

ESSAYS IN RELATIONSHIP BANKING:
The Efficiency of Savings-linked
Relationship Lending and Credit
Information Sharing

Dissertation

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Steffen Kirsch

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1. Bericht: Prof. Dr. Hans-Peter Burghof
2. Bericht: Prof. Dr. Dirk Hachmeister

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1. Prüfer: Prof. Dr. Hans-Peter Burghof
2. Prüfer: Prof. Dr. Dirk Hachmeister

Vorsitzender der Prüfungskommission: Prof. Dr. Holger Kahle

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Prologue

Informational asymmetries are considered to be the major cause of inefficiencies in commercial banking. Contemporary research in the field of financial intermediation identifies relationship banking to be elementary for reducing these inefficiencies (for a comprehensive survey see Boot, 2000; Elyasiani and Goldberg, 2004). The key tool of relationship banking to mitigate informational asymmetries is to produce private information about a customer while in a banking relationship with that customer. Concepts of relationship banking differ, first, in the way information is produced and, second, in the way this information is utilized by the bank.

When engaging in relationship lending, the most studied and most regarded field in relationship banking,¹ banks can utilize private information to improve the lending business. With more and better information about a borrower, banks should be able to make a more sophisticated assessment of creditworthiness and borrower quality. This can reduce problems of adverse selection and moral hazard, which should lead to higher credit availability, more quality-adequate credit rates, or decreased requirements of costly collateral. The extensive empirical literature studying the effects of relationship lending on market outcomes finds that relationship lending increases the availability of funds (Petersen and Rajan, 1994; Cole, 1998), especially for borrowers of small banks (Cole et al., 2004; Berger et al., 2005), and leads to decreased collateral requirements (Berger and Udell, 1995; Degryse and Van Cayseele, 2000). The results concerning the effect on credit rates are more ambiguous. While some studies find that relationship lending lowers credit rates (Berger and Udell, 1995; Bharath et al., 2011), others find increasing credit rates with the duration of the relationship (Degryse and Van Cayseele, 2000) or no significant effects

¹Relationship banking and relationship lending are often used interchangeably in the literature. However, there are, of course, also non-lending banking products and services that can profit from private information about customers.

of relationship lending on the price of credit (Petersen and Rajan, 1994; Elsas and Krahnén, 1998).

The mixed empirical evidence for the effect of relationship lending on credit rates is explained by theoretical research. Boot and Thakor (1994) predict decreasing credit rates with the length of a bank-firm relationship. They argue that banks dynamically optimize borrowers' incentives by taxing borrowers in the early stage of a long-term relationship with above-cost credit rates and subsidizing borrowers with decreasing credit rates as the relationship matures. Greenbaum et al. (1989), Sharpe (1990), and Petersen and Rajan (1995), however, show that hold-up problems can arise due to relationship lending that lead to increasing credit rates. They argue that relationships provide inside lenders with an informational advantage over outside lenders. Due to information asymmetries between outside lenders and relationship borrowers, the latter may be informationally captured by inside lenders. That allows inside lenders to exploit their informational advantage to charge higher credit rates.

Another possible shortcoming of relationship lending is the soft budget constraint problem (Dewatripont and Maskin, 1995; Bolton and Scharfstein, 1996). To increase the probability of recovering the previous loan, inside lenders may extend credit to distressed borrowers who cannot get funds by outside lenders. If borrowers, however, anticipate this possibility to renegotiate their contracts *ex post*, bad incentives can arise *ex ante*. Besides these results, the possibility to renegotiate a credit contract can also be an efficiency-enhancing flexibility of relationship lending in complex real-world environments where contracts cannot capture every possible state of the world. In this way, relationship lenders can provide liquidity insurance in situations of unexpected temporary deterioration of borrowers liquidity (Elsas and Krahnén, 1998).

To subsume, private information about borrowers are apparently known to be uttermost important for the lending business. Producing such information to reduce problems that arise because of informational asymmetries constitutes a

raison d'être of banks. Nonetheless, the most recent financial crisis of 2007/2008 was catalyzed by lax lending standards that increasingly ignored borrower quality. Lending was conveniently rather based on collateral in times of steady rising house prices and nearly boundless possibilities of credit securitization.² That was especially the case for household lending. As a result, aggregate household debt, default rates of household loans, and sizes of defaults have significantly increased over the past years (Athreya et al., 2012). Hence, the question arises how relationship lending to private households can contribute to counter these effects and the depicted experience of the financial crisis.

Most of the relationship lending literature concerns firm lending and the results cannot always be transferred to household lending without restrictions. The most direct way to produce credit-relevant private information about a borrower is to engage in repeated lending. While this approach seems to work very well for firm lending, it often promises very limited learning potential when it comes to household lending. Households usually do not at all engage in substantial lending as frequently and numerous as firms. Hence, households may never possess extensive credit track records, especially for high volume loans like in mortgage lending, and numerous loan demanding households actually do not possess credit records at all. For relationship lending to households, banks are often limited to learn about borrowers by interacting with them in different products. I argue that repeated interactions in saving relationships exceedingly qualify for the purpose of identifying borrower qualities, foremost because saving substantially requires the same individual characteristic as repaying a loan: the ability to regularly abstain from consumption. This view is confirmed by recent empirical evidence: Puri et al. (2017) show that saving relationships prior to lending can provide information that help to reduce loan defaults of households. Theoretical research is yet missing.

The contribution of the first part of my thesis is to provide theoretical research

²The lax lending practice was, of course, also caused by other factors, e.g. by loose monetary policy and regulation failure.

on relationship lending that is based on information production about borrowers in preceding saving relationships. To analyze the efficiency of savings-linked relationship lending, I develop a multi-period partial equilibrium model of lending to private households and compare savings-linked relationship lending with arm's-length lending. I derive competitive contract designs and derive market characteristics for which savings-linked relationship lending can be an equilibrium choice and enhance the efficiency of the financing market.

One important result of my model is that savings-linked relationship lending is particularly well suited and economically beneficial for housing finance of private households. Further, the competitive contract design of savings-linked relationship lending that I derive in the model shares major characteristics with *Bauspar* contracts. The concept of *Bausparen* (in English: contractual saving for housing) can therefore be seen as an implementation of savings-linked relationship lending. *Bausparen* features a contractual saving stage before the loan is made and on which lending is based, but it also contains several other specifics. These include contract-inherent options regarding the loan granting and the loan repayment, and a contractually fixed interest rate for the saving stage as well as a fixed credit rate for the future loan.

Despite being a widespread and important product of housing finance in Europe and existing since about a century, research about *Bausparen* is utterly scarce and the available economic explanations are insufficient. Existing research mainly concentrates on explaining the value of *Bausparen* in terms of the hedging effects that it provides towards future interest rate changes. But this cannot explain the specific savings-linked design of *Bausparen* since this is not required for effective hedging. My model is capable of filling this gap and provides, to my knowledge, the first rigorous theoretical relationship lending explanation for *Bausparen* and highlights its savings-linkage.

The hitherto discussion establishes the result that producing and possessing proprietary information about borrowers is valuable for lenders. *Prima facie*, it

therefore seems contradictory that lenders in most countries engage in sharing at least some of their proprietary information with competitors by using publicly regulated or private credit registries (Jappelli and Pagano, 2002; Djankov et al., 2007). Theoretical research provides several explanations for credit information sharing. One of them is that information sharing can discipline borrowers and reduce moral hazard (Vercammen, 1995; Padilla and Pagano, 2000). This result is based on reputation effects. If borrowers fail to repay their loan with one lender, information sharing between lenders provides this information to every other lender who incorporate it in their assessment of borrower quality. Borrowers may therefore lose their reputation with every lender when failing with one lender, which can induce incentives to perform. The problem is, however, that reputation effects diminish if there is less to learn about agents. That means, the more comprehensive the credit registry becomes, the weaker the reputation effects of information sharing get. To prevent the reputation effects of credit information sharing from diminishing and finally disappearing, the previous literature suggests to restrict credit reporting. This, however, seems hard to realize and comes at the expense of efficiency-enhancing effects that information sharing can produce on other levels (Pagano and Jappelli, 1993; Padilla and Pagano, 1997; Bouckaert and Degryse, 2006).

The contribution of the second part of my thesis is to show that credit information sharing can induce borrower discipline beyond “passive” reputation effects if the information shared is actively used. I provide a new approach to discipline borrowers on the basis of credit information sharing. This approach is adapted from classical disciplining which is the principle that undesired outcomes need to inevitably provoke unfavorable consequences. In a multi-period model of repeated lending in a market with established credit information sharing, I analyze and compare two different ways of disciplining. First, disciplining by *pro rata* credit rationing, where lenders punish defaulting borrowers by decreasing next period’s credit volumes. Second, disciplining by credit rate tightening, where lenders punish defaulting borrowers by increasing next period’s credit

rates. I can show that disciplining is not sensitive to increasing informativeness of credit registries and therefore also possible in case of an informationally comprehensive credit reporting. That means that disciplining can even work when reputation effects break down. This is especially true when defaulters are punished with lower credit volumes, which constitutes a rare case of efficient equilibrium credit rationing that is not a case for government intervention.³

The remainder of my thesis is organized as follows. Part I analyzes the efficiency of savings-linked relationship lending. Part II revisits the effect of credit information sharing on borrower discipline. Concluding remarks are given in the epilogue.

³This complements a substantial literature about inefficient credit rationing (Jaffee and Russell, 1976; Keeton, 1979; Stiglitz and Weiss, 1981, 1983; Williamson, 1986). An exception is de Mesa and Webb (1992).

Part I

The Efficiency of Savings-linked Relationship Lending

Abstract

In a multi-period partial equilibrium model of lending to private households, I compare arm's-length lending with relationship lending that is based on information production about borrowers in preceding saving relationships. These are often the only source of private information that lenders possess about loan demanding households. The model shows that savings-linked relationship lending leads to a Pareto improvement or an increasing allocative efficiency of the financing market compared to arm's-length lending in markets of low time preference or low average borrower quality. In these markets, savings-linked relationship lending can overcome financing market failure due to adverse selection, especially for financing volumes that are large in comparison to households' periodic savings or incomes. Thus, the model shows that savings-linked relationship lending is particularly well suited and economically beneficial for housing finance of private households and is able to increase home ownership rates. Competitive savings-linked relationship lending, as derived in the model, shares major characteristics with contractual saving for housing which is a widespread and important product of housing finance in Continental Europe. My model therefore provides, to my knowledge, the first theoretical relationship lending explanation for contractual saving for housing. Further, my results add a novel economic explanation for synergies between the two main activities of traditional commercial banking, deposit-taking and lending.

1 Introduction

In relationship lending, lenders utilize private information that they have produced in a relationship with a client to improve the lending business with the same client. Information is generally produced over time by repeated interaction with a client either in the same product or in a different product. Adding the temporal dimension, approaches of relationship lending can be divided into information production by (a) sequential interaction in the same product, (b) sequential interaction in different products, or (c) simultaneous interaction in different products. The information that is produced by a creditor while simultaneously interacting with the debtor in different products is useful for credit monitoring. Mester et al. (2007), for instance, show empirically that transaction accounts help financial intermediaries to monitor commercial borrowers. The approach to learn from sequential interaction in the same product has been examined theoretically essentially for the case of firm lending (Petersen and Rajan, 1995; Sharpe, 1990; von Thadden, 2004), where lenders learn about the quality of a firm from its debt service behavior in preceding lending relationships and utilize the information they have gathered for upcoming lending relationships with the same firm. This kind of learning from preceding credit relationships for subsequent credit relationships with the same debtor is conceptually also conceivable for lending to private households. But, as already outlined in the prologue, households do usually not at all engage in substantial lending as frequently and numerous as firms. Hence, households may never possess extensive credit track records, especially for high volume loans like in mortgage lending, and numerous loan demanding households actually do not possess a credit record at all. This radically restricts the possible extent of learning from repeated lending. Consequently, information production that is required for relationship lending to private individuals is often limited to sequential interaction in different products.⁴

⁴An advantage of this approach over the others is that some adverse selection problems of lending can only be mitigated if private information is produced prior to loan granting,

I argue that repeated interactions in saving relationships exceedingly qualify for the purpose of identifying individual borrower qualities for two reasons. First, saving track records are quite easy to build and, second, saving behavior is a highly relevant proxy for individual borrower quality. The latter is true because saving substantially requires the same basic individual characteristic as debt service: the ability to regularly abstain from consumption. This view is supported by recent research of Puri et al. (2017). They present empirical evidence that saving relationships prior to lending can provide information that help to reduce loan defaults of households.

