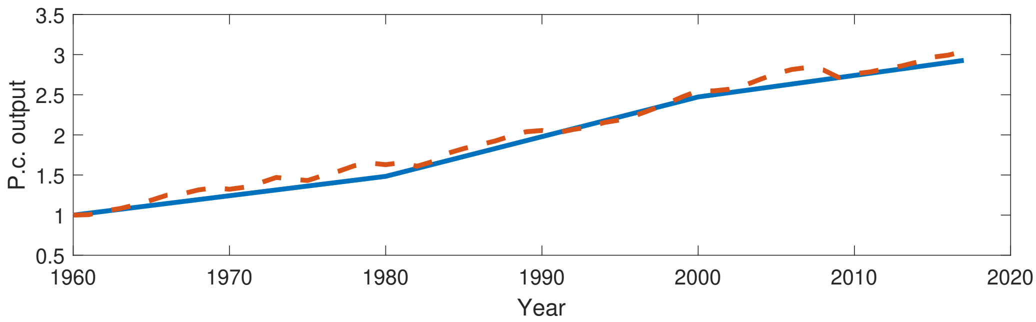


Figure 3.2: Evolution of p.c. output (model prediction: solid blue line; data: dashed red line)

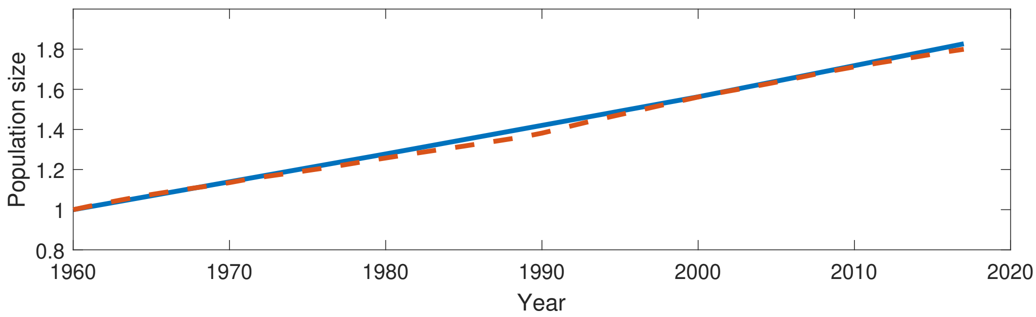


Figure 3.3: Evolution of the population size (model prediction: solid blue line; data: dashed red line)

A critical examination, however, calls for caution. The stagnant U.S. life expectancy is not driven by stagnant economic development (as the model would suggest) but by several other aspects such as increased rates of deaths of despair and unequal access to health services (Case and Deaton, 2015; Dowell et al., 2017; Harper et al., 2017) that are out of the scope of this paper. Given the positive effect of life expectancy on economic growth, discussed in Section 3.5, and also supported by other studies (Lorentzen et al., 2008; Cervellati and Sunde, 2013; Bloom et al., 2014), this result should rather be an incentive for governments to improve health services in order to not only decrease mortality but to also promote long-run growth. This conclusion is consistent with Baldanzi et al. (2019) where increases in life expectancy increase both productivity and individual utility derived from living longer. We further elaborate on that in the next section.

Figure 3.5 visualizes the fertility effects of changes in the lifetime horizon. Fertility data are transformed into the number of unisex adults to make them comparable to the model results. In the data, there is a sharp decrease in fertility until the mid 1970's visible. Afterwards, fertility fluctuates just below the replacement level. Obviously, in the model,

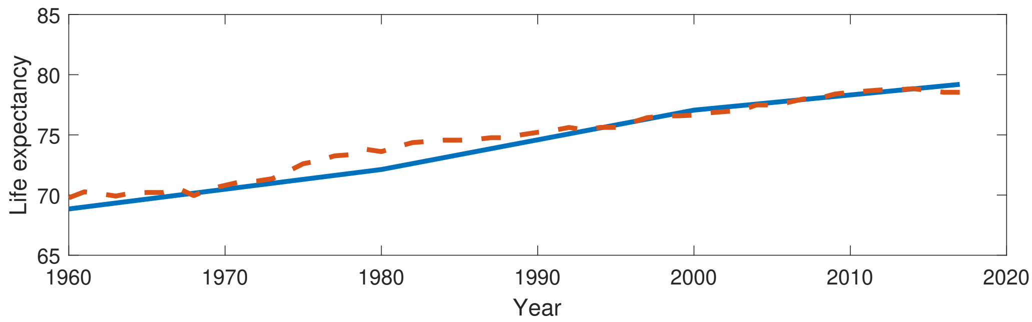


Figure 3.4: Evolution of life expectancy (model prediction: solid blue line; data: dashed red line)

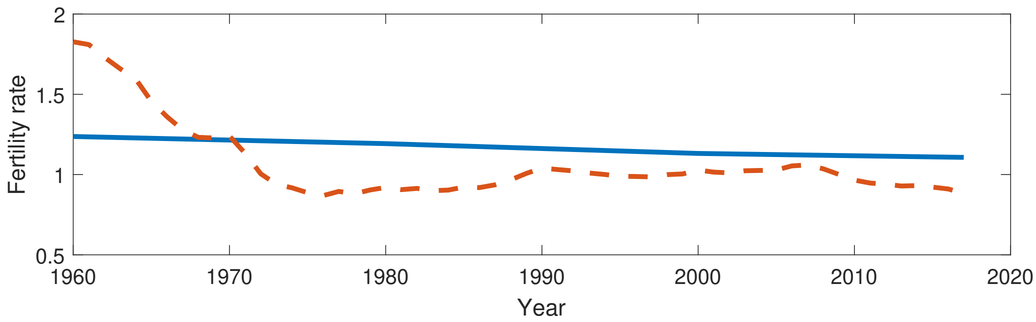


Figure 3.5: Evolution of fertility (model prediction: solid blue line; data: dashed red line)

the longevity-induced fertility decline is not able to explain the sharp decrease we see in the data. However, the life expectancy effect becomes visible. For the chosen parameter values, over time, the model fertility rate declines to 0.91 along the BGP and approaches the fertility rate that we observe in the data. Remembering that the change in the model fertility is solely driven by changes in life expectancy, there is another possible takeaway. The unisex U.S. fertility rate has fallen by 0.95, while the model fertility rate has fallen by 0.13. Accordingly, the model captures approximately 14% of the changes in fertility over that time horizon, which leads us to Remark 3.3.

Remark 3.3. *The model result suggests that over the period 1960–2017, 14% of the observed decline in the U.S. fertility rate is due to increases in life expectancy.*

This result can be explained as follows. The quantity-quality trade-off initiated through increases in income is usually assumed to be the main driver for reductions in fertility (Becker and Lewis, 1973). Through the design of our model economy, this trade-off is non-existent over time and the only time-dependent effect remaining is how a change in the lifetime horizon affects individual choices. In our case, as life expectancy increases, individuals require higher savings for old-age consumption. In a standard overlapping generations setting, reduced adult consumption would be the only effect. In our setting, individuals additionally have the opportunity to increase their lifetime earnings by working more, i.e., by increasing the labor force participation rate. Therefore, they choose to have fewer children in order to increase labor income. Although, this is typically not the main motivation for having fewer children, it is one piece of the puzzle to explain fertility behavior and its economic implications.

3.5 Comparative statics

From the previous analysis it has become clear that, in our framework, life expectancy exhibits growth effects through savings, fertility and labor force participation. However, their magnitudes and interrelations have yet to be discussed. In a dynamic general equilibrium setting, as in the present work, it is next to impossible to exactly determine the overall effect analytically. To provide an intuitive example, consider an increase in the health tax τ. We have shown that, as a response, immediately life expectancy and, with it, labor force participation and the saving rate increase. Therefore, higher available human capital, $\tilde{H}_t = \Omega_t L_t$, raises productivity and income for the next period. Given the dynamic effects on $\tilde{H}$ over time, the change in the size of the labor force needs to be considered, too. An increase in life expectancy decreases fertility and, thus, the future size of the labor force. Assuming, for now, that life expectancy remains constant in the next period, the effect of a longevity-induced change in fertility in period t on the aggregate stock of human capital in period $t+1$ can be calculated as

$$
\begin{aligned}
\tilde{H}_{t+1} &= (1 - n_t\psi - n_t\eta e)n_t L_t h_{t+1}, \\
\frac{\partial \tilde{H}_{t+1}}{\partial n_t} &= (1 - 0.358 n_t) L_t h_{t+1},
\end{aligned}
$$

where we use the parameter values from Section 3.4. Only for a model fertility rate of $n > 2.79$, the positive effect of higher labor force participation will outweigh the negative effect of a smaller labor force. Since this corresponds to an actual fertility rate of 5.58, one could expect that, for reasonable fertility rates, aggregate available human capital decreases in life expectancy and exerts a negative impulse on economic development. This, however, ignores longevity-induced changes in period $t+1$. Given that an increase in life expectancy raises labor force participation and savings in period t, productivity and capital available for investments in period $t+1$ will be higher. This, in turn, raises period $t+1$ production and therefore also $t+1$ life expectancy. The question at hand is whether the longevity-induced increases in technology and capital can outweigh the negative effects on labor supply. This strongly depends on many other aspects, such as further endogenous changes in labor force participation or the effects of the reduction in savings caused by a higher health tax.