The approach to use saving relationships to improve the lending business further contributes to the literature on the economics of traditional commercial banking. In this context my work provides a novel theoretical relationship lending explanation for economic synergies between the main activities of traditional commercial banking, namely deposit-taking and lending (Kashyap et al., 2002).

I develop a multi-period partial equilibrium model of unsecured⁵ lending to private households in order to study relationship lending that is based on information production in prior saving relationships with private individuals. I compare such savings-linked relationship lending with arm's-length lending and autarkic savings accumulation in a competitive financing market with individuals that differ in their ability to abstain from consumption, that is, in borrower quality.

Different from existing approaches in the relationship lending literature, I consider that relationship lending is not just a free byproduct of customer relationships. Information production in relationships is costly as it at least requires time. Savings-linked relationship lending is therefore only applicable with temporal delay compared to arm's-length lending. This imposes costs on

which necessarily requires learning from different products.

⁵This assumption is to set the focus on private information about borrowers which are more relevant if credit is less secured. For research about the effects of collateral see Coco (2000).

savings-linked relationship lending in a market with individuals that feature positive time preference.

I show that savings-linked relationship lending is an equilibrium choice in markets of low time preference or in markets of low average borrower quality. In these markets, savings-linked relationship lending leads to a Pareto improvement or an increasing allocative efficiency of the financing market compared to arm's-length lending. It is further shown that savings-linked relationship lending can overcome financing market failure that is due to adverse selection and enable financing when arm's-length lending is not viable in equilibrium. This is especially true for financing volumes that are large in comparison to households' periodic savings or incomes since savings-linked relationship lending shows, in contrast to arm's-length lending, to be robust to *ceteris paribus* changes in financing volume. Hence, savings-linked relationship lending is particularly well suited and economically beneficial for housing finance of private households and is able to increase home ownership rates.

The model has a direct application to contractual saving for housing (CSH)⁶, a widespread, important, and longtime successful product of housing finance in Continental Europe, especially in Germany, Austria, France, and the Czech Republic. CSH can also be found in Belgium, Croatia, Hungary, Italy, Luxembourg, Poland, Romania, Slovakia, and has also been considered to be implemented in further European countries, e.g. the Netherlands and Belarus. Inspired by the extensive and successful use of CSH in Europe, China and India experiment with contractual saving for housing since 2003 and 2004. In Germany CSH substantially exists since the 1920s and about 36 percent of the population owned a CSH contract in the year 2015; in Austria and the Czech Republic the market penetration in 2015 reached about 59 and 40 percent, respectively, and was even greater in the past.⁷ In its simplest form, the actual design of CSH contracts is akin to the competitive contract design of savings-linked relation-

⁶In Germany and Austria the traditional term *Bausparen* is used.

⁷Data sources: Verband der Privaten Bausparkassen, OeNB, EFBS, eurostat.

ship lending derived in the model: in a contractual saving stage, savings are regularly transferred to the contract-providing bank over a particular time span before a loan is made contingent on the saving behavior. Since savings-linked relationship lending can be identified to be a central element of CSH, CSH shares the economic efficiency that my model identifies for savings-linked relationship lending. My work is to my best knowledge the first to give a pure relationship lending explanation for CSH and to explain the economic benefits of its savings-linked design. Previous attempts to economically explain CSH in the very thin literature mainly focus on arguments of interest rate hedging since the credit rate of the future contingent loan is fixed in the contract terms of CSH (Cieleback, 2002; Plaut and Plaut, 2004).⁸ These works, however, fail to explain the specific savings-linked design of CSH since this is not required for effective hedging.

Lea and Renaud (1995) recognize the feature of CSH to learn from individuals' saving behavior about their borrower qualities, but they verbally conclude that CSH is merely adequate for transition economies. This is in sharp contrast with the prevalence of CSH in some highly developed economies in Central Europe. My results, however, disclose that contractual saving for housing can indeed be beneficial in transition economies to overcome substantial informational asymmetries. But CSH can also be beneficial in developed economies if time preference is sufficiently low, that is, if individuals regard the quality of housing to be more important than the time of its possession.

The remainder of Part I is organized as follows. I develop the basic theoretical model in Section 2. I solve it for the benchmark case of perfect information, then derive the asymmetric information equilibrium, first, for a market with arm's-length lending and, second, for a market that additionally features savings-linked relationship lending. Subsequently, parametric results are presented and discussed, and the economic efficiency of savings-linked relationship lending

⁸Zietemann (1987) and Scholten (1999) evaluate CSH from a capital budgeting perspective, which is, however, not capable of explaining an inherent economic value of CSH.

is derived. Section 3 introduces modifications to the basic model. Section 4 first applies the model to housing finance and to contractual saving for housing and then discusses the effects of state subsidy for savings-linked relationship lending. I close with a conclusion in Section 5.

2 The Model

I develop a T -period model of risk-neutral⁹ and utility maximizing private individuals who desire to consume a particular good G . Individuals have no initial endowment but they can save a fraction of their income every period for the purpose of acquiring G . There are two quality types of individuals. High quality individuals, H , are able to save 1 monetary unit every period with certainty. Low quality individuals, L , manage to save 1 monetary unit with probability p and to save nothing with converse probability $1 - p$, where $e^{-1} \leq p < 1$. The main results of this work do not depend on the assumption that the success probability has a lower bound of e^{-1} but this assumption makes the mathematics much more convenient. A general version of the model for a lower bound of zero and its proof is provided in Appendix C. The proportion of type H individuals in the market is denoted by θ and $1 - \theta$ is the proportion of low quality types, where $0 < \theta < 1$.

Let individuals live $T \geq 2$ periods. As alternative interpretation of the model's total number of periods, T , regard it as the time span in which individuals are able and willing to regularly assign savings to payments that are related to good G . Good G is only acquired once in an individual's life at time t and no quality upgrades subsequent to the purchase are possible. Let us assume for simplicity that individuals do not receive any utility from savings that are not used for G .¹⁰ Individuals have a positive time preference and therefore prefer

⁹The assumption of risk-neutrality is appropriate to isolate my results from hedging arguments.

¹⁰This assumption may seem stronger than it is since it basically just standardizes the utility from an alternative consumption to zero.

to consume G as soon as possible.¹¹ To which extent they do so is reflected by the exogenous per-period time preference factor δ , where $0 < \delta < 1$. While I use t to refer to the time of purchasing good G , I use τ to generally refer to a point in the time frame of the model.

As savings are individuals' only source for debt service, individuals' saving behavior also directly determines their borrower quality. Banks can therefore learn about an individual's type by observing her saving behavior which cannot be observed publicly but only by the particular inside bank that is in a saving relationship with the individual. Let us abstract from preexisting saving accounts. Individuals can choose to save by themselves in a money box or they can save using a saving account of one and only one bank of their choice. I rule out that the total amount saved can be used for signaling. That means that individuals cannot credibly convey information about their quality by exposing the amount saved to an outside bank.¹²

Individuals can choose autarky (marked by A) and accumulate savings until time $t \in (0, T]$ to purchase G . But they can also apply for a loan on the competitive financing market at time $t \in [0, T)$, where risk-neutral and profit maximizing banks offer unsecured loans and zero-interest saving accounts to individuals. Banks know the general utility function of the individuals, the proportion θ , and the saving probabilities of the different types but they have no initial information about the quality type of a particular individual.

Banks can offer two different types of credit to individuals: arm's-length debt and relationship loans. While arm's-length lending is not based on information production about individuals' quality, relationship lending is. In my model, banks can probabilistically learn about an individual's quality by observing

¹¹See Olson and Bailey (1981) for arguments in favor of a positive pure time preference.

¹²The justification of this assumption is, first, that banks cannot assess if the amount of money that is presented to them actually consists of regular savings for G and not of income that is finally not dedicated to be used for G . Second, banks may not find out about that since the amount of equity used for purchasing G is not of interest in an unsecured loan.

her saving behavior in a saving relationship and use this information to offer savings-linked relationship loans. Therefore, relationship lending is costly since information production requires time.

Savings-linked relationship lending can generally be implemented in two different ways. First, the link between loan terms and saving behavior can be merely an implicit agreement that is not contractually fixed. Effects of reputation, e.g., need to prompt banks to take individuals' saving behavior into account when loan terms are offered. The second implementation, however, contractually defines the loan terms contingent on the saving behavior in the preceding saving relationship. Such a savings-linked contingent loan contract is entered simultaneously with a saving relationship at time $\tau = 0$. The contract specifies the time t when the loan is made and predefines the contingent loan terms. Let us assume that individuals can quit the saving relationship at any time and opt for an alternative financing. Further, let us entirely abstract from costs and frictions of contingent contracting.

Individuals receive utility from the quality¹³ of G they expect to be able to afford at time t .¹⁴ The expected quality, Q_t , of G that individuals of type $\omega \in \{H, L\}$ can afford at time t is composed of the expected amount saved until t and of the financing ℓ_t they can take out at time t , thus $Q_t = \ell_t + E[S_{0,t}(\omega)]$. Let us define $S_{\tau_1, \tau_2}(\omega)$ as the amount that individuals of type ω save between time τ_1 and time τ_2 , where $\tau_1 < \tau_2$. The expected total amount that a type H individual can save between τ_1 and τ_2 is $(\tau_2 - \tau_1)p$, for a type L individual it is

$$E[S_{\tau_1, \tau_2}(L)] = \sum_{i=0}^{\tau_2 - \tau_1} \text{prob}(i, \tau_2 - \tau_1) i = (\tau_2 - \tau_1)p, \quad (1)$$

where, following from the binomial process of the savings, $\text{prob}(i, n)$ is the probability that a type L individual saves exactly the total amount i over the

¹³Depending on context, an interpretation as quantity is also possible.

¹⁴I presume a strong proportionality between price and quality of good G .

next n periods,

$$\text{prob}(i, n) := \binom{n}{i} p^i (1-p)^{n-i}. \quad (2)$$

Since individuals desire more quality of G over less and obtain no utility from savings that are not used for G , competition between banks leads to an equilibrium loan contract with a repayment that consists of all the upcoming savings until T and hence the highest possible loan volume.¹⁵ From this follows that there is no need to model the loan rate explicitly, the loan volume is sufficient. Thus, with the risk-free market interest rate standardized to zero, individuals' utility is generally described by

$$u(Q_t, t) = Q_t \delta^t. \quad (3)$$

The expected utility that an individual of type ω receives from a particular financing option, ϕ , is denoted by U_ω^ϕ . I write ℓ_t^ϕ for the greatest feasible financing volume in financing option ϕ that can be granted at time t . In the peculiar case of autarky I maintain this notation and naturally obtain the financing volume $\ell_t^A \equiv 0$ at t .

Individuals always prefer more utility over less. The decision of individuals in indifference situations follows the tiebreaker rule that is defined hereinafter.