To shed some light on the directions and the magnitudes of the described channels, we test how p.c. output in the calibrated model changes if life expectancy is kept completely constant, respectively, if only single channels are switched off. As displayed in Figure 3.6, in the baseline case with a fully endogenous survival probability, p.c. output increases by a factor of 2.93. To asses the overall effect, we keep the economy-wide life expectancy in 1960 constant and observe a reduction in p.c. output of 11.9% over 58 years. This result comes as expected and is in line with the central conclusions of many other works (see, for example, Chakraborty, 2004; Cervellati and Sunde, 2011; Prettner, 2013; Prettner and Trimborn, 2017). Without any longevity-induced effects over time, there is less innovation and less capital, which drags down growth in the economy.

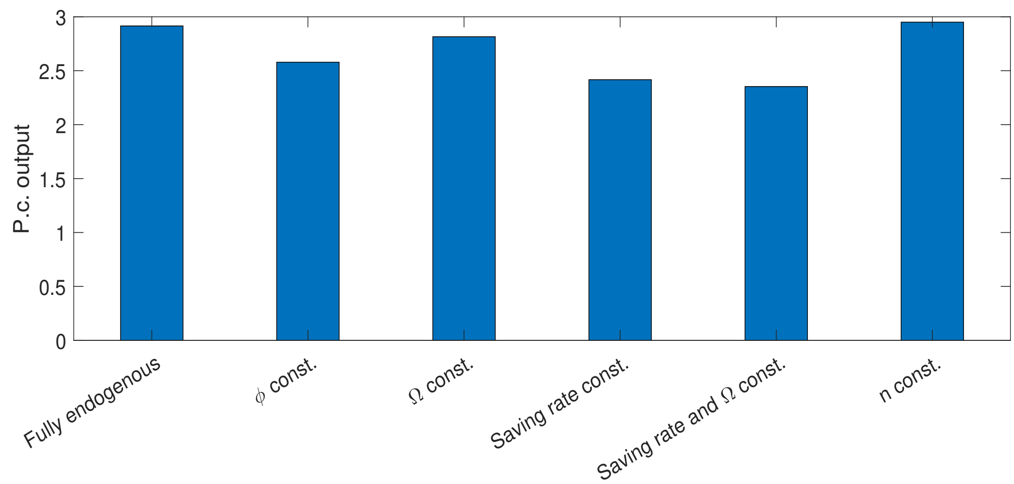


Figure 3.6: Level of p.c. output keeping different channels constant

In a second step, we are interested in disentangling the aggregate life expectancy effect. We do so by keeping the survival probability in savings, labor force participation and fertility constant, separately and sequentially. First of all, we notice that all channels point in the expected directions. As life expectancy is switched off in labor force participation, p.c. output decreases to 2.82. The effect is even stronger for savings, where p.c. output in 2017 would only be at 2.42 times the level in 1960. Combining both channels, the reduction in p.c. output amounts to 0.57 or 19.5% in total. For fertility, the opposite effect can be observed. Ignoring the effect of changes in longevity on the fertility decision, p.c. output increases by 0.008 because of a larger workforce, which translates into a higher growth rate of new ideas. This change seems reasonably small considering that only longevity-induced changes in fertility are being accounted for. We summarize the findings of this comparative static exercise in the following remark.

Remark 3.4. *Through capital accumulation and the larger number of scientists and workers, life expectancy induces economic growth. The positive effects of higher savings and higher labor force participation outweigh the negative one of lower fertility.*

In principle, there are further potential effects that could be considered in future works. Just to mention a few, including doctors and nurses as a third type of human capital would impose an additional negative effect of life expectancy on economic development since longer lives would reduce the share of human capital available for production and R&D. One potential positive effect that has been abstracted from in this analysis is the income-dependent quantity-quality trade-off. Incorporating longevity- or income-determined education as in de la Croix and Licandro (1999) and Strulik et al. (2013) would weaken the negative fertility effects and could, potentially, even outweigh them.

So far, the average U.S. health share of 11.4% between 1960–2017 has been used in the analysis. To gain additional insights into the relationship between life expectancy and economic development, in Figure 3.7 we investigate the maximum attainable 2017 life expectancy and level of p.c. output for $\tau \in [0, 1]$.

As expected, we find a hump-shaped relationship. Also, for $\tau = 0$ and $\tau = 1$ zero production takes place. For zero health investments, the survival probability and the

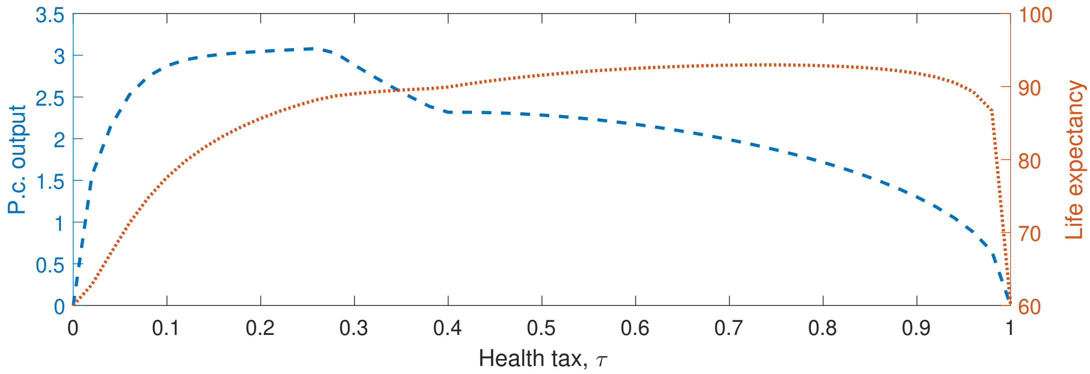


Figure 3.7: p.c. output and life expectancy for $\tau \in [0, 1]$ (p.c. output: dashed blue line; life expectancy: dotted red line)

saving rate are zero and, thus, the capital stock and production are zero, too. For small health shares, the gains in output as well as in life expectancy are large. According to our model, a maximum output of $y^{max} = 3.08$ can be reached with a tax rate of $\tau = 0.271$. This corresponds to a life expectancy of 88.6 years. For larger health shares, there is a steep decline in output. The logic behind is that, at some point, the positive effects of increasing longevity are outweighed by the resource demand of health care. Interestingly, life expectancy continues to increase way beyond the growth maximizing tax rate. For $\tau = 0.734$, a maximum life expectancy of 92.9 years can be reached. The trade-off in terms of economic development is, however, huge. Compared to the growth-maximizing size of the health care sector, an additional 3.85 years of life expectancy come at the cost of reducing p.c. output by 38.2%.

Remark 3.5. *Increasing the health share beyond its growth-maximizing size comes at the cost of a steep decline in p.c. output, while the additional gains in life expectancy are rather small.*

Our results also contribute to the discussion surrounding rising health shares and the consequent effects on economic development. Given that the average OECD health share has increased from 9.3% to 12.6% over the period 2000–2017 (World Bank, 2019), we can, at least, be guardedly optimistic that this increase in absolute and relative health expenditures did not only extend individual's lives but also fostered economic growth. This result contrasts Kuhn and Prettner (2016) who include health care personnel and find a smaller optimal size of the health sector of 8.61% along the BGP. Since in our framework, the optimal health share decreases over time and remembering that we abstract from human capital employed in health care, a larger optimal size of the health sector is corollary.

Turning to the welfare effects of longevity-driven economic growth, Kuhn and Prettner (2016) conduct an analysis on the trade-off between life and growth and find that it can be optimal to increase the size of the health care sector beyond its growth-maximizing size. In principle, our results do not rule out this conclusion, nonetheless, a thorough analysis including more generations would be necessary to properly assess the welfare effects within our framework.

3.6 Conclusions

We introduce an endogenous survival probability into a growth framework driven by purposeful R&D. The frequency at which new ideas are developed increases in the level of aggregate savings and in the number of scientists employed. Higher savings imply higher operating profits since more machines can be produced. This raises the demand for and, thus, the wage rate of scientists, resulting in higher R&D employment and faster economic progress.

Life expectancy increases in the public resources devoted toward health, which, in turn, raises the incentives to save and to work more, while it induces a negative effect on the fertility decision. We disentangle the separate channels and are able to show that, overall, life expectancy exerts positive growth effects. The reduction in the size of the labor force over time is overcompensated by higher labor force participation and, especially, by higher savings and investments. Additionally, we show that the growth-maximizing size of the health care sector is way smaller than the size that would maximize life expectancy. Given that within the OECD, the average share of the health care sector is substantially smaller than our growth-maximizing size, devoting further resources toward health care might not only increase life expectancy but also foster economic development.

Our model approach allows for several interesting extensions. We abrogate from income-dependent education, which would, most likely, spur longevity-induced growth effects. Also, including human capital employed in health care might provide additional interesting insights, as increases in life expectancy would draw away labor from production and R&D and exert a potential negative effect on economic growth. Related to that, dividing the health sector into health care personnel and medical researchers could be a promising avenue for future research because it would allow for a more realistic modeling of aging.

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CHAPTER 4

The Scientific Revolution and Its Role in the Transition to Sustained Economic Growth*

*This chapter is joint work with Klaus Prettner.

Abstract

We propose a Unified Growth model that analyzes the role of the Scientific Revolution in the takeoff to modern economic growth. Basic scientific knowledge is a necessary input in the production of applied knowledge, which, in turn, fuels productivity growth and leads to rising incomes. Eventually, rising incomes instigate a fertility transition and a takeoff of educational investments and human capital accumulation. In regions where scientific inquiry is severely constrained (for religious reasons or because of oppressive rulers), the takeoff to modern growth is delayed or might even not occur at all. The novel mechanism that we propose for the latent transition towards a takeoff could contribute to our understanding of why sustained growth emerged first in Europe.