Definition I.1 (Tiebreaker rule). Consider the individuals' decision options $a = (Q_a, t_a)$ and $b = (Q_b, t_b)$ that provide the expected utilities $U(a)$ and $U(b)$, respectively. If $U(a) = U(b)$, individuals first choose loans that are granted immediately, that is, at $t = 0$ over loans with $t > 0$. Second, individuals choose pooling contracts over separating contracts.¹⁶ If the indifference is still unsolved, individuals choose a over b if $t_a < t_b$, then if $Q_a > Q_b$.

¹⁵This is possible if there are no costs of default, which I shall presume.

¹⁶This assumption can be considered a social disposition of individuals if no individual disadvantages emerge.

2.1 First Best Solution

In a world of perfect information every bank knows about the quality of every particular individual. That allows banks to selectively offer contracts with different loan terms to different quality types of individuals. Pooling contracts, instead, offer by definition the same loan terms to different types of individuals and therefore result in high quality individuals cross-subsidizing low quality individuals. Thus, pooling contracts can clearly never be better for high quality individuals than type-adequate loan terms. As type H individuals can and will choose the utility maximizing contract (that features zero cross-subsidization) in a competitive market environment, no pooling contracts are found in the perfect information equilibrium and the selectively offered loan contracts feature type-adequate loan terms.

A bank offers type-adequate standard loans (SL)—denoted by LSL for types L and HSL for types H —to individuals if it can expect to at least break-even. Banks do so if they offer loan volumes that do not exceed individuals' expected debt service potential which is the expected total savings of the future. Type H individuals have a certain total debt service potential of $S_{t,T}(H) = T - t$ at time t and type L individuals have an expected total debt service potential of $E[S_{t,T}(L)] = (T - t)p$ at time t . Since utility maximizing individuals prefer the maximum loan volume they can get at time t , banks' break-even constraints are binding in the competitive equilibrium and they offer $\ell_t^{\text{HSL}} = T - t$ and $\ell_t^{\text{LSL}} = (T - t)p$. This leads to the utilities $U_H^{\text{HSL}} = T\delta^t$ and $U_L^{\text{LSL}} = Tp\delta^t$, which are obviously both maximized for $t = 0$.

Information production is, of course, irrelevant under perfect information. Thus, saving relationships prior to a loan are meaningless as they cannot lead to more accurate loan terms but impose costs of deferral of consumption.

If individuals choose autarky and do not to take out a loan at all, the quality of G that individuals can afford at time t is equal to their accumulated savings

until t . Thus, types H have an expected utility of $U_H^A = t\delta^t$ and types L of $U_L^A = pt\delta^t$ in case of autarky. The optimal time of consumption is given by $t = t_A := \min[-1/\ln(\delta), T]$ for both types. The individuals' utility from autarkic consumption can therefore obviously never be better than from the best type-adequate loan contract. Proposition I.1 summarizes.

Proposition I.1. *In the first best equilibrium, individuals of type H (L) consume good G of quality T (Tp) immediately, financed with type-adequate standard loans.*

2.2 Arm's-Length Lending

In presence of informational asymmetries and absence of information production, banks cannot distinguish between different types of individuals. Therefore, type-adequate loan terms are not possible and banks can only offer the same loan contracts to all the individuals or to random individuals without knowing their particular quality. If these loan contracts aim to attract all the individuals, we have pooling contracts, while separating contracts intend to be solely accepted by individuals of a particular type.

2.2.1 Pooling Arm's-Length Loan Contracts

The total repayment of a standard loan contract that is granted at time t is equal to $\min[D_t, S_{t,T}(\omega)]$, where D_t is the contractual total repayment. When offering a pooling arm's-length loan (denoted by PAL) to random individuals at time t , banks' participation constraint is given by

$$\ell_t \leq \theta D_t + (1 - \theta) \sum_{i=0}^{T-t} \text{prob}(i, T - t) \min(i, D_t). \quad (4)$$

The maximum utility is reached for the greatest feasible loan at time t which can be offered for the maximum repayment $D_t = T - t$ (since individuals do by assumption not gain any utility from unused savings) and a binding

participation constraint of banks. Equation (4) becomes

$$\ell_t = \ell_t^{\text{PAL}} := \theta(T - t) + (1 - \theta)(T - t)p. \quad (5)$$

The expected utility that the different types of individuals receive from a PAL that is made at time t follows to be

$$U_H^{\text{PAL}} = (\ell_t^{\text{PAL}} + t)\delta^t = [\theta(T - t) + (1 - \theta)(T - t)p + t]\delta^t, \quad (6)$$

$$U_L^{\text{PAL}} = (\ell_t^{\text{PAL}} + tp)\delta^t = [\theta(T - t) + (1 - \theta)(T - t)p + tp]\delta^t. \quad (7)$$

Since $\partial U_L^{\text{PAL}}/\partial t < 0$, types L prefer to take out a PAL at time $t = 0$. The utility maximizing PAL for type L individuals is therefore given by $\ell_0^{\text{PAL}} = T(\theta + p - \theta p)$. For types H the optimal time t for consuming G with a pooling arm's-length loan can be stated with respect to the time preference measure,

$$\arg \max_{t \in [0, T)} U_H^{\text{PAL}} = \begin{cases} 0 & \text{if } \delta \leq \delta_{\text{PAL:lo}} := \exp\left(\frac{-(1-\theta)(1-p)}{T(\theta+p-\theta p)}\right), \\ t_{\text{PAL}} & \text{if } \delta \in (\delta_{\text{PAL:lo}}, \delta_{\text{PAL:hi}}), \\ t \rightarrow T & \text{if } \delta \geq \delta_{\text{PAL:hi}} := \exp\left(\frac{-(1-\theta)(1-p)}{T}\right). \end{cases} \quad (8)$$

$t = t_{\text{PAL}} \in (0, T)$ is a finite real-valued and unique maximum of U_H^{PAL} if $\delta \in (\delta_{\text{PAL:lo}}, \delta_{\text{PAL:hi}})$, where

$$t_{\text{PAL}} := T - T[(1 - \theta)(1 - p)]^{-1} - \ln(\delta)^{-1}. \quad (9)$$

The preferences of the different types of individuals apparently differ regarding pooling contracts. In a competitive financing market, the preferences of the individuals generally determine which break-even loan contracts are offered. For pooling contracts, the preferences of types H are crucial. If banks offer loan contracts with pooling terms that are actually not chosen by types H , they face losses if the intended pooling contracts are accepted by low quality individuals. It follows that type L individuals cannot expect to receive the

pooling contract of their choice since banks can only offer pooling terms if high quality individuals (or no one) accept. Contracts that are only chosen by types L , however, require LSL terms to let banks break even.

Banks can offhandedly offer standard loans with LSL terms. Further, individuals can always opt for autarky. It can be seen from Eq. (8) that type H individuals choose autarky over a pooling arm's-length loan if $\delta \geq \delta_{\text{PAL:hi}}$. But if $\delta < \delta_{\text{PAL:hi}}$, types H prefer a PAL over autarky since $U_H(\ell_t^{\text{PAL}}) - U_H(\ell_t^{\text{A}}) = (T - t)[\theta + (1 - \theta)p]\delta^t > 0 \forall t \in [0, T)$. A pooling arm's-length loan also always offers greater utility than a loan with LSL terms to high quality individuals since $U_H(\ell_t^{\text{PAL}}) - U_H(\ell_t^{\text{LSL}}) = (T - t)\theta(1 - p)\delta^t > 0 \forall t \in [0, T)$.

The maximum utility that type L individuals receive from an LSL or from autarky can be regarded as their lower limit of utility, denoted by \bar{U}_L . As the financing options LSL and autarky are not affected by informational asymmetries, the previous first-best analysis is still valid and from Proposition I.1 follows that $\bar{U}_L = U_L(\ell_0^{\text{LSL}})$. Low quality individuals only accept a pooling arm's-length loan contract if it provides greater utility than their lower limit of utility, which cases are specified by Lemma I.1.

Lemma I.1. $\ell_t^{\text{PAL}} \stackrel{L}{\succ} \ell_0^{\text{LSL}}$ if and only if $t < t_\alpha$, where

$$t_\alpha := \frac{T(\theta + p - \theta p)}{\theta - \theta p} + \ln(\delta)^{-1} W\left(\frac{Tp}{\theta p - \theta} \ln(\delta) \delta^{\frac{T(\theta + p - \theta p)}{\theta p - \theta}}\right) \quad (10)$$

and $W(\cdot)$ denotes the Lambert W -function.

Proof. U_L^{PAL} is strictly monotonic decreasing in t , while \bar{U}_L is constant. Further it is $U_L(\ell_0^{\text{PAL}}) > \bar{U}_L$ and $\lim_{t \rightarrow T} U_L(\ell_t^{\text{PSL}}) < \bar{U}_L$. Thus, there is a unique intersection point $t_\alpha \in (0, T)$ for given (p, δ, T) . For $t = t_\alpha$, the tiebreaker rule in Definition I.1 leads to $\ell_0^{\text{LSL}} \succ \ell_{t_\alpha}^{\text{PAL}}$. ■

To subsume this subsection, pooling arm's-length lending is not accepted by high quality types if $\delta \geq \delta_{\text{PAL:hi}}$, and it not accepted by low quality types if the loan is made at $t \geq t_\alpha$. Since these two conditions differ for the different types,

it becomes apparent that pooling contracts cannot be offered offhandedly by banks in a competitive market as banks only break even in intended pooling loan contracts if the actual pool of borrowers is not riskier than the dedicated target pool.

2.2.2 *Separating Arm's-Length Loan Contracts*

In my model the two different types of individuals are homogeneous groups. That means that all individuals of a particular type act the same way: either every individual of type ω prefers and chooses a particular financing option or none of this type. But individuals of a different type can make different choices. A pooling contract is, however, no longer pooling different qualities if not all the individuals choose it regardless of their quality type. If only one group of individuals chooses an arm's-length loan contract that is offered to everyone or to random individuals, we have a separating arm's-length loan (SAL) contract. Separating contracts require that individuals do not only differ in relevant aspects but also know their own characteristic concerning these aspects. Thus, in my model, the individuals' knowledge of their own type is the key assumption to make separating contracts possible. It is, however, also important that banks, while initially not knowing about the individuals' type, are aware of the individuals' knowledge of their own type.

From Lemma I.1 follows that there is no pooling in an intended PAL with $t \geq t_\alpha$ since types L prefer their best alternative, ℓ_0^{LSL} , in that case. Hence, the intended PAL transforms to a separating contract that is only accepted by high quality individuals if $\delta < \delta_{\text{PAL:hi}}$. But a contract with PAL terms and $t \geq t_\alpha$ is regularly not the most efficient separating contract possible which I derive in the following.

The first condition for a separating contract is that it must be feasible and worse for type L individuals than their best alternative; let us refer to this condition as *first separating condition* in the following. The second condition

for a separating contract is that it must be preferred by type H individuals over their best alternative;¹⁷ let us refer to this as *second separating condition*. Both conditions can only be met if $t \geq t_\alpha$, as explained in the following. To satisfy the first separating condition, a separating contract at $t < t_\alpha$ would require worse terms than a PAL that is provided at the same t , as follows straight from Lemma I.1. But such an intended separating contract would not be preferred over the PAL by types H and therefore not satisfy the second separating condition. For $t \geq t_\alpha$, however, a separating contract with (weakly) better terms than a PAL can be offered to attract types H while still not attracting types L .

Let us analyze the first separating condition in more detail in the remainder of this subsection. A separating contract that provides the loan ℓ_t^{SAL} at time t generally satisfies the first separating condition if $U_L(\ell_t^{\text{SAL}}) \leq \bar{U}_L$ which is equal to $(\ell_t^{\text{SAL}} + tp)\delta^t \leq Tp$. Rearranging the equation results in the constraint

$$\ell_t^{\text{SAL}} \leq \ell_t^{\text{sep}} := Tp\delta^{-t} - tp. \quad (11)$$

If this constraint is met, types L are not attracted to an SAL, and if someone is attracted at all, it is only types H . Therefore, the banks' participation constraint in an SAL is equal to the one in an HSL under perfect information, that is, $\ell_t^{\text{SAL}} \leq \ell_t^{\text{HSL}} = T - t$. The stricter of both constraints is binding in equilibrium, hence we obtain $\ell_t^{\text{SAL}} = \min(\ell_t^{\text{sep}}, \ell_t^{\text{HSL}})$. Since ℓ_t^{sep} is strictly monotonic increasing in t , while ℓ_t^{HSL} is strictly monotonic decreasing in t , and it is $\ell_0^{\text{sep}} < \ell_0^{\text{HSL}}$ but $\ell_T^{\text{sep}} > \ell_{t \rightarrow T}^{\text{HSL}}$, there is one unique intersection point of ℓ_t^{sep} and ℓ_t^{HSL} at $t_{\text{SAL}} \in (0, T)$ for given (p, δ, T) , where

$$t_{\text{SAL}} := \frac{T}{1-p} + \ln(\delta)^{-1} W\left(\frac{Tp\delta^{T/(p-1)} \ln(\delta)}{p-1}\right). \quad (12)$$

Thus, to satisfy the first separating condition, the relevant constraint for the

¹⁷Otherwise the contract would not be accepted by anyone and therefore not separating anything.

SAL contract is $\ell_t^{\text{SAL}} \leq \ell_t^{\text{HSL}}$ for $t \geq t_{\text{SAL}}$, and for $t < t_{\text{SAL}}$ the relevant constraint is $\ell_t^{\text{SAL}} \leq \ell_t^{\text{sep}}$. This is illustrated in Fig. 1a.

Now let us determine the utility maximizing SAL for types H that satisfies these constraints, as illustrated in Fig. 1b. Since $U_H(\ell_t^{\text{HSL}})$, the utility that types H obtain from the contract ℓ_t^{HSL} , is strictly monotonic decreasing in t , the best HSL terms that can be achieved in a separating contract are given by $\ell_{t_{\text{SAL}}}^{\text{HSL}}$. The utility $U_H(\ell_t^{\text{sep}})$ is concave in t and has a unique maximum at $t = \min[-1/\ln(\delta), T] = t_A$. As t_{SAL} is also the intersection point of the utilities $U_H(\ell_t^{\text{HSL}})$ and $U_H(\ell_t^{\text{sep}})$ for given (p, δ, T) , it follows that the utility maximizing separating loan is made at time t_{SAL} if $t_{\text{SAL}} \leq t_A$ and at time t_A if $t_{\text{SAL}} > t_A$. Lemma I.2 proves that $t_{\text{SAL}} \leq t_A$ always holds under the assumptions of the model. Hence, the utility maximizing separating arm's-length loan contract for type H individuals is $\ell_{t_{\text{SAL}}}^{\text{SAL}}$.

Lemma I.2. $t_{\text{SAL}} \leq t_A$.

Proof. The inequality $t_{\text{SAL}} \leq t_A$ can be reduced to

$$W\left(\frac{Tp\delta^{T/(p-1)}\ln(\delta)}{p-1}\right) \geq \frac{T\ln(\delta)}{p-1} - 1. \quad (13)$$

Since $y = W(x)$ is a solution to $ye^y = x$, it is $W^{-1}(y) = ye^y$. Then Eq. (13) reduces further to

$$1 - p + T\ln(\delta)(1 - pe) \geq 0 \quad (14)$$

which is certainly true if $1 - pe \leq 0$. This is equivalent to $p \geq e^{-1}$ and therefore always satisfied according to the model's assumptions. \blacksquare

2.2.3 Arm's-length Lending Equilibrium

While the first separating condition ensures that type L individuals are not attracted by SAL terms, the second separating condition addresses the question when such a separating contract is chosen by type H individuals. To answer this question it is required to compare the best separating contract that has

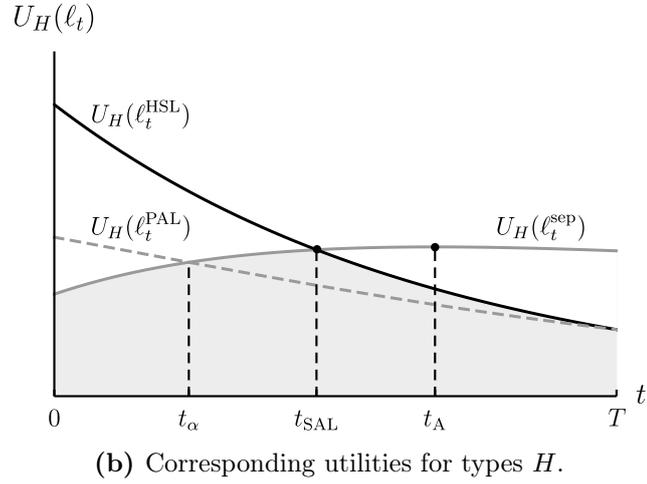
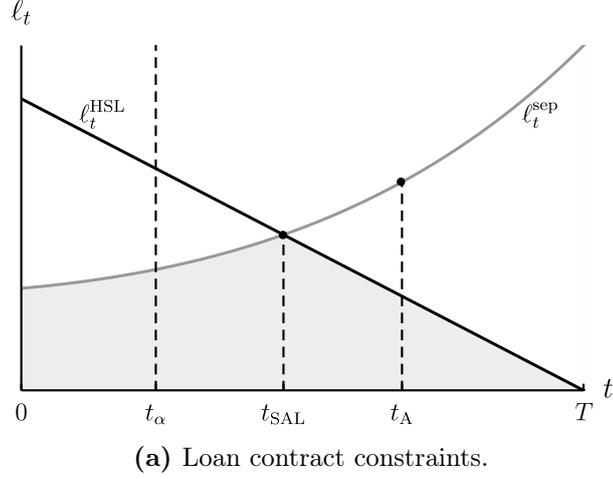


Figure 1: Constraints of separating arm’s-length loan contracts.
The gray filling shows the space (including borders) where the first separating condition is satisfied.

been derived above with the alternative options of high quality individuals. These alternatives have been reduced previously to ℓ_0^{PAL} if $\delta \leq \delta_{\text{PAL:lo}}$, $\ell_{t_{\text{PAL}}}^{\text{PAL}}$ if $\delta \in (\delta_{\text{PAL:lo}}, \delta_{\text{PAL:hi}})$, and ℓ_T^{A} if $\delta \geq \delta_{\text{PAL:hi}}$.

We can easily assert that autarky with consumption at time T is strictly dominated by the best separating arm’s-length loan. There is always an SAL with $t = t_{\text{SAL}} < T$ that provides types H with the utility of an HSL. As obviously $U_H(\ell_{t_{\text{SAL}}}^{\text{HSL}}) = T\delta^{t_{\text{SAL}}} > U_H(\ell_T^{\text{A}}) = T\delta^T$, the autarky option is strictly inferior.

Comparing the best SAL with a pooling arm's-length loan at time $t = 0$, we can reduce $U_H(\ell_0^{\text{PAL}}) \geq U_H(\ell_{t_{\text{SAL}}}^{\text{SAL}})$ to $\delta \leq \delta_l$, where

$$\delta_l := (\theta + p - \theta p)^{\frac{\theta + p - \theta p}{T\theta}}. \quad (15)$$

$\delta_l < \delta_{\text{PAL:lo}}$ is always true for the parameter definitions of the model.¹⁸ Thus, a PAL at time $t = 0$ is both the optimal PAL and preferred over the best SAL if and only if $\delta \leq \delta_l$. If, however, $\delta \in (\delta_{\text{PAL:lo}}, \delta_{\text{PAL:hi}})$ and the optimal PAL is therefore given by $\ell_{t_{\text{PAL}}}^{\text{PAL}}$, Lemma I.3 proves that type H individuals prefer the best SAL in that case, given the parameter definitions of the model.

Lemma I.3. $\ell_{t_{\text{SAL}}}^{\text{SAL}} \overset{H}{\succ} \ell_{t_{\text{PAL}}}^{\text{PAL}}$ if $\delta \in (\delta_{\text{PAL:lo}}, \delta_{\text{PAL:hi}})$.

Proof. See Appendix A.

Low quality individuals always prefer a PAL at $t = 0$ over their best alternative according to Lemma I.1. And they naturally always choose their best alternative (that is, LSL terms at $t = 0$) whenever types H choose an SAL, according to the first condition of a separating contract. Even though banks have no initial information about individuals' borrower quality, type L individuals can at best receive the type-adequate loan terms in that case because an SAL is perfectly separating and type L individuals' choice of a different loan is therefore perfectly type-revealing.

The market equilibrium is finally recorded in Proposition I.2. It shows that individuals are either both financed with pooling loan contracts at $t = 0$, or types H are financed with optimal separating arm's-length loan contracts, while types L receive type-adequate loan terms. Even though the equilibrium is formulated with respect to the time preference measure, which equilibrium actually emerges depends on all the parameters of the model since δ_l depends on all of them.

¹⁸ $\delta_l < \delta_{\text{PAL:lo}}$ directly follows from Lemma I.3: if $\delta > \delta_{\text{PAL:lo}}$, type H individuals cannot prefer a PAL at $t = 0$ over the optimal SAL when the latter is even superior to the best PAL for that δ .

Proposition I.2. *The competitive arm's-length lending equilibrium is given by*

$$(\ell_t^\phi(H), \ell_t^\phi(L)) = \begin{cases} (\ell_0^{\text{PAL}}, \ell_0^{\text{PAL}}) & \text{if } \delta \leq \delta_t, \\ (\ell_{t_{\text{SAL}}}^{\text{SAL}}, \ell_0^{\text{LSL}}) & \text{if } \delta > \delta_t. \end{cases} \quad (16)$$

2.3 Savings-linked Relationship Lending

Banks can probabilistically produce information about individuals' borrower qualities by observing their saving behavior in a saving relationship. This information can be used to form groups of individuals on the basis of their saving behavior over a particular period of time and selectively assign different loan terms to these different groups. There are numerous conceivable designs of such savings-linked relationship lending. First, the formation of different groups of individuals on the basis of their saving behavior can be undertaken in plenty of different ways. And, second, there are infinitely many ways to selectively set loan terms for these different groups for which banks' participation constraint holds. But whatever design of savings-linked relationship lending is chosen, there is one fundamental aspect that always has to be considered: the amount of cross-subsidization between different quality types of individuals. In savings-linked relationship lending, cross-subsidization can occur inside groups if there is pooling of different qualities in the groups defined. Or it can occur between groups if the different loan terms that are assigned to the different groups are not matching the particular qualities of the groups.

Since the high quality types cross-subsidize the low quality types of a pool, the former prefer that pooling inside groups is reduced to a minimum by differentiating between different quality types as precisely as possible when groups are formed. If it is possible to reduce pooling by a better differentiation, there is a bank actually doing so in a competitive financing market to attract the high quality types. Thus, there will be no unnecessary pooling inside groups in equilibrium. It is shown in the following that pooling inside groups is reduced

to a minimum if two groups of individuals are defined on the basis of the saving behavior.

The first group, labeled group F , consists of all the individuals that fail to save in a saving relationship with a bank. Since type H individuals always manage to save under the assumptions of the model, not having a flawless saving track record unambiguously exposes individuals to be of low quality. Thus, individuals who fail to save during a saving relationship form a homogeneous group consisting of low quality individuals only.