4.1 Introduction

Though the world does not change with a change of paradigm, the scientist afterward works in a different world.
(Thomas S. Kuhn, 1970)

The introductory quote of famous twentieth century historian and philosopher Thomas Kuhn, who established the notion of the *Scientific Revolution*, could be transferred easily to popular works of modern growth economics. In these R&D-based models of economic growth as pioneered by Romer (1990), Aghion and Howitt (1992), and Jones (1995)[1], scientists build upon the existing stock of knowledge such that inventions made in the past facilitate the future R&D-process. Of course, the historical period of the Scientific Revolution that paved the way for the Age of Enlightenment and for subsequent social and economic progress has a much broader interpretation. It comprises not only the stock of past knowledge that facilitates future research such as Isaac Newton's laws of motion, the law of universal gravitation, and the subsequent research on vacuums in the seventeenth century, but also the paradigm shifts in the foundations of scientific inquiry (the scientific method), and changes in the possibilities to disseminate knowledge. For example, the "Republic of Letters", a group of scientists and intellectuals who discussed and shared ideas intensively changed the way of knowledge dissemination and finally led to the establishment of the first journals.

The deepening of knowledge how nature works, the emergence of the scientific method, and the faster dissemination of ideas that are all rooted in the Scientific Revolution inspired tinkerers and engineers over the next centuries to build upon this knowledge base in moving the technological frontier forward (Mokyr, 2016). Without the knowledge on motion and vacuums, without painstakingly experimenting, and without the possibility to share ideas, the steam engine might not have been developed and improved by Denis Papin, Thomas Savery, Thomas Newcomen, and James Watt (Rosen, 2010).

[1] For a non-exhaustive list of contributions in endogenous, semi-endogenous, and Schumpeterian growth theory, see, for example, Grossman and Helpman (1991), Kortum (1997), Dinopoulos and Thompson (1998), Peretto (1998), Segerström (1998), Young (1998), Howitt (1999), Dalgaard and Kreiner (2001), Strulik (2005), Bucci (2008), Peretto and Saeter (2013), Strulik et al. (2013), and Prettner (2014).

The Unified Growth Theory as developed in the seminal works of Galor and Weil (2000) and Galor (2005, 2011)[2], led to a better understanding among economists on the mechanisms that triggered the escape from the Malthusian trap, resulting in the Industrial Revolution and in the takeoff toward sustained modern economic growth. This strand of literature usually emphasizes the quality-quantity tradeoff that affects the size and the education of the labor force and, with it, the rate at which new ideas are developed. What these models do not consider is the above explained scientific basis that is necessary for productive applied R&D to take place. Prettner and Werner (2016) include a basic scientific research sector into an endogenous growth framework of the Jones (1995) type and analyze the extent to which basic research influences modern economic growth. However, Prettner and Werner (2016) do not focus on the interactions between basic scientific research and applied research over the very long run and how these interactions facilitate a takeoff toward the phase of sustained economic growth.

We aim at contributing to the literature by analyzing the extent to which the Scientific Revolution could have influenced the following escape from Malthusian stagnation. We do this by merging the two strands of Unified Growth Theory and R&D-based endogenous growth theory that includes both basic scientific knowledge and applied patentable knowledge. As is standard in the Unified Growth literature, the model features utility-maximizing households with a quality-quantity tradeoff regarding the number of children and their education. An increase of income over time leads the economy up to a point at which investments in education become positive and and a fertility transition sets in (see, for example, Strulik et al., 2013). The associated increase in human capital accumulation is then one of the central divers of the takeoff toward sustained economic growth.

In contrast to the standard Unified Growth literature, however, there is an additional engine for the takeoff toward sustained economic growth that provides the basis for for the rise in the income level that leads to the fertility transition in the first place. This second driving force is represented by the evolution of the stock of basic scientific knowledge,

[2]Other prominent contributions include the works of Jones (2001), Kögel and Prskawetz (2001), Hansen and Prescott (2002), Galor and Moav (2002, 2004, 2006), Doepke (2004), Cervellati and Sunde (2005, 2011), Strulik and Weisdorf (2008), and Strulik et al. (2013).

which is a necessary input in the production of applied knowledge in a purposeful R&D sector along the lines of Romer (1990) and Jones (1995). Applied R&D only becomes profitable and operative once a large enough stock of basic scientific knowledge in a society exists. Only then does the applied research sector start to produce the patents that are needed in the intermediate goods sector to produce the differentiated machines that are, in turn, required in the final goods sector to produce the consumption aggregate.

This structure of the model makes clear that the takeoff of applied R&D is a central driver of long-run economic development that enables the fertility transition later on. The increase in applied R&D, however, cannot occur if there is no basic scientific knowledge base in the economy. This mechanism is our proposed formal modeling of the contribution of the Scientific Revolution as a major trigger of the later Industrial Revolution and the takeoff to modern economic growth as described by Wootton (2015) and Mokyr (2016). We believe that the suggested novel approach enables a more sophisticated understanding of the growth process over the very long run and of the economic importance of the interaction between the basic scientific knowledge stock of a society and the accumulation of applied knowledge in the transition from stagnation to sustained long-run economic progress.

The paper is organized as follows. In Section 4.2, we introduce the basic model assumptions, the structure of the household side, and the properties of the production side of the economy. In Section 4.3, we derive the balanced growth path analytically. In Section 4.4, we present the model simulation and discuss comparative statics with regards to the timing of the Scientific Revolution and its effect on the timing of the Industrial Revolution. Finally, in Section 4.5, we summarize our findings and provide suggestions for future research.

4.2 The model

In this section we describe the basic knowledge-driven growth framework in the vein of Romer (1990) and Jones (1995) into which we incorporate an endogenous fertility-education decision along the lines of Becker and Lewis (1973), Galor and Weil (2000), and

Strulik et al. (2013) and a basic science sector in which the laws of nature are deciphered and the foundations for applied knowledge are laid along the lines of Prettner and Werner (2016).

4.2.1 Basic assumptions

Consider a small open economy that is populated by three overlapping generations: children, adults, and retirees. Children receive consumption from their parents and retirees consume out of their savings accumulated in adulthood. At the end of old-age, individuals die with certainty.[3] Adults are modeled as single-sex parents[4] and make all economically relevant decisions on i) consumption during adulthood and old-age, ii) the number of their children, and iii) the education investments in each child. The resulting consumption-saving decision impacts on intermediate goods production and thereby on the incentives to develop new blueprints in applied R&D. A necessary input in applied R&D is at least a basic knowledge of the laws of nature, on scientific inquiry, and on the way to disseminate insights. This knowledge is generated in a basic scientific sector by thinkers who decipher how nature works. The better a society's understanding of the laws of nature, of scientific inquiry, and of knowledge dissemination are, the more productive is applied R&D. Since applied R&D is one of the main drivers of long-run economic growth, basic scientific knowledge acts as a catalyst of the takeoff to sustained economic growth.

The fertility decision of adults determines the evolution of the population size, whereas the education decision determines individual human capital accumulation. There is a quality-quantity tradeoff of parents in the sense that they can increase the number of their children but at the expense of lower investments in the education of each child (and vice versa). For low levels of economic development, education investments are a luxury good and parents find it optimal to choose the corner solution of no education investments

[3]For simplicity, we abstract from pension schemes and from a changing life expectancy because pension schemes are rather a 20th century development (Boersch-Supan and Wilke, 2004) and the implementation of endogenous life expectancy would complicate the model without altering the central results (for a Unified Growth Model that takes changing mortality into account, see Cervellati and Sunde, 2005).

[4]The assumption of single-sex adults is made to abstract from modeling intra-family bargaining processes, which allows us to focus on the macroeconomic effects. For contributions that investigate the intra-household decision process in more detail see, for example, de la Croix and Vander Donckt (2010), Bloom et al. (2015), Prettner and Strulik (2017), and Doepke and Kindermann (2019).

and high fertility. Once income surpasses a certain threshold, investments in education of children sets in, which triggers a quality-quantity substitution of increasing education investments and falling fertility. As in standard Unified Growth Models, this is another main engine for the takeoff to sustained economic growth.

4.2.2 Consumption side

Individuals derive utility from consumption during adulthood, c_t, from consumption during retirement, $c_{t+1} = s_t(1 + \bar{r})$, where s_t are savings and $\bar{r}$ is the rate of return, from having children, n_t, and from the education investments in their children, e_t. For simplicity, we assume a small open economy such that the capital rental rate is determined on the world market. Utility is logarithmic and determined according to the following function

$$u_t = \log(c_t) + \beta \log[s_t \cdot (1 + \bar{r})] + \xi \log(n_t) + \theta \log(e_t + \bar{e}), \tag{4.1}$$

where β refers to individual impatience[5], ξ represents the preferences of parents for the number of children, and θ are the preferences of parents for children's education. The parameter $\bar{e}$ represents a minimum informal education level that children acquire through observation and learning-by-doing even if parents do not invest in the education of their children at all (see Strulik et al., 2013). This parameter ensures that education is a luxury good and it does not pay off for poor societies to invest in formal education. This formulation captures the situation in agrarian pre-industrial societies well in which children mainly learned by working alongside their parents and peers on the fields.

The lifetime budget constraint is given by

$$(1 - \psi n_t) w_t h_t = c_t + s_t + \eta e_t n_t, \tag{4.2}$$

where w_t is the wage rate per unit of human capital, h_t. The price of a unit of education

[5] The parameter β induces a similar individual behavior as a probability to die between adulthood and old age. Thus, a small β can also be interpreted as having a relatively short retirement phase, which fits most of human history (Chakraborty, 2004; Baldanzi et al., 2019b).

is given by η, whereas ψ denotes the fraction of parental time that each child requires to be raised (Galor and Weil, 2000; Galor, 2005, 2011). The product $w_t h_t$ is labor income per worker for a given level of individual human capital and $1 - \psi n_t$ represents the labor force participation rate. Individuals save s_t of their wage income for old-age consumption. The reminder is spent on consumption during adulthood, c_t, and on children's education, $\eta e_t n_t$. Expenditures on education depend, in turn, on the cost of each unit of education, η, the quantity of education, e_t, and the number of children, n_t. Overall, this setting implies a quality-quantity tradeoff: on the one hand, more children increase utility; on the other hand, more children decrease the amount of resources that can be devoted to the education of each child. This decrease in education per child has then a negative effect on utility.[6]

Maximizing (4.1) subject to (4.2) yields the following optimality conditions for consumption, savings, fertility, and education

$$c_t = \frac{w_t h_t}{1+\beta+\xi}, \qquad n_t = \frac{(\xi-\theta) w_t h_t}{(1+\beta+\xi)(\psi w_t h_t - \eta \bar{e})},$$

$$e_t = \frac{\theta \psi w_t h_t - \xi \eta \bar{e}}{\eta(\xi-\theta)}, \qquad s_t = \frac{\beta w_t h_t}{1+\beta+\xi}.$$

We observe that consumption and savings increase with income, while consumption decreases with the discount factor and savings increase with the discount factor. In addition, we observe that fertility stays constant in the long-run limit even for rising income, which is in line with the literature (Galor and Weil, 2000; Galor, 2005, 2011; Strulik et al., 2013). For fertility to be positive, $\xi > \theta$ and $w_t h_t > \eta \bar{e}/\psi$ have to hold. These parameter restrictions are reasonable because they rule out the situation in which parents would want to invest in the education of their children before choosing to have children at all and they ensure a minimum level of income that is needed for positive fertility (i.e., to ensure that the population does not become extinct in the next generation). Education investments

[6] If, instead, the costs of fertility were given by a fixed amount of resources, fertility would increase perpetually with rising income, which is counterfactual. Since, in this case, education would also rise with income, the quality-quantity tradeoff vanished.

cannot be negative such that the possibility of a corner solution exists

$$e_t = \begin{cases} 0 & \text{for } w_t h_t < \xi\eta\bar{e}/\theta\psi \\ \frac{\theta\psi w_t h_t - \xi\eta\bar{e}}{\eta(\xi-\theta)} & \text{otherwise.} \end{cases}$$

Altogether, parents only invest in the education of their children after the wage income has surpassed the given threshold $\xi\eta\bar{e}/\theta\psi$.

4.2.3 Human capital

Children's education determines the next generation's level of human capital when the children of the previous period become adults and supply their time on the labor market. To derive adult's human capital, we set the parental expenditures on education equal to the costs of education (the salaries of teachers) and isolate the implied employment level of teaching personnel. Aggregate educational expenditures of parents are given by $\eta e_t n_t L_t$, where L_t is the number of workers/households in period t. Educational expenditures are thus education expenditures per child $(\eta \cdot e_t)$, multiplied by the number of children (n_t), and aggregated over all households that invest in education (L_t). The costs of education are the wages of teachers given by $H_t^E w_t$, with H_t^E being the aggregate human capital employed in education. Equating educational expenditures with educational costs and solving for human capital employment in the schooling sector yields

$$H_t^E = \frac{\eta e_t n_t L_t}{w_t}.$$

Assuming that the human capital of the next generation depends on the educational resources invested in each child and denoting the productivity of teachers by μ, individual human capital at time $t+1$ can be derived as

$$h_{t+1} = \frac{\mu H_t^E}{L_{t+1}} + \bar{e}.$$

In this expression, μH_t^E describes the economy-wide offered schooling. Dividing the economy-wide offered schooling by the number of pupils in period t (i.e., the number of adults in period $t+1$), yields the educational resources devoted to each child, which represents the quality of schooling. In case of a low parental income, education expenditures are zero and no teachers will be employed in the economy. Pupils would then solely learn by observing their parents and peers such that individual human capital stayed equal to the costless informal education that each child obtains, $\bar{e}$.

4.2.4 Production side

There are four sectors, the final goods sector, the intermediate goods sector, the applied R&D sector, and the basic scientific research sector. The aggregate final good is produced under perfect competition using workers and an intermediate good as input. The intermediate good, in turn, is produced under Dixit and Stiglitz (1977) monopolistic competition using one unit of final output to produce one unit of the intermediate good, x_t (cf. Aghion and Howitt, 2009). For the monopolist to produce the intermediate good, a blueprint needs to be bought from the applied research sector. The necessary funds are collected by issuing shares that can be purchased using household's savings. For simplicity, we abstract from physical capital in the production process. Its inclusion would not alter our main findings but it would complicate the model substantially (see also Galor and Weil, 2000).

The accumulation of applied knowledge (in the form of patents/blueprints) follows Romer (1990) and Jones (1995) after the takeoff to modern economic growth occurred. Applied knowledge is produced in a purposeful R&D sector in which profit-driven firms invest in the creation of the new patents/blueprints to derive a stream of profits from the associated monopolistic competition. This setting is augmented by a basic scientific sector as in Prettner and Werner (2016), in which the laws of nature are deciphered and the methods of scientific inquiry are invented. The stock of accumulated knowledge in this sector provides the basis for applied research. Since the laws of nature and the way of performing science cannot be patented, the output of this sector is non-excludable such that this sector is not profit-driven. In addition, the ideas that are generated in this sector are non-rival such that its use by one researcher in applied research does not impinge on the productivity of the idea when other applied researchers are using them.

We conceptualize the non-excludability of the results of scientific inquiry in the sense that great minds are either i) intrinsically motivated to think about how nature works or ii) that they do it because it might raise a thinker's reputation among her peers. In modern times, basic research is typically funded by governments and conducted in research institutes and universities.[7] Since we do not want to overburden our model, we abstract from the public financing of modern basic science and focus on the potential way how basic scientific discoveries could historically have occurred and contributed to the takeoff to modern knowledge-based economic growth. The underlying assumption is that the number of eureka moments increases with the size of the population (Kremer, 1993) and with its education level (Strulik et al., 2013). Scientists might also form societies/journals to disseminate their thoughts and ideas such that the knowledge they create diffuses to other parts of the society and can be used by the scientists in the applied research sector to create new patents/blueprints (Mokyr, 2002, 2005, 2016; Wootton, 2015). More generally, the output that this sector produces could be thought to comprise everything that makes it easier to discover new technologies and accumulate more basic and applied knowledge. In that sense the output of the basic scientific sector can be interpreted as an important part of the *Culture of Growth* (Mokyr, 2016) that is necessary for a society to engage in the creation of new ideas and thereby to foster *progress* (Wootton, 2015).

The aggregate final good is produced according to the Cobb-Douglas production function

$$Y_t = (H_t^Y)^{1-\alpha} \sum_{i=1}^{A_t} (x_t^i)^\alpha,$$

where H^Y is human capital employed in final goods production (i.e., the stock of knowledge of workers in the final goods sector), x^i is the amount of intermediate good i used in production, $\alpha \in (0, 1)$ is the elasticity of final output with respect to the employment of

[7]For the modeling of a modern basic research sector along these lines, see, for example, Gersbach et al. (2012), Gersbach and Schneider (2015), Akcigit et al. (2013), Prettner and Werner (2016), and Gersbach et al. (2018).

intermediate goods, and A_t refers to the stock of blueprints available in period t. Thus, there are A_t different intermediate goods used in the production of the final good.

Perfect competition ensures that all production factors are paid their marginal value products. The wage per unit of human capital of final goods producers and the price of intermediate good i are therefore given by

$$\begin{aligned} w_t^Y &= (1-\alpha)\frac{Y_t}{H_t^Y}, \\ p_t^{Y,i} &= \alpha \left(H_t^Y\right)^{1-\alpha} \left(x_t^i\right)^{\alpha-1}. \end{aligned}$$

Using the second expression, the profit function in the intermediate goods sector i becomes

$$\pi_t^{x,i} = p_t^{Y,i} x_t^i - x_t^i.$$

Because the intermediate goods producer utilizes a one-for-one technology, the costs of production are equal to the amount of final output employed in the production process. Profit maximization then leads to the optimal pricing rule

$$p_t^i = \frac{1}{\alpha}.$$

In the standard Romer (1990) framework, the price of intermediate good i additionally depends on the capital rental rate. Since we abstract from any sort of physical capital in our model economy, the capital rental rate drops out. The mark-up of the monopolist only depends on the elasticity of final output with respect to intermediates. An immediate implication is that all intermediate goods producers charge the same mark-up over the price that obtains in a perfectly competitive market such that prices do not depend on the machine variety i anymore. The total quantity of intermediate goods produced can then be expressed as

$$x_t = H_t^Y \alpha^{\frac{2}{1-\alpha}}.$$

Aggregate output, operating profits in the intermediate goods sector, and the wage rate per unit of human capital in the final goods sector thus simplify to

$$
\begin{aligned}
Y_t &= A_t H_t^Y \alpha^{\frac{2\alpha}{1-\alpha}}, \\
\pi_t^x &= \frac{1-\alpha}{\alpha} \alpha^{\frac{2}{1-\alpha}} H_t^Y, \\
w_t^Y &= (1-\alpha) A_t \alpha^{\frac{2\alpha}{1-\alpha}}.
\end{aligned}
$$

The applied research sector follows Prettner and Werner (2016). The stock of patents increases according to the production function

$$
A_{t+1} - A_t = \delta A_t^{\chi} B_t^{\sigma} H_t^A.
$$

As in Romer (1990) and Jones (1995), the development of new ideas depends on the stock of already existing ideas, A_t, on the amount of human capital employed in applied research, H_t^A, and on the productivity of scientists in this sector, δ. To analyze the effect of the Scientific Revolution, we also include basic scientific knowledge, B_t, as a necessary input for applied knowledge production. In this setting, χ measures the extent of intertemporal knowledge spillovers (standing on shoulders externality) in the production of applied knowledge, while σ measures the extent of intersectoral knowledge spillovers from basic scientific knowledge to applied research. To focus on a meaningful economic solution, human capital employed in R&D (H_t^A) needs to be non-negative. Thus, the stock of ideas cannot decrease over time.