The second group, labeled group S , consists of all the individuals that exhibit a flawless saving history in the saving relationship with a bank. Since type L individuals with a flawless track record cannot be distinguished from types H on the basis of the information produced, the second group is heterogeneous in quality and characterized by pooling that cannot be further reduced.

The quality-adequate loan terms for individuals of group F are LSL terms. Worse loan terms are not feasible because individuals can always receive loans with LSL terms in the market. Loan terms superior to quality-adequate loan terms for group F , however, require cross-subsidization between groups at the expense of group S to satisfy banks' break-even constraint. This kind of cross-subsidization is not feasible in equilibrium in a competitive financing market.

Since LSL terms are the quality-adequate loan terms for type L individuals, the time when the loan is made to group F is a degree of freedom and not affecting the terms for group S . According to earlier analysis, individuals of group F prefer to take out a loan with LSL terms as soon as possible, that is, in this case, at the time they fail to save in a saving relationship, t_f .

We can subsume that, in equilibrium, savings-linked relationship lending is characterized by as less cross-subsidization inside and between groups as possible, and quality-adequate loan terms for individuals that fail to save in the saving

relationship with a bank. Thus, the competitive savings-linked relationship loan, ℓ_t^{RL} , follows to be of the general form

$$\ell_t^{\text{RL}} = \begin{cases} \ell_t^{\text{RL}|s} & \text{if saving history is flawless,} \\ \ell_{t_f}^{\text{RL}|f} = \ell_{t_f}^{\text{LSL}} & \text{if saving failure at time } t_f \in (0, t]. \end{cases} \quad (17)$$

In the model, contingent contracting dominates implicit agreements for the purpose of implementing savings-linked relationship lending. Whether a bank actually stands to an implicit agreement depends on the incentives to do so because there is no explicit contractual obligation. The model framework with only one generation of individuals and one financing in an individual's life does (intentionally) not establish reputation effects that are able to enforce implicit contracts to be binding. As the inside bank is the only one to have access to private information about an individual's saving behavior, a so-called informational lock-in of the individuals can occur which allows banks to earn profits instead of offering the best possible loan terms to the individuals (Greenbaum et al., 1989; Sharpe, 1990; Petersen and Rajan, 1995). Contingent contracting, however, is comprehensively possible under the assumptions of the model and leads to ex ante competition between banks over the utilization of the information that can be produced in a saving relationship. This results in the best possible loan terms for individuals (as in Boot and Thakor, 1994). Savings-linked relationship lending is therefore always implemented with contingent contracts in the equilibrium of the model market.

2.3.1 Pooling Relationship Loan Contracts

A pooling relationship loan contract is by definition accepted by both types of individuals. Since the loan terms of individuals of group F are quality-adequate, the loan terms of individuals with impeccable saving histories also require to be quality-adequate. The conditional probability that an individual with flawless

saving track record (that is, savings of n monetary units over n periods) is of high quality is given by

$$\text{prob}(n|H) = \frac{\theta}{\theta + (1 - \theta)p^n}, \quad (18)$$

where naturally $\text{prob}(n|L) = 1 - \text{prob}(n|H)$. Hence, the maximum loan volume of a pooling relationship loan (denoted by PRL) that can be granted at time t in case of a flawless saving history is given by

$$\begin{aligned} \ell_t^{\text{PRL}|s} &= \text{prob}(t|H) (T - t) + \text{prob}(t|L) (T - t)p \\ &= (T - t) \frac{\theta + (1 - \theta)p^{t+1}}{\theta + (1 - \theta)p^t}. \end{aligned} \quad (19)$$

Since type H individuals receive the relationship loan $\ell_t^{\text{PRL}|s}$ with certainty, their utility from a pooling relationship loan is given by $U_H^{\text{PRL}} = (\ell_t^{\text{PRL}|s} + t)\delta^t$. The utility maximizing PRL for types H is made at time $t_{\text{PRL}} := \arg \max_{t \in (0, T)} U_H^{\text{PRL}}$. If there is no finite real-valued t_{PRL} , a pooling relationship loan contract is dominated by a pooling arm's-length loan or by autarky since $\lim_{t \rightarrow 0} U_H^{\text{PRL}} = U_H(\ell_0^{\text{PAL}})$ and $\lim_{t \rightarrow T} U_H^{\text{PRL}} = U_H(\ell_T^{\text{A}})$.

Type L individuals achieve a flawless saving track record over n periods with probability p^n . The utility they receive from a PRL contains the expected utility of the contingent loan in case of an impeccable saving history (that is, loan terms $\ell_{t_{\text{PRL}}}^{\text{PRL}|s}$) and the expected utility of the loan with LSL terms they take out right after failing in the saving stage at t_f . This average LSL (denoted by ALSL), weighted with the particular probabilities of occurrence, is given by

$$U_L^{\text{ALSL}} := \sum_{t_f=1}^t (1 - p)p^{t_f-1} [(T - t_f)p + t_f - 1] \delta^{t_f} \quad (20)$$

$$= \frac{\delta p(1 - p)(\delta^t p^t - 1)(1 + T\delta p - T - \delta)}{(\delta p - 1)^2} + \frac{\delta^{t+1} p^t (1 - p)^2 t}{\delta p - 1}. \quad (21)$$

The expected total utility that type L individuals receive from a savings-linked

pooling relationship loan is therefore given by $U_L^{\text{PRL}} = p^t U_H^{\text{PRL}} + U_L^{\text{ALSL}}$.

2.3.2 Separating Relationship Loan Contracts

As for arm's-length lending, separating relationship loans (denoted by SRL) need to be considered besides pooling relationship loan contracts. Like in the case without relationship lending, the first condition for a separating contract is that the contract must be feasible and worse for type L individuals than their best alternative. This cannot be attained by changing the loan terms for group F to terms that are inferior to $\ell_{t_f}^{\text{LSL}}$ as it does not change the attractiveness for types L at all. That is true because every individual can receive LSL terms at every outside bank at any time, as argued before. Hence, the loan terms for individuals with flawless saving history, $\ell_t^{\text{RL}|s}$, need to be addresses to design a separating relationship loan contract.

A separating contract that provides the loan $\ell_t^{\text{SRL}|s}$ at time t to individuals of group S satisfies the first separating condition if $U_L(\ell_t^{\text{SRL}|s}) \leq \bar{U}_L$ which is equal to $p^t \delta^t (\ell_t^{\text{SRL}|s} + t) + U_L^{\text{ALSL}}(t) \leq Tp$. This rearranges to

$$\ell_t^{\text{SRL}|s} \leq \ell_t^{\text{sep}|s} := \frac{Tp - U_L^{\text{ALSL}}(t)}{p^t \delta^t} - t. \quad (22)$$

If this constraint is satisfied and, thus, no low quality individuals are attracted by the contract, banks' participation constraint for the separating relationship loan is given by $\ell_t^{\text{SRL}|s} \leq \ell_t^{\text{HSL}}$. The stricter of both constraints is crucial for meeting the first separating condition, and it is binding in equilibrium. Thus, we obtain $\ell_t^{\text{SRL}|s} = \min(\ell_t^{\text{sep}|s}, \ell_t^{\text{HSL}})$.

Let us now derive the utility maximizing separating relationship loan contract that satisfies the first separating condition. $U_H(\ell_t^{\text{HSL}})$ is strictly monotonic decreasing in t , while $U_H(\ell_t^{\text{sep}|s})$ is strictly monotonic increasing in t , where $U_H(\ell_{t \rightarrow 0}^{\text{sep}|s}) < U_H(\ell_0^{\text{HSL}})$ and $U_H(\ell_{t \rightarrow T}^{\text{sep}|s}) > U_H(\ell_{t \rightarrow T}^{\text{HSL}})$. Let us denote the unique intersection point of these utility functions as $t_{\text{SRL}} \in (0, T)$ for given (p, δ, T) .

The maximum feasible utility that type H individuals obtain from a separating relationship loan is therefore given by $\ell_{t_{\text{SRL}}}^{\text{SRL}|s}$ and provides HSL terms. Since $\ell_{t_{\text{SRL}}}^{\text{SRL}|s} = \ell_{t_{\text{SRL}}}^{\text{HSL}} = \ell_{t_{\text{SRL}}}^{\text{sep}|s}$, the utility maximizing SRL for types H satisfies the first separating condition.

2.3.3 Relationship Lending Equilibrium

The market equilibrium is derived by comparing the optimal relationship loan contracts derived above with each other and with the optimal choice in case of arm's-length lending (see Proposition I.2). First, it can be shown that the best separating arm's-length loan, $\ell_{t_{\text{SAL}}}^{\text{SAL}}$, is dominated by the optimal separating relationship loan, $\ell_{t_{\text{SRL}}}^{\text{SRL}}$. Both loans offer HSL terms, but the SRL offers them at a sooner time since $t_{\text{SRL}} < t_{\text{SAL}}$, which is proven in Lemma I.4. As $U_H(\ell_t^{\text{HSL}})$ is monotonic decreasing in t , an earlier provision of a loan with HSL terms results in a greater utility for type H individuals.

Lemma I.4. $t_{\text{SRL}} < t_{\text{SAL}}$.

Proof. The difference between $U_H(\ell_t^{\text{sep}|s})$ and $U_H(\ell_t^{\text{HSL}})$ is naturally zero at $t = t_{\text{SRL}}$. Further, the difference is strictly monotonic increasing in t since $\partial[U_H(\ell_t^{\text{sep}|s}) - U_H(\ell_t^{\text{HSL}})]/\partial t > 0$. At $t = t_{\text{SAL}}$, the difference can be detected to be strictly positive, $U_H(\ell_{t_{\text{SAL}}}^{\text{sep}|s}) - U_H(\ell_{t_{\text{SAL}}}^{\text{HSL}}) > 0$. Thus, it follows $t_{\text{SRL}} < t_{\text{SAL}}$. ■

The remaining three loan contracts form the equilibrium choice of high quality individuals: the optimal separating relationship loan, the best pooling relationship loan, and the pooling arm's-length loan at $t = 0$. Type H individuals prefer the best SRL over a PAL at $t = 0$ if $U_H(\ell_{t_{\text{SRL}}}^{\text{SRL}}) > U_H(\ell_0^{\text{PAL}})$ which reduces to the condition

$$\delta > \delta_S := (\theta + p + \theta p)^{1/t_{\text{SRL}}}. \quad (23)$$

The optimal SRL is always preferred over the best PRL by types H if $t_{\text{PRL}} \geq t_{\text{SRL}}$ since $\partial U_H(\ell_t^{\text{HSL}})/\partial t < 0$ and the optimal SRL provides HSL terms, whereas a PRL can, due to imperfect probabilistic learning, only converge to HSL terms

with increasing t . In general, $U_H(\ell_{t_{\text{SRL}}}^{\text{SRL}}) > U_H(\ell_{t_{\text{PRL}}}^{\text{PRL}})$ is true whenever $\delta > \delta_{\text{RL}}$, where I define

$$\delta_{\text{RL}} := \begin{cases} \left(\frac{(1-\theta)(T-t_{\text{PRL}})p^{1+t_{\text{PRL}}} + (1-\theta)p^{t_{\text{PRL}}}t_{\text{PRL}} + T\theta}{T(\theta + p^{t_{\text{PRL}}} - \theta p^{t_{\text{PRL}}})} \right)^{1/(t_{\text{SRL}} - t_{\text{PRL}})} & \text{if } t_{\text{PRL}} < t_{\text{SRL}}, \\ 0 & \text{otherwise.} \end{cases} \quad (24)$$

The best PRL provides greater utility than a PAL at $t = 0$ if $U_H(\ell_{t_{\text{PRL}}}^{\text{PRL}}) > U_H(\ell_0^{\text{PAL}})$ which transforms to the condition

$$\delta > \delta_{\text{P}} := \left(\frac{T(\theta + p - \theta p)(\theta + p^{t_{\text{PRL}}} - \theta p^{t_{\text{PRL}}})}{(1-\theta)p^{t_{\text{PRL}}}[Tp + (1-p)t_{\text{PRL}}] + T\theta} \right)^{1/t_{\text{PRL}}}. \quad (25)$$

The equilibrium is finally stated in Proposition I.3. Apparently we find pooling arm's-length loans instantly provided to all the individuals in equilibrium if their time preference is sufficiently large (and the time preference measure, δ , therefore sufficiently small). Otherwise, pooling or separating relationship loan contracts prevail in equilibrium. Whether the relationship loan contracts are pooling or separating depends heavily on the borrower quality in the market, that is, on θ and p .