Already from this formulation, the importance of the Scientific Revolution for the Industrial Revolution becomes obvious. Overall productivity of applied research is given by $\delta A_t^{\chi} B_t^{\sigma}$, which determines the profitability of this sector and the amount of labor that it employs. Without any knowledge of the laws of nature, or, for that matter, with a culture that does not foster scientific inquiry, applied scientists are unproductive and new blueprints/patents cannot be discovered. As a consequence, no applied scientists are employed by firms, which reduces the frequency at which new ideas are developed to zero. This approximates, from a formal perspective, the historical state of economies before the Scientific Revolution (Wootton, 2015). Nature is still arcane and profit-driven R&D is non-existent.

Once this state is overcome and a positive stock of basic scientific knowledge exists, applied knowledge production becomes feasible. In more recent times, applied R&D firms maximize their profits

$$\pi_t^A = p_t^A \delta A_t^\chi B_t^\sigma H_t^A - w_t^A H_t^A,$$

where the first term on the right-hand side is the revenue of selling ideas at the price p_t^A and the second term is the cost of employing human capital H_t^A, which is the wage w_t^A per unit of human capital employment. Maximizing profits with respect to the employment of applied scientists, H_t^A, yields the following relation between wages in applied research and effective productivity in the applied research sector

$$w_t^A = p_t^A \delta A_t^\chi B_t^\sigma.$$

Clearly, if applied R&D firms can charge higher prices, p_t^A, for the blueprints that they sell, the wages of applied scientists are higher such that this sector could attract more employment and, thus, produce more ideas. If scientists were more productive (δ were higher), a similar argument held true and employment of applied scientists and thereby technological progress would be faster. Finally, a greater stock of basic scientific knowledge, B_t, also fosters applied research productivity and leads to faster technological progress and faster economic growth.

As argued above, if $B_t = 0$ holds, then the wages of applied scientists were zero and no technological progress would take place. As the stock of basic scientific knowledge increases, $(B_t - B_{t-1} > 0)$, the productivity of applied knowledge creation rises gradually, such that wages and employment of applied scientists also rise. This, in turn, fosters technological progress and economic growth and catalyzes a takeoff toward sustained knowledge-driven economic development.

Labor market clearing implies that the wage rates of workers in the final goods sector and those of scientists in the applied research sector have to equalize. Considering that prices of patents, p_t^A, are paid for by discounted operating profits, $\pi_t^x/(1+\bar{r})$, the amount of human capital employed in final goods production can be expressed as

$$H_t^Y = \frac{(1+\bar{r})A_t^{1-\chi}}{\alpha\delta B_t^\sigma}.$$

Given that human capital is employed in production, education and R&D, H_t^A can be derived as

$$H_t^A = (1-\psi n_t)H_t - H_t^Y - H_t^E.$$

Turning to the basic scientific research sector, the knowledge base increases according to the production function

$$B_{t+1} - B_t = \kappa H_t^\lambda,$$

where, unlike in the applied research sector, deciphering the laws of nature is not compensated.[8] We follow Kremer (1993) and Strulik et al. (2013) in the assumption that the discovery of new basic scientific knowledge depends on the overall number of thinkers in the economy and on their education, i.e., on the stock of aggregate human capital. We also include a stepping-on-toes externality as represented by the inverse of λ, to account for potential duplication of research effort as in Jones (1995). Finally, κ is the research productivity in the basic science sector. A situation in which $\kappa = 0$ could be interpreted as capturing a society in which religion or oppressive institutional settings prevent scientific inquiry. Thus, in the words of Mokyr (2016), the "Culture of Growth" would be absent.

Putting all the information together, we arrive at the following system of equations

[8]As argued above, introducing compensation of basic scientific knowledge creation via public funding and taxes is possible but it complicates the model substantially without leading to new insights. For the workings of the model for a modern economy in which basic scientific knowledge is created in publicly funded universities and research facilities see Prettner and Werner (2016). However, these authors fully abstract from the takeoff to modern economic growth, the Scientific Revolution, and a Unified Growth setting.

that fully describes the evolution of our model economy over time

$$A_{t+1} = A_t + \delta A_t^{\chi} B_t^{\sigma} H_t^A, \tag{4.3}$$

$$B_{t+1} = B_t + \kappa H_t^{\lambda}, \tag{4.4}$$

$$h_{t+1} = \frac{\mu H_t^E}{n_t} + \bar{e}, \tag{4.5}$$

$$n_{t+1} = \frac{(\xi - \theta) w_{t+1} h_{t+1}}{(1 + \beta + \xi)(\psi w_{t+1} h_{t+1} - \eta \bar{e})}, \tag{4.6}$$

$$L_{t+1} = n_t L_t, \tag{4.7}$$

$$w_{t+1} = (1 - \alpha) A_{t+1} \alpha^{\frac{2\alpha}{1-\alpha}}, \tag{4.8}$$

$$H_{t+1}^Y = \frac{(1 + \bar{r}) A_{t+1}^{1-\chi}}{\alpha \delta B_{t+1}^{\sigma}}, \tag{4.9}$$

$$H_{t+1}^E = \frac{\eta L_{t+1} n_{t+1}}{w_{t+1}} \frac{\theta \psi w_{t+1} h_{t+1} - \xi \eta \bar{e}}{\eta(\xi - \theta)}, \tag{4.10}$$

$$H_{t+1}^A = (1 - \psi n_{t+1}) h_{t+1} L_{t+1} - H_{t+1}^Y - H_{t+1}^E, \tag{4.11}$$

Here, Equation (4.3) refers to the equilibrium evolution of the stock of applied knowledge that is needed for the production of differentiated intermediate goods that are, in turn, used in the production of the final output. Equation (4.4) refers to the evolution of the stock of basic scientific knowledge that is an essential input in the production of applied knowledge and lays the foundation for a takeoff toward modern knowledge-based economic growth. Equation (4.5) describes the evolution of individual human capital depending on the knowledge that children acquire by observing their parents and peers and by the purposeful education investments of parents. The latter only become positive once an economy has surpassed a certain income threshold, facilitating the takeoff toward sustained economic growth. Equation (4.6) refers to the fertility choice of households that determines population growth. In line with empirical observations, fertility decreases after a certain stage of economic development is reached and then converges to a lower but positive level. Equation (4.7) captures the evolution of the workforce. Equation (4.8) delivers the wage rate per unit of human capital that increases with the stock of applied knowledge in the economy. Finally, Equations (4.9)–(4.11) express employment of human capital in final goods production, education, and R&D, respectively.

In the next section, we use this system to derive the balanced growth path (BGP) analytically. Afterwards, we solve the model numerically to analyze the extent to which basic scientific knowledge drives the takeoff toward sustained long-run growth.

4.3 Balanced growth path

Along the BGP, the growth rates of all variables and the employment shares remain constant. In the following, we denote the growth rate of a variable x between periods t and $t+1$ by $g_{x,t} = (x_{t+1} - x_t)/x_t$.

We observe that positive growth implies ever rising incomes ($\lim_{t\to\infty} w_t h_t = \infty$), such that fertility and educational investments along the BGP are equal to

$$n = \frac{\xi - \theta}{(1+\beta+\xi)\psi}, \tag{4.12}$$

$$e_t = \frac{\theta\psi w_t h_t}{\eta(\xi - \theta)}. \tag{4.13}$$

Along the BGP, fertility is constant and education is growing with $w_t \cdot h_t$. Considering that consumption, c_t, and savings, s_t, also grow with $w_t \cdot h_t$, the BGP growth rates of individual human capital and of the wage rate need to be determined. Evolution of individual human capital follows the equation

$$h_{t+1} = \frac{\mu\eta e_t n_t L_t}{w_t L_{t+1}} + \bar{e}.$$

Substituting e_t from Equation (4.13) and using that $L_{t+1}/L_t = n_t$, we arrive at

$$h_{t+1} = \frac{\mu\theta\psi h_t}{\xi - \theta} + \bar{e}. \tag{4.14}$$

Along the BGP, $\bar{e}$ becomes negligibly small compared with formal schooling as represented by the first term in Equation (4.14). Therefore, the BGP growth rate of individual human capital can be expressed as

$$g_h = \frac{\mu\theta\psi}{\xi - \theta} - 1. \tag{4.15}$$

Wage growth solely depends on growth in productive ideas as we know from Equation (4.8). We therefore turn towards examining the growth rate of A_t in the next step. From Equation (4.3) we get

$$g_{A,t} = \frac{\delta B_t^{\sigma} H_t^A}{A_t^{1-\chi}}. \tag{4.16}$$

By definition, the growth rate of A must be constant along the BGP, i.e., we have that $g_{A,t} = g_{A,t+1}$ holds for all t. This occurs if

$$g_{A,t} = \left(\frac{B_{t+1}}{B_t}\right)^{\frac{\sigma}{1-\chi}} \left(\frac{H_{t+1}^A}{H_t^A}\right)^{\frac{1}{1-\chi}} - 1, \tag{4.17}$$

is fulfilled such that the numerator and the denominator of Equation (4.16) grow at the same rate. In addition, also the growth rate of B must be constant, i.e., we must have $g_{B,t} = g_{B,t+1}$, which holds for

$$\frac{B_{t+1}}{B_t} = \left(\frac{H_{t+1}}{H_t}\right)^{\lambda}. \tag{4.18}$$

Next, we derive the expression H_{t+1}/H_t in Equation (4.18) by substituting in the expressions for aggregate human capital such that

$$\frac{H_{t+1}}{H_t} = \frac{L_{t+1}h_{t+1}}{L_t h_t}. \tag{4.19}$$

Using that fertility is constant along the BGP and taking advantage of Equation (4.15), Equation (4.19) can be rewritten as

$$\frac{H_{t+1}}{H_t} = n\frac{\mu\theta\psi}{\xi - \theta}. \tag{4.20}$$

Inserting Equation (4.20) into Equation (4.18), the growth factor of scientific knowledge along the BGP becomes

$$\frac{B_{t+1}}{B_t} = \left(n\frac{\mu\theta\psi}{\xi-\theta}\right)^{\lambda}. \tag{4.21}$$

Finally, the BGP expression for H^A_{t+1}/H^A_t has to be determined. Along the BGP, the share of human capital employed in applied research is constant. Therefore, $g_{H^A} = g_H$ has to hold, which implies

$$\frac{H^A_{t+1}}{H^A_t} = \frac{H_{t+1}}{H_t}. \tag{4.22}$$

Using equations (4.20), (4.21), and (4.22) in Equation (4.17), the growth rate of applied knowledge along the BGP follows as

$$g_A = \left(n\frac{\mu\theta\psi}{\xi-\theta}\right)^{\frac{1+\lambda\sigma}{1-\chi}} - 1.$$

Substituting in the fertility rate from Equation (4.12), we arrive at the following expression for the long run BGP growth rate in the modern growth regime

$$g_A = \left(\frac{\theta\mu}{1+\beta+\xi}\right)^{\frac{1+\lambda\sigma}{1-\chi}} - 1. \tag{4.23}$$

In the expression for the long-run growth rate, Equation (4.23), household behavior is represented by the term in brackets and mainly relates to the choice of having more children versus educating them well. From this expression, a number of intuitive results that are in line with the standard literature (cf. Strulik et al., 2013; Prettner and Werner, 2016; Baldanzi et al., 2019a) follow. The preference parameter for education, θ, raises individual human capital accumulation of the next generation and reduces fertility, whereas the reverse holds true for the preference parameter for the number of children, ξ. In line with Strulik et al. (2013), the negative effect of decreasing fertility on aggregate human capital accumulation is overcompensated by the positive effect of accumulating human capital faster. The reason is that a decline in fertility sets free additional resources via the budget constraint that can be used to invest in education. Thus, economic growth increases with θ and decreases with ξ. There is an additional positive effect represented by μ, which is the productivity of teachers. If teachers are more productive, then, for a given investment in education, human capital accumulates faster. This does not affect fertility and only raises human capital accumulation. Thus, technological progress and income growth increase. We summarize these effects in the following proposition.

Proposition 4.1.

i) An increase in education investments and a decline in fertility, as triggered by an increase in the parameter θ or a decrease in the parameter ξ, unambiguously raise long-run economic growth because the positive effects of greater education investments on aggregate human capital accumulation outweigh the negative effects of lower fertility.

ii) An increase in teaching productivity, μ, unambiguously raises long-run economic growth.

On top of these results, the long-run growth rate increases with the standing on shoulders effect, λ, because it determines the rate at which basic scientific knowledge accumulates. The long-run growth rate also increases with the intersectoral knowledge spillovers, σ, because they increase the importance of basic scientific knowledge in the production of new patents. Both of these effects increase the productivity of human capital employed in applied research and thereby raise the rate at which new patents are developed. This, in turn, raises final goods production and income growth. We summarize these results in the following proposition.

Proposition 4.2. *For $\chi < 1$, long-run economic growth increases unambiguously with faster accumulation of basic scientific knowledge as represented by the terms λ and σ. Thus, basic scientific knowledge is an important driver of economic prosperity.*

This proposition shows the importance of basic scientific knowledge for long-run economic growth in the modern regime. Irrespective of the assumption $\chi < 1$, which usually implies that long-run growth is only a function of the parameters that determine population growth and education (as in Jones, 1995), our result shows that basic scientific knowledge accumulation and human capital accumulation attain crucial roles in determining economic prosperity.

4.4 Simulation

4.4.1 Data

The simulation resembles developments of total factor productivity (TFP), basic scientific knowledge, wage income, the net fertility rate, and individual human capital. Our aim is to use long term data from the United Kingdom that reach back before the Industrial Revolution. We choose the UK as a reference because it is an important forerunner in both the Scientific Revolution and the Industrial Revolution (Galor, 2005, 2011; Wootton, 2015; Mokyr, 2016). In addition, the data coverage and the data quality for the UK tends to better over such a long time horizon than for other countries.

We take the data on TFP from FRED (2017) that contains annual TFP growth rates from 1761 onward. Using 25-years averages to eliminate business-cycle fluctuations, we derive the change in the level of TFP over time. We approximate basic scientific knowledge by means of the annual number of cited references (Ware and Mabe, 2015) with data from 1651 onward. As explained in Section 4.2.4, basic scientific knowledge is useful for applied research without, however, being patentable, i.e., it is non-rival and non-excludable. We are well aware of the fact that the number of citations is only a crude indicator for scientific activity but it is the best that we have at our disposal. In addition, more citations would surely imply a higher rate of knowledge diffusion and, thus, indicate a higher use of basic scientific research in applied research.

Since we abstract from physical capital in the production process, a direct indicator for economic development in terms of income growth is the wage per worker. As a proxy for this wage rate in the UK, we refer to the real wage of UK craftsmen during 1700–2000 reported by Clark (2005). Given that the majority of the population was low-skilled historically, this is arguably an acceptable proxy.

In our model, fertility is the number of children per unisex adult. Choosing fertility in the UK as a comparison would be misleading because of high rates of child mortality, especially before the twentieth century (see Doepke, 2005). We therefore combine the data set of Ajus and Lindgre (2015) on fertility rates in the UK with the data set of Johansson et al. (2015) on child mortality in the UK to calculate the net reproduction rate. The resulting time series on the net reproduction rate per woman is then equal to the unisex net fertility rate as used in our model and it covers the period 1800-2000.

Finally, education and with it individual human capital is one of the main driving forces of the transition to sustained economic growth. Thus, our simulation should match the corresponding data. We therefore use the time series on mean years of schooling in the UK from Madsen and Murtin (2017) and apply a Mincer equation as in Prettner et al. (2013) and Hall and Jones (1999) to transform the education data from 1700–2000 into units of human capital.

4.4.2 Simulation results

For our simulation we have data covering up to 300 years. We choose the following parameter values and initial conditions to match these data. The elasticity of final output with respect to intermediates is set to $\alpha = 0.3$, which is in line with the literature (Jones, 1995; Acemoglu, 2009). Similar to Strulik et al. (2013), the time costs for raising one child are 8%, i.e., $\psi = 0.08$. The yearly individual discount rate is approximately 3%, which corresponds to a discount factor of $\beta = 0.3$ over 40 years (Cropper et al., 2014). All other parameter values are set to fit the data as precisely as possible. In so doing, we set $\xi = 0.35$, $\bar{e} = 0.5$, $\theta = 0.23$, $\eta = 0.1$, $\delta = 1.15$, $\kappa = 0.4$, $\chi = 0.59$, $\mu = 5.4$, $\sigma = 0.15$, and $\lambda = 1$.[9] The initial values for productivity, basic scientific knowledge, and the size of the workforce are taken as $A_0 = 10$, $B_0 = 10$, and $L_0 = 1$.

Figure 4.1 shows the evolution of TFP over time, with the data (dashed red line) and the model results (solid blue line) being normalized to unity in 1820. Broadly consistent

[9]Note that the intertemporal spillovers, χ, are substantially larger than the intersectoral spillovers, σ. By that we avoid a situation in which basic scientific knowledge is the main driver for economic progress.

with existing works, TFP is stagnant for decades until the mid-nineteenth century, when the Industrial Revolution altered production possibilities in a fundamental way (Galor and Weil, 2000; Galor, 2005, 2011; Mokyr, 2005; Strulik et al., 2013). Not only does our TFP calibration match the onset of the Industrial Revolution, it also predicts the length and the magnitude of the takeoff as well as the phase of sustained economic growth from the twentieth century onward reasonably well.