Proposition I.3. *The competitive market equilibrium with savings-linked relationship lending is given by*

$$(\ell_t^\phi(H), \ell_t^\phi(L)) = \begin{cases} (\ell_0^{\text{PAL}}, \ell_0^{\text{PAL}}) & \text{if } \delta \leq \min(\delta_{\text{P}}, \delta_{\text{S}}), \\ (\ell_{t_{\text{PRL}}}^{\text{PRL}}, \ell_{t_{\text{PRL}}}^{\text{PRL}}) & \text{if } \delta_{\text{P}} < \delta \leq \delta_{\text{RL}}, \\ (\ell_{t_{\text{SRL}}}^{\text{SRL}}, \ell_0^{\text{LSL}}) & \text{otherwise.} \end{cases} \quad (26)$$

2.4 Parametric Results and Economic Efficiency

To interpret the model's solution, let us calculate and examine the equilibrium in the full parameter space $(\theta \times p)$ and for reasonable values of time preference in this section. Regarding the time preference, it is important to recall that

the time preference parameter, δ , is modeled as a per-period measure. And the time span of a period is by definition given by individuals' saving frequency whose unit is not specified. For manageability, however, type L individuals are modeled to save one monetary unit or nothing in one period, no matter how this period is defined. That means that the unit of the saving frequency does not influence the probabilistic process of saving in the model and the quality of the saving signal is not affected. Because of this invariance of the stochastic process of saving, a *ceteris paribus* comparison of equilibria that arise for different units of the saving frequency is not smoothly possible in my model. The model is rather designed to study savings-linked relationship lending in a multi-period context for a given saving frequency. For a saving frequency of n years, the time preference measure for a given annual rate of individuals' time preference, $r_\delta^{\text{p.a.}}$, generally calculates as $\delta = (1 + r_\delta^{\text{p.a.}})^{-n}$. Since household incomes, major expenses, savings, and credit repayments are in fact mostly cash-flows that occur on a monthly (or sometimes weekly) basis over several years, the model specifically aims to analyze a monthly (or weekly) saving frequency. Thus, although I generally solved the model for $T \geq 2$, the model is intended to analyze situations of much larger T , for instance monthly saving and debt repayment over several years.

If we consider a monthly saving frequency, the corresponding annual time preference rates, $r_\delta^{\text{p.a.}}$, to some particular time preference factors, δ , are given in Table 1. Different empirical studies find strongly varying rates of time preference, which presumably is due to differences in elicitation techniques (see Frederick et al., 2002). Although, we can uncritically assess that, for a monthly saving frequency, focusing on results for $\delta \geq 0.9$ in the parametric analysis of this section entails no loss of generality.

Figure 2 illustrates the equilibria of the model according to Propositions I.2 and I.3 in the parameter space $(\theta \times p)$ by showing isoquant maps of the equilibrium-determining variables. The arm's-length lending equilibrium is

δ	0.9	0.95	0.98	0.99	0.995	0.999
$r_\delta^{\text{p.a.}}$	254.1%	85.1%	27.4%	12.8%	6.2%	1.2%

Table 1: Annual rate of time preference, $r_\delta^{\text{p.a.}}$, and per-period measure of time preference, δ , for a monthly saving frequency.

presented in Figure 2a by the isoquant map of δ_l (solid black lines), the corresponding level is noted above the particular line. Fillings visualize the equilibrium for $\delta = 0.99$. For a given δ , I find separating arm's-length lending, $(\ell_{t_{\text{SAL}}}^{\text{SAL}}, \ell_0^{\text{LSL}})$, below a particular isoquant of δ_l (or in the darkgray space for $\delta = 0.99$) and pooling arm's-length lending, $(\ell_0^{\text{PAL}}, \ell_0^{\text{PAL}})$, above a particular isoquant of δ_l (or in the white space for $\delta = 0.99$).

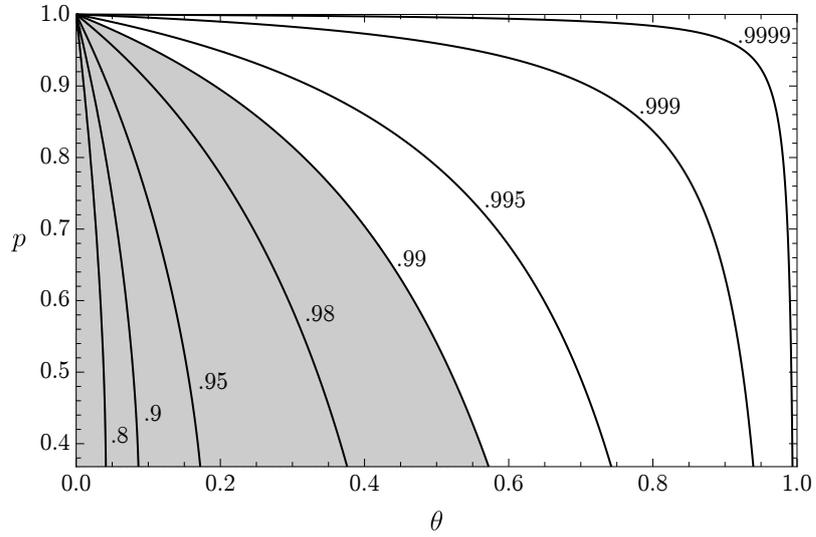
Figure 2b illustrates savings-linked relationship lending equilibrium by showing the relevant¹⁹ parts of the isoquant maps of δ_S (solid black lines), of δ_P (solid gray lines), and of δ_{RL} (dashed black and solid black lines)²⁰. The graph is calculated for $T = 40$ but the relationship lending equilibrium hardly differs for any $T \geq 25$; to illustrate this, Appendix B presents the graph for a range of different T . That means that, in terms of a monthly saving frequency, the results based on Figure 2b are thoroughly valid for any loan with a credit period of more than two years.²¹

The dotted gray line in Figure 2b marks all the points where the different isoquants of the same level intersect; the level is noted at the corresponding intersection point. Given a particular level of time preference δ , according to Proposition I.3 we find separating relationship lending in equilibrium in the parameter space that is left of the particular dashed black and solid black line, pooling relationship lending is chosen in the parameter space that is right of (and on) the particular dashed black line while also being below the particular solid gray line, and pooling arm's-length loans are the equilibrium choice in the

¹⁹The 'relevant' part of a particular isoquant is the part that is an actual border between different equilibrium choices in the parameter space (θ, p) , pursuant to Proposition I.3.

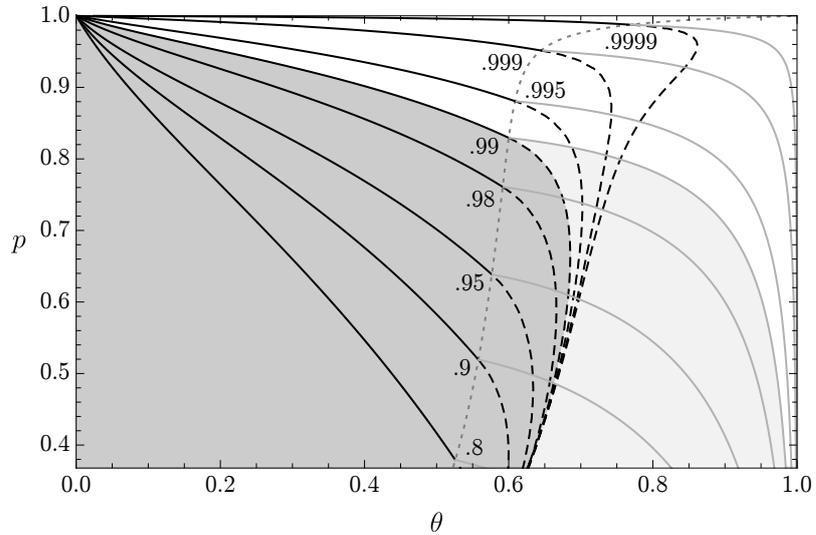
²⁰While the relevant part of δ_S obviously lies exactly on δ_{RL} , the non-relevant part does not.

²¹Although the graph differs somehow for very small $T < 25$, some main conclusions still remain valid even in this case.



(a) Arm's-length lending equilibrium.

Isoquant map of δ_v (solid black), fillings conform to $\delta = 0.99$ (darkgray: SAL, white: PAL).



(b) Savings-linked relationship lending equilibrium.

Relevant parts of the isoquant maps of δ_S (solid black), δ_P (solid gray), and δ_{RL} (dashed black and solid black) for $T = 40$. Fillings conform to $\delta = 0.99$ (darkgray: SRL, lightgray: PRL, white: PAL).

Figure 2: Parametric illustration of the model equilibria (for $T = 40$).

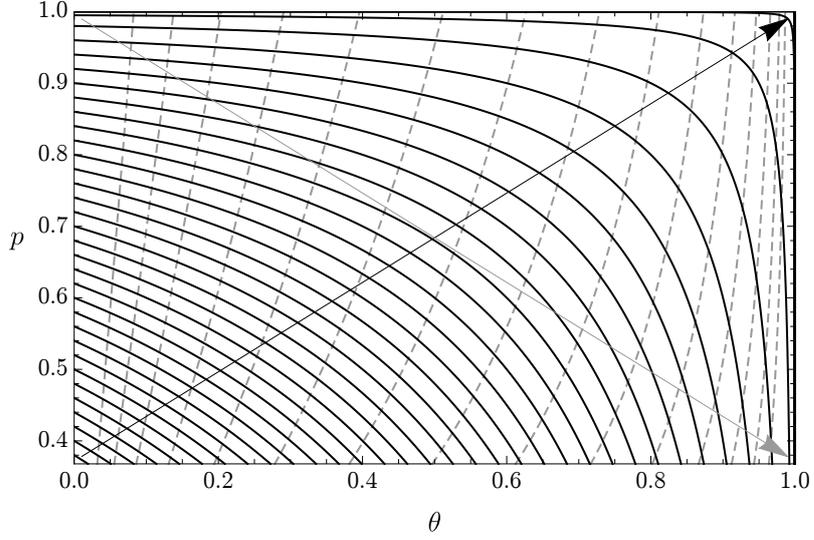


Figure 3: Isoquant map of average and relative borrower quality.

Solid black: average borrower quality, *dashed gray:* relative borrower quality. Similar colored arrows point in the particular direction of increase.

parameter space above (and on) the particular solid black and the solid gray line. Filled areas visualize the equilibrium for $\delta = 0.99$, where the darkgray region conforms to equilibria with separating relationship lending, $(\ell_{t_{\text{SRL}}}^{\text{SRL}}, \ell_0^{\text{LSL}})$, the lightgray colored parameter space is the one where pooling relationship lending occurs, $(\ell_{t_{\text{PRL}}}^{\text{PRL}}, \ell_{t_{\text{PRL}}}^{\text{PRL}})$, and the white space covers equilibria of pooling arm's-length lending, $(\ell_0^{\text{PAL}}, \ell_0^{\text{PAL}})$. One can directly recognized that, the larger δ (that is, the lower the time preference of the individuals), the larger the parameter space where savings-linked relationship lending is preferred over arm's-length lending in equilibrium.