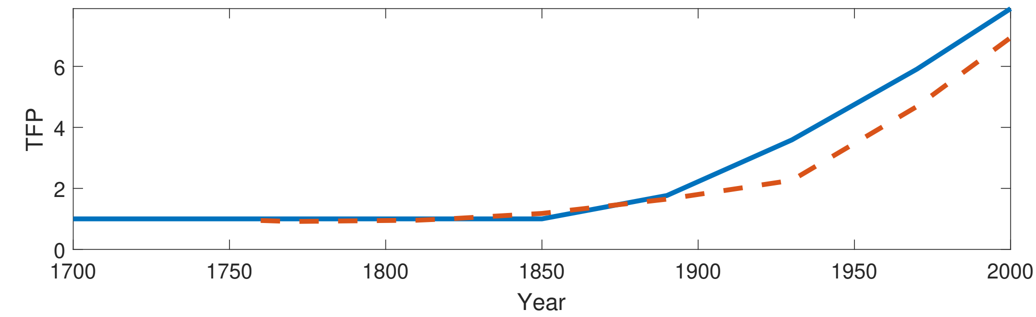

Figure 4.1: Evolution of TFP (model prediction: solid blue line; data: dashed red line)

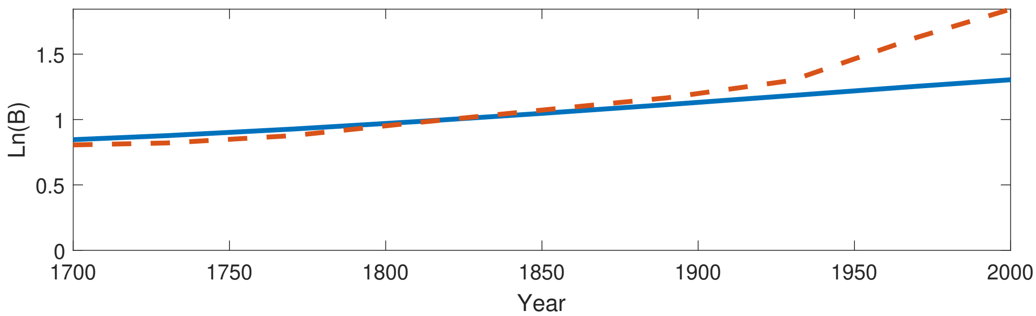

Figure 4.2: Logarithm of the stock of basic scientific knowledge (model prediction: solid blue line; data: dashed red line)

Which dynamics pave the way to sustained economic growth? Before the onset of the Industrial Revolution, wage income is low. Accordingly, educational investments are low, whereas the fertility rate is high. Productive R&D increases with the stock of existing blueprints, with the stock of basic scientific knowledge, and with the amount of human capital devoted to applied research. For early stages of development, productivity and basic scientific knowledge are small, as is the stock of aggregate human capital. Scientists in the applied research sector are relatively unproductive, which is why the labor force is employed in final goods production, leaving productivity stagnant. A growing population and almost constant education slowly but gradually raise the aggregate stock of human capital. Due to decreasing marginal productivity in the final goods sector and a slow increase in the stock of basic scientific knowledge that comes with the rise in the population size, productivity of applied researchers increases and becomes high enough for researchers to be attracted into applied research. This is the time when productivity levels start to rise slowly at first and at a faster pace later.

Additional insights are obtained from Figure 4.2 by taking a closer look at the role of basic scientific knowledge in the process towards the takeoff. While the Industrial Revolution and with it productivity growth started around the turn of the nineteenth century (Ashton, 1997), the takeoff in basic scientific discoveries occurred about one century before. The increase in the growth rate of citations is stronger in the data than the increase in the growth rate of basic scientific knowledge in the model. The main reason for that difference is that in our model all basic scientific discoveries are productive, i.e., they raise productivity in applied research immediately. However, as we all tend to know only too well from personal experience, not all scientific research is relevant for applications. In particular, over time, basic scientific research has broadened. While in the past, the share of research in the natural sciences was comparatively high, it has decreased as other disciplines, such as economics, have gained importance. Therefore, over time, the share of scientific research that is useful for applied research might have decreased, which could explain the gap between the model predictions and the data.

Wage income is depicted in Figure 4.3 and is also normalized to unity in 1820. The value derived from the simulation is the available income per worker. As for TFP, we predict the takeoff approximately right. The income gap that emerges during the twentieth century can be attributed to the presence of skilled workers in the workforce and an associated increase in the skill premium (Acemoglu, 1998). Since our model incorporates production workers as well as scientists, one would expect a steeper increase in wages compared to craftsmen's wages.

In Figure 4.4, the fertility rate in the model decreases over time and the quantity-quality trade-off induces an even stronger decrease after the takeoff in income growth. Comparing the model outcome to UK data, a similar trend can be observed. Importantly,

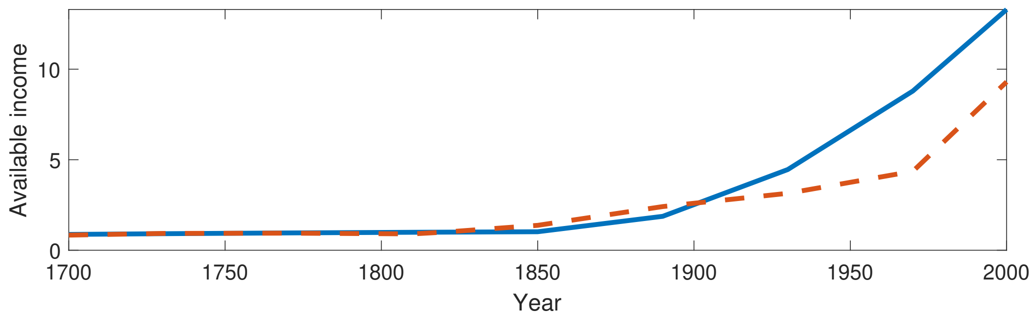

Figure 4.3: Evolution of available income (model prediction: solid blue line; data: dashed red line)

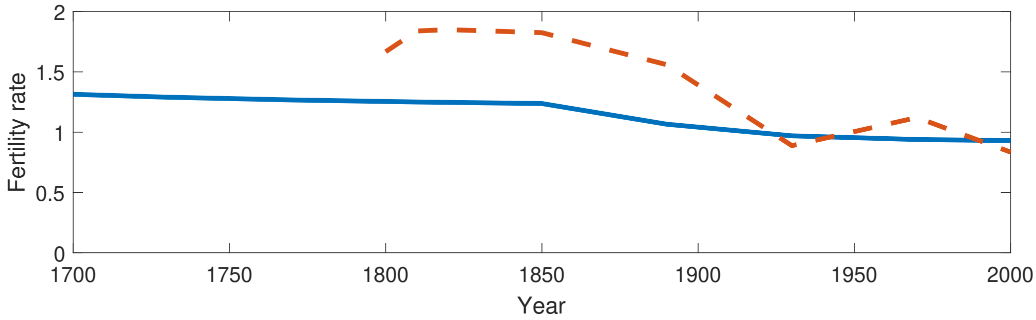

Figure 4.4: Evolution of fertility (model prediction: solid blue line; data: dashed red line)

the fertility rate is high for low levels of development and it decreases below replacement fertility at the end of the twentieth century. The main differences between the series are due to changes in life expectancy over time that our model does not capture. High mortality rates before the onset of the demographic transition slowed down population growth in the UK and in the rest of the world (Human Mortality Database, 2019). This negative pressure on the population size is not present in our model because life expectancy is assumed to be constant. Therefore, for the pre-industrialization area, the model fertility rate can be smaller than the fertility rate from the data.

Finally, inspecting Figure 4.5, individual human capital in the data and in the model increases at the same rate until the Industrial Revolution, after which an increase in the growth rate in the data can be observed that the model does not match fully. One important reason is the absence of differential skills, which would induce higher investments in education of some parts of the population (Acemoglu, 1998). Another reason for the discrepancy might also be that the data only reflect the quantity of schooling without controlling for quality, which the model captures.

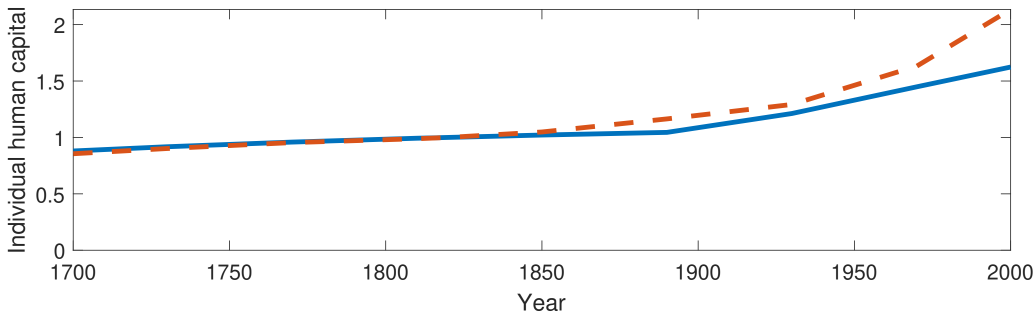

Figure 4.5: Individual human capital (model prediction: solid blue line; data: dashed red line)

4.4.3 Comparative statics

So far, we have shown the importance of the Scientific Revolution for long-run economic growth from an analytical and numerical perspective. Exploiting the calibrated model, it is possible to better understand the implications for the timing of the takeoff toward sustained long-run growth by employing a comparative statics analysis. Changing the evolution of the stock of basic scientific knowledge, respectively its importance for applied research, ceteris paribus, we can analyze how a different timing of scientific discoveries might have altered economic progress and the timing of the takeoff.

In Figure 4.6, we show the evolution of wages given different assumptions on the productivity of thinkers in the basic scientific research sector. With the exception of κ and $B_0 = 10$, all parameter values and initial values are as in Section 4.4. The baseline case of $\kappa = 0.4$ is displayed as the red line. By varying κ, basic scientific knowledge accumulates at a different rate, which, in turn, affects the productivity of scientists working in applied R&D and, with it, economic progress.

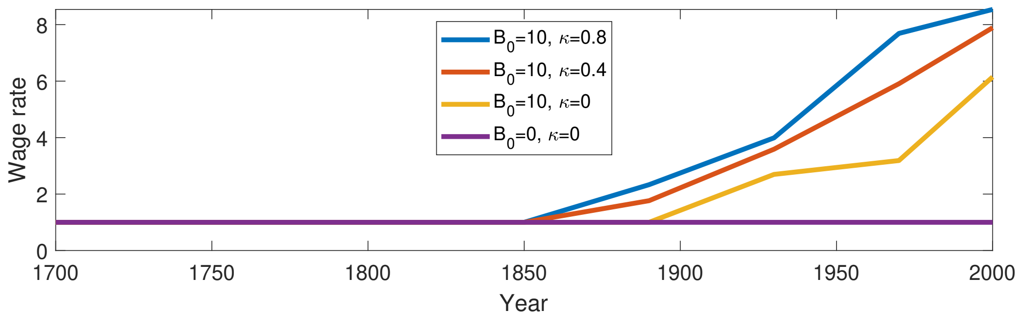

Figure 4.6: Wages for different values of κ and initial levels of B_0

Overall, the rate of economic growth increases with κ such that the takeoff to long-run growth is steeper. The logic behind is that more basic scientific knowledge is available, which makes applied research more profitable. By contrast, the timing of the Industrial Revolution is postponed with a decrease of κ. In the extreme case of $\kappa = 0$, $B_0 = 10$ is constant over time. In this case, the takeoff is postponed by one generation (as shown by the yellow line). Since productivity of scientists in the applied research sector is determined not solely by scientific knowledge but also by education, i.e., human capital, the economy reaches the threshold at which applied research becomes profitable later. Eventually, better educated scientists are able to compensate the lack of growth in basic scientific knowledge and the Industrial Revolution takes its course. While a setback of one generation might seem little over the course of human history, such a setback would imply that we had an income level today similar to the one in 1980, which is substantially less.

Changing the intersectoral spillovers, σ, and keeping everything else constant, also affects wages and follows a very similar logic. As obvious from Figure 4.7, the timing of the takeoff crucially hinges on the degree of inclusion of scientific knowledge into applied knowledge production. For low spillovers, i.e., if scientific advancements are less integrated into the development of productive R&D (e.g. in case of poor knowledge diffusion or for cultural reasons), the takeoff in wages occurs later. Again, the reason is that the application of basic scientific knowledge increases the productivity of applied researchers. If there is a fast rate of scientific discoveries but these discoveries are not considered in applied research, the productivity in and profitability of developing new blueprints is low, which delays the takeoff toward sustained economic growth. These observations directly lead to the following remark.

Remark 4.1. *Basic scientific research and with it the Scientific Revolution play a crucial role in the timing of the Industrial Revolution. A postponement of the Scientific Revolution or a weak integration of basic scientific knowledge in applied research would have delayed economic progress severely.*

As discussed in Remark 4.1, growth in basic scientific knowledge is not necessary for

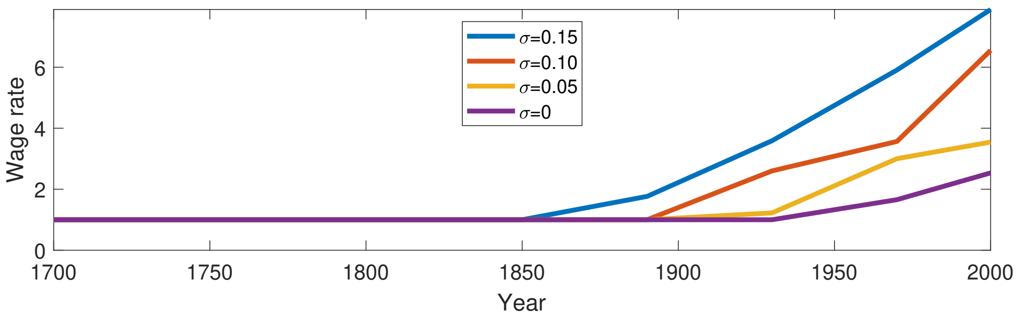

Figure 4.7: Wages for different values of σ

the economy to take off eventually but it can postpone the takeoff substantially. However, this is only true for an initially positive stock of scientific knowledge. If g_B were zero over the course of human history, B_0 would be zero too. Such a scenario is shown in Figure 4.6 (purple line). The economy would not take off at all because without any understanding of the natural laws and scientific inquiry, no productive R&D would be possible, leaving the economy stagnant indefinitely. We emphasize this in the following remark.

Remark 4.2. *Scientific knowledge is indispensable for an economy to take off because productive R&D requires scientists to have, at least, a basic understanding of the laws of nature and of scientific inquiry.*

4.5 Conclusions

We propose a novel Unified Growth model that sheds light on the role of the Scientific Revolution in the process of the convergence toward a takeoff to sustained economic growth. We show that the accumulation of basic scientific knowledge (comprising knowledge about the laws of nature, knowledge about the scientific method, and knowledge about the ways to disseminate ideas) and its application in applied research is a crucial driver of economic progress in the long run. If the stock of scientific knowledge does not grow or if the application of scientific achievements in applied research is too weak, the takeoff to sustained economic growth will be delayed. In the extreme case in which scientific inquiry is prevented altogether, e.g., for religious reasons or by oppressive rulers, the takeoff to sustained growth might be delayed indefinitely.

This theory can explain why some countries and regions experienced the fertility transition and the takeoff to modern economic growth much later than others. For example, China was technologically more advanced than European countries in the middle ages but then the Ming Dynasty decided on isolationist policies. Science did not progress as quickly as previously and China was eventually overtaken by Europe, where the Industrial Revolution occurred first. In fact, China, which was among the richest countries in the world around 1000 AD, became one of the poorest countries in the world in the midst of the twentieth century (Morris, 2010). We believe that our proposed framework can be helpful in understanding the reasons why this was the case.

As far as promising avenues for further research are concerned, a need exists for better data on the calibration of the model for the time period 1500 onward. Particularly helpful would be a database that allowed the quantification of major scientific insights and major applied knowledge over that time period. Another interesting topic is to analyze the extent to which institutions and knowledge interacted in the emergence of the *Culture of Growth*.

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CHAPTER 5

General Conclusions

Demographic change, mainly driven by changing fertility and mortality, has taken place in all industrialized countries over the past decades and is not expected to slow down. The individual gains such as longer lifespans and higher education levels are indisputably positive. However, there exist concerns about the long-run macroeconomic effects. This dissertation contributes to the debate by extending the literature on the growth effects of demographic change. The main questions posed in Chapter 1 were about the fertility and saving effects of changes in the lifetime horizon and the associated economic implications over the long run. Additionally, the welfare effects of higher life expectancy were to be analyzed. Chapters 2–4 each contribute to these questions from a different angle by developing models of endogenous economic growth that incorporate different demographic structures.

Chapter 2 focuses on the long-run economic growth effects of exogenous changes in mortality and the consequent welfare implications. Higher survival induces individuals to save more for retirement. Higher aggregate savings place downward pressure on the market interest rate and increase the present value of innovations. As a result, employment in R&D rises, leading to faster innovation and a higher long-run economic growth rate. The welfare effects of increased life expectancy are disentangled by distinguishing between longevity-induced utility gains due to higher consumption versus utility gains due to living longer. It is shown that the direct welfare gains of higher life expectancy, usually, outweigh the indirect welfare gains of faster economic development.

The model proposed in Chapter 3 provides additional insights into the relationship between life expectancy, fertility and economic development by endogenizing the individual survival probability and by introducing endogenous fertility and education. The probability to survive to retirement increases in the governmental resources devoted toward adult health. Similar to Chapter 2, there exists a positive relationship between life expectancy and savings. However, due to the feedback effects between life expectancy, savings, fertility and labor force participation, the overall effects on innovation and economic growth are ambiguous and cannot be derived analytically. Calibrating the model to U.S. data, the different channels are quantified. As the central result it is concluded that, according to the model, life expectancy effects are positive and contributed with 11.9% to increases in the U.S. real GDP p.c. over the time period 1960–2017. Also, the growth-maximizing size of the health care sector might be larger than what is observed in industrialized countries, nowadays. Combined with the finding from Chapter 2 that the lifetime-maximizing size of the health care sector is way larger than the size that maximizes income, Chapter 3 supports the view to not only consider the growth effects of health care.

Finally, Chapter 4 analyzes how the Scientific Revolution contributed to the takeoff toward sustained economic growth. Basic scientific knowledge is an essential input in applied R&D and increases in the number of tinkerers and in their level of education. For low levels of development, fertility is high and educational investments are zero. Education only starts to rise once income surpasses a certain threshold, which spurs innovation, initiates the fertility transition and paves the way to sustained economic growth. The rate at which basic scientific knowledge accumulates as well as its availability in applied research is shown to be crucial in determining the timing and the magnitude of the economic takeoff. The reason for the given relationship is that scientific knowledge raises the productivity and the profitability of applied research. These findings provide one possible explanation why some regions experienced the Industrial Revolution and the economic takeoff later than others.

In a nutshell, this dissertation identifies several channels through which demographic change exerts positive growth effects and can at least partly mitigate the negative consequences of population aging. Potential policy measures include higher subsidization of both applied and basic research, higher health spending, better education and improved incentives to save and invest in firms and projects that drive the technological frontier forward.