Figure 3 illustrates isoquant maps of the average borrower quality (solid black lines) and the relative borrower quality (dashed gray lines). Let us define the average borrower quality, \bar{q} , of the market to be the expected repayment of a simple loan of volume one that is provided to random individuals. Let the relative borrower quality, q_r , be defined as the ratio of this repayment's fraction that is expected to be made by high quality individuals' to the repayment's

fraction that is expected to come from low quality individuals. Formally,

$$\bar{q} := \theta + (1 - \theta)p, \quad (27)$$

$$q_r := \theta/(p - \theta p). \quad (28)$$

An isoquant of \bar{q} , as plotted in Fig. 3, contains all points (θ, p) that result in the same expected repayment of the simple loan of volume one. An isoquant of q_r contains all combinations (θ, p) that result in the same ratio of repayment fractions of the simple loan. Thus, an isoquant at level \bar{q} or q_r is given by $p(\bar{q}) = (\bar{q} - \theta)/(1 - \theta)$ or $p(q_r) = \theta/(q_r - \theta p)$. The level of average borrower quality increases towards the upper right hand corner, $(1, 1)$, and the level of relative borrower quality increases towards the lower right hand corner, $(1, 0)$.

Comparing Fig. 2b and Fig. 3 discloses a distinct relationship between market equilibrium and borrower quality. While the average borrower quality essentially determines if the equilibrium is composed of arm's-length lending or relationship lending, the relative borrower quality basically determines if we find pooling or separating relationship lending in equilibrium. Hence, if the market is characterized by high average borrower quality, we can only find savings-linked relationship lending in equilibrium if individuals possess a low rate of time preference (that is, a large δ). If the market, however, is characterized by low average borrower quality, savings-linked relationship lending is already viable for higher rates of time preference. A high (low) relative borrower quality resolves the further choice between pooling and separating relationship lending in favor of the former (latter). This leads to Theorem I.1.

Theorem I.1. *Savings-linked relationship lending is generally an equilibrium choice in markets of low average borrower quality or in markets of low time preference. Further, savings-linked relationship lending is rather a separating (pooling) equilibrium if relative borrower quality is low (high).*

Savings-linked relationship lending requires a saving relationship with the individuals prior to the provision of the loan. That means that individuals

who enter such a contract agree to postpone their consumption although they could receive an arm's-length loan instantly. Rational individuals only do so if relationship lending enables more or better, albeit later, consumption. Thus, the time preference measure, δ , acts as a trade-off parameter between time of consumption and quality of consumption. Individuals who choose savings-linked relationship lending prefer quality of consumption over time of consumption.

When comparing the savings-linked relationship lending equilibrium (Fig. 2b) as stated by Proposition I.3 with the arm's-length lending equilibrium (Fig. 2a) of Proposition I.2, we can assess the effects of savings-linked relationship lending on economic efficiency which are subsumed in Theorem I.2 and amplified thereafter.

Theorem I.2. *In equilibrium, savings-linked relationship lending leads to a Pareto improvement or an increasing allocative efficiency of the financing market compared to arm's-length lending.*

Savings-linked relationship lending constitutes a Pareto improvement over separating arm's-length contracts since high quality individuals would not choose relationship lending in equilibrium if it was inferior to separating arm's-length loans, of course. Low quality types obtain their lower limit of utility whenever high quality types choose a separating arm's-length loan and can therefore not get worse.

Savings-linked relationship lending does not lead to a Pareto improvement in comparison to pooling arm's-length lending in the basic model. But information production from individuals' saving behavior prior to lending allows banks to offer more quality-adequate or even perfectly quality-adequate loan terms. Thus, the inherent cross-subsidization from high to low quality individuals in pooling arm's-length lending can be reduced. This naturally reduces the utility of low quality types and increases that of high quality ones in comparison to pooling arm's-length lending where cross-subsidization is maximized. The importance of the increasing allocative efficiency that relationship lending can

induce is not salient enough in the basic model since there is always financing for every individual and, therefore, the distribution of wealth is addressed more than the formation of social welfare. In Section 3.2 the model is modified in this regard to highlight the relevance of efficient allocation of capital in the context of this work.

3 Modifications to the Model and Robustness

3.1 Symmetric Unawareness of Borrower Quality

If we change the model's assumptions with regard to individuals' knowledge of their own type, the results change significantly. Let us assume that individuals are not aware of their own type but, like banks, only of the fraction θ of high quality type individuals in the population. This assumption of symmetric unawareness of borrower quality causes separating loan contracts to be impossible. The expected utility that individuals receive from financing alternative ϕ is calculated as a weighted average of the utility of low quality types and that of high quality types:

$$\mathbb{E}[U(\ell_t^\phi)] = \theta U_H(\ell_t^\phi) + (1 - \theta) U_L(\ell_t^\phi). \quad (29)$$

As $\partial \mathbb{E}[U(\ell_t^{\text{PAL}})] / \partial t = T \log(\delta)(\theta + p - \theta p)\delta^t < 0$, the expected utility that type-unaware individuals obtain from a pooling arm's-length loan is maximized for the lower limit of t which is $t = 0$. Thus, we obtain $\ell_0^{\text{PAL}} \succ \ell_t^{\text{PAL}} \forall t \in (0, T)$ for type-unaware individuals. Comparing the expected utility from that optimal pooling arm's-length loan with the expected utility from autarky, we can assess the difference of the particular expected utilities, $\mathbb{E}[U(\ell_0^{\text{PAL}})] - \mathbb{E}[U(\ell_t^{\text{A}})] = (T - t\delta^t)(\theta + (1 - \theta)p)$, to be obviously always positive for the parameter definitions in this model. Hence, type-unaware individuals have the preference relation $\ell_0^{\text{PAL}} \succ \ell_t^{\text{A}} \forall t \in [0, T]$.

It follows that in a market of arm's-length lending to type-unaware individuals, only instantly provided pooling arm's-length loans, ℓ_0^{PAL} , are found in equilibrium in the whole parameter space of the model. And this result is still valid if savings-linked relationship lending is considered, as Proposition I.4 proofs.

Proposition I.4. *Type-unaware individuals prefer arm's-length lending over savings-linked relationship lending.*

Proof. It is $\lim_{t \rightarrow 0} \mathbb{E}[U(\ell_t^{\text{PRL}})] = T(\theta + p - \theta p) = \mathbb{E}[U(\ell_0^{\text{PAL}})]$. The utility function $\mathbb{E}[U(\ell_t^{\text{PRL}})]$ has exactly one unique global maximum at some $t^* \in \mathbb{R}$ for the parameter definitions in this model. Thus, if the first partial derivative of $\mathbb{E}[U(\ell_t^{\text{PRL}})]$ with respect to t is negative at $t = 0$, it is $\ell_{t_1}^{\text{PRL}} \succ \ell_{t_2}^{\text{PRL}} \forall t_1 < t_2$, where $t_1, t_2 \in (0, T)$, and the optimal PAL, ℓ_0^{PAL} , is preferred. There can only be a feasible PRL, $\ell_{t>0}^{\text{PRL}}$, that provides greater utility than the best PAL, ℓ_0^{PAL} , if the mentioned first partial derivative at $t = 0$ is positive. Solving $(\partial \mathbb{E}[U(\ell_t^{\text{PRL}})] / \partial t)|_{t=0} > 0$ for T gives the condition $T < T_\kappa(\theta, p, \delta)$, where

$$T_\kappa(\theta, p, \delta) := \frac{(1 - \theta)(1 - p)(1 - \delta)[1 - \delta p + \delta p(\log(\delta) + \log(p))]}{(\delta p - 1)[(1 - \theta)(1 - \delta)p \log(p) + (\theta + p(1 - \theta - \delta)) \log(\delta)]}. \quad (30)$$

Maximizing $T_\kappa(\theta, p, \delta)$ in the parameter space of the model yields $\hat{T}_\kappa(\theta \rightarrow 0, p = e^{-1}, \delta \rightarrow 1) = e - 2$. In the multi-period model we have $T \geq 2$ and thus it is always $T > T_\kappa(\theta, p, \delta)$. ■

From Proposition I.4 follows that savings-linked relationship lending is irrelevant in a market with type-unaware agents. Individuals' knowledge of their own quality type is therefore a necessary condition for savings-linked relationship lending to be meaningful. That also means that separating contracts cannot be neglected whenever savings-linked relationship lending is considered.

3.2 Adverse Selection

The basic model, as formulated and solved above, does not contain adverse selection problems. There is always financing for every individual in equilibrium.

This is true in the first best solution, in the arm's-length lending equilibrium, and as well in the equilibrium with savings-linked relationship lending. I intentionally solved the model neglecting factors that lead to adverse selection, that is, situations where financing is not possible for or not accepted by one or both groups of individuals. The reason for doing so is that a supplementary introduction and discussion of such factors might provide a more transparent and more versatile insight to the problem. This section seeks to slightly modify the basic model to create the possibility of non-financing equilibria due to adverse selection. This can be achieved by introducing a reservation utility for high quality individuals and a quality floor of good G into the basic model.

Let us define RU_H to be the reservation utility of type H individuals. If a financing option ϕ provides weakly less utility than the reservation utility RU_H , types H do not choose financing option ϕ . It seems reasonable to define the reservation utility relative to the market environment, that is, the reservation utility differs for different values of the parameters (θ, p, δ, T) .²² To have an impact on the equilibrium, the reservation utility of types H needs to be at least as large as their lower limit of utility, \bar{U}_H , which is given by a pooling arm's-length loan made at $t = 0$, thus $\bar{U}_H = U_H(\ell_0^{\text{PAL}})$. Further, the reservation utility of types H should be assumed not to exceed the utility that they receive from the first best solution, otherwise the consumption of good G is not preferable at all for high quality individuals. Hence, we obtain $RU_H \in [\bar{U}_H, T)$. With this definition, the following two situation can emerge for a given parameter set (θ, p, δ, T) if such a reservation utility is introduced. First, neither arm's-length lending nor relationship lending is accepted by type H individuals in equilibrium. Second, arm's-length lending is not possible in equilibrium but relationship lending is. As a result, if the reservation utility of high quality individuals is sufficiently large to affect the equilibrium for a given parameter set (θ, p, δ, T) ,

²²Defining the reservation utility as a constant absolute number, independent from the market environment, is basically equivalent to just cutting out the parameter space that is characterized by generally lower utility levels. I do not see a serious economic application to this approach.

financing can only occur in the form of savings-linked relationship lending in equilibrium.

Introducing a reservation utility for low quality individuals is not capable of affecting the equilibrium since their reservation utility would require to be above their quality-adequate utility to have an impact, which seems to be a surreal situation. Let us therefore just define $RU_L = \bar{U}_L$ in order to get a clear analysis. Instead, to create an impact on the equilibrium, we can address types L by introducing a quality floor of good G , denoted Q_{\min} . That simply means that it is not possible, not reasonable, or not desirable to consume less than Q_{\min} of G . For notational convenience let us assume that the consumption quality of good G needs to strictly exceed Q_{\min} . Such a quality floor only impacts the equilibrium if it weakly exceeds the consumption quality that type L individuals obtain from their lower limit of utility, which is equivalent to \bar{U}_L .²³ To avoid affecting types H with the introduction of a quality floor and overriding their reservation utility that I have currently introduced, let the quality floor be below the quality that types H obtain in their lower-limit case, which happens to be \bar{U}_H . We finally get $Q_{\min} \in [\bar{U}_L, \bar{U}_H)$. With such a quality floor of good G that has an impact on the equilibrium, low quality individuals can only purchase and consume good G in pooling equilibria with sufficient cross-subsidization from types H . Further, if $Q_{\min} > \bar{U}_L$, separating contracts become possible for lower t . They are therefore more attractive for high quality individuals and found in equilibrium in a larger parameter space than before.

With these two modifications—reservation utility for types H and quality floor of good G —the model entails the following typical literature cases of adverse selection problems: first, low quality agents cannot be financed on their own and, second, collective pooling of all agents (or random agents) does not work because of adverse selection. The minimum extent of modification

²³Type L individuals' lower limit of utility is a type-adequate loan provided at $t = 0$. Thus, there is neither discounting nor saving and the consumption quality is equal to $\bar{U}_L = U_L(\ell_0^{\text{LSL}}) = \ell_0^{\text{LSL}}$.

to achieve this is given by the borderline cases $RU_H = \bar{U}_H$ and $Q_{\min} = \bar{U}_L$ which allow a particularly straightforward comparison with the equilibria of the basic model. In the first best solution, low quality individuals can no longer be financed since they ask for a loan of at least Q_{\min} that cannot be granted to them; types H still receive financing with type-adequate loan terms at time $t = 0$. In the arm's-length lending equilibrium, financing can now only occur in separating contracts. There is a financing market failure whenever a pooling loan would be the utility maximizing contract since high quality individuals do not accept general pooling terms and separating contracts require too costly signaling in that case. When adding savings-linked relationship lending, we find financing either with separating relationship contracts or with pooling relationship contracts in equilibrium. The latter case is special because it is the only case, given the modifications of this section, where low quality individuals can receive financing in the model equilibrium (if they successfully save in the saving relationship prior to the loan, of course).²⁴

It becomes evident that, with the constraints of this section, savings-linked relationship lending does always result in a Pareto improvement compared to arm's-length lending. In the parameter space where relationship lending did not result in a Pareto improvement but in an improved allocative efficiency in the basic model, this higher allocative efficiency now results in a Pareto improvement under the new constraints. Additionally and recorded in Theorem I.3, savings-linked relationship lending can overcome the market failure of arm's-length lending that the modifications of this section induce. This follows from the fact that the best separating arm's-length contract is strictly dominated by savings-linked relationship lending (see Lemma I.4 and subsequent elaboration) and, thus, there surely is a non-empty parameter space that allows savings-linked relationship lending (separating or pooling) in equilibrium while arm's-length financing breaks down.

²⁴This case, however, does not exist if the reservation utility of types H is even too large to have them accept the semi-pooling loan terms of pooling relationship lending.

Theorem I.3. *Savings-linked relationship lending can overcome financing market failure due to adverse selection and enable financing when arm's-length lending is not viable in equilibrium.*

3.3 Robustness to Financing Volume

Let us regard the model's total number of periods, T , as time span in which individuals are able and willing to regularly assign savings to payments that are related to good G .²⁵ If individuals desire to acquire a higher priced good G , they have to assign savings over a longer time span because the greater T , *ceteris paribus*, the more funds are available in total for the purchase of good G with or without credit financing. Large T therefore correspond to financing volumes that are large compared to households' periodic saving potential which, in turn, can be considered to be positively related to income. To examine the robustness of savings-linked relationship lending and its efficiency to changes in financing volume, I analyze the model's equilibria for different T in the following.

As stated in Proposition I.2, the arm's-length lending equilibrium is determined by δ_l which is functionally dependent on T . It is $\partial\delta_l/\partial T > 0$, and $\lim_{T \rightarrow 0} \delta_l = 0$, $\lim_{T \rightarrow \infty} \delta_l = 1$. It follows that δ_l not only increases with increasing T but also changes significantly over its whole domain of definition. Thus, the arm's-length lending equilibrium also changes significantly for changes in T , that is, for different financing volumes. The higher T , the more predominant are pooling contracts in the arm's-length lending equilibrium. Figure 4 illustrates the arm's-length lending equilibrium for different T on the right-hand side. The left-hand side of Fig. 4 shows the savings-linked relationship lending equilibrium for different T . Lines and fillings are used similar to those in Fig. 2. The lines in each subfigure are isoquants of the equilibrium-determining variables at the levels marked in Fig. 4b.

²⁵Using T as life span in the basic model is solely for modeling convenience since the model does not aim at studying the relationship between life span and relationship lending.

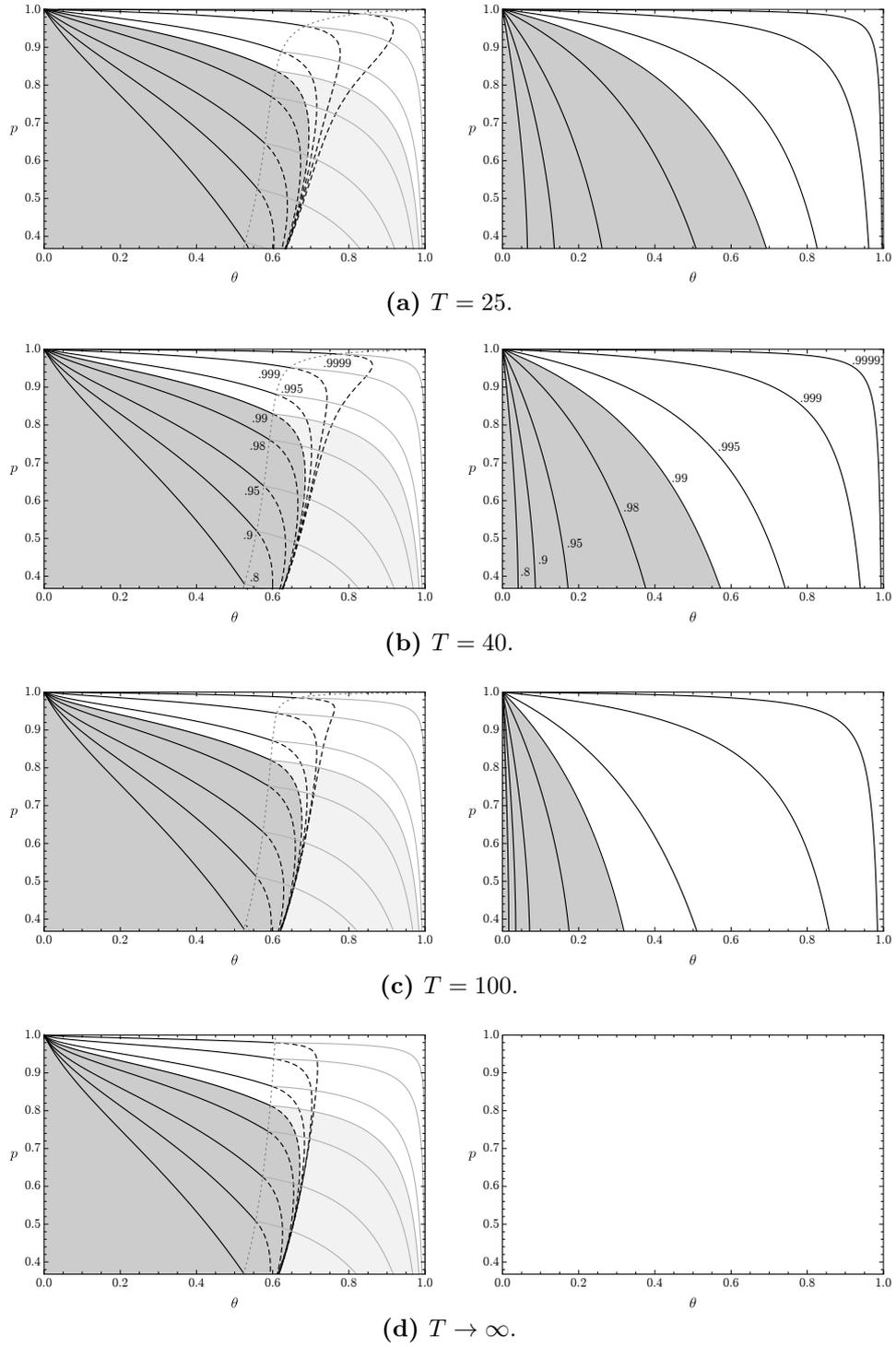


Figure 4: Equilibria for savings-linked relationship lending (*left*) and arm's-length lending (*right*) for different T .

Fillings (for $\delta = 0.99$): *darkgray left*: SRL, *lightgray*: PRL, *white*: PAL, *darkgray right*: SAL. Lines: *solid black left*: δ_S , *solid gray*: δ_P , *dashed black and solid black left*: δ_{RL} , *solid black right*: δ_L .

In contrast to arm's-length lending, the savings-linked relationship lending equilibrium is obviously very robust to changes in T and does hardly alter for any $T \geq 25$. When considering constraints that lead adverse selection problems, as done in Section 3.2, financing fails whenever pooling arm's-length lending is predominant (that is, in the white parameter space in Fig. 4 for $\delta = 0.99$). We see that arm's-length lending fails increasingly for increasing relative financing volumes, while the savings-linked relationship lending equilibrium is robust to increasing T , *ceteris paribus*. Hence, savings-linked relationship lending shows to be of exceeding economic value for financing volumes that are large in comparison to households' periodic savings or income. Theorem I.4 subsumes.

Theorem I.4. *Savings-linked relationship lending can overcome financing market failure due to adverse selection particularly for financing volumes that are large in comparison to households' periodic savings or incomes.*

4 Savings-linked Relationship Lending for Housing Finance

4.1 Application of the Model and Implications

Housing can be considered a good in the sense of my model as it is an expenditure that requires saving or lending and is generally not acquired on a regular basis in an individual's life. Plus, demand for housing is well known to show a positive income elasticity, which is in line with my assumptions. Of course, housing finance is usually not operated completely unsecured in contrast to the model's assumption. But this assumption allows to highlight the role of private information in relationship lending and to isolate the result from well-known effects of collateral. My model and its results can therefore generally be applied to housing finance. In Section 3.3 I found savings-linked relationship lending to be particularly well suited for housing finance. That is because housing usually requires large expenditures compared to households'

periodic savings or incomes and, while arm's-length lending is more likely to break down in this scenario due to effects of adverse selection, savings-linked relationship lending remains hardly affected.

The idea of savings-linked relationship lending is applied in the real world in the form of contractual saving for housing. CSH contracts are basically very similar to the equilibrium contracts of savings-linked relationship lending in my model: in a contractual saving stage, savings are regularly transferred to the providing bank over a particular time horizon before a loan is made contingent on the saving behavior. Since savings-linked relationship lending can be identified to be a central element of CSH, CSH shares the economic efficiency that my model identifies for savings-linked relationship lending. My work is to my best knowledge the first to rigorously explain CSH and its economic value purely in terms of relationship lending. Further, the model can be used to evaluate CSH for different markets. This is of interest because CSH actually exists in highly developed economies (like Austria, France, Germany) as well as in transition economies (like China, Croatia, Czech Republic, Hungary, Poland, Romania). To give a more contextual interpretation of the model's results, let us presume a positive correlation between the developedness of an economy and its average borrower quality. Savings-linked relationship lending is therefore likely to be an economically beneficial equilibrium choice in transition economies where the average borrower quality is typically rather low. But the model also shows that savings-linked relationship lending can be an economically beneficial equilibrium choice in highly developed economies with high average borrower quality if time preference regarding housing is sufficiently low. This, for instance, is a quite appropriate characterization of the German market where CSH flourished over many centuries.²⁶ These results sharply contrast the view of Lea and Renaud (1995) that “a CSH-system would have no justification in fully developed and competitive financial markets today.”

²⁶A recent study by Wang et al. (2016) of time preferences in 53 countries indicates that time preference in Germany is comparatively low.

Products of housing finance for private households are politically often also evaluated in terms of their coverage in society. With adverse selection problems due to the constraints of Section 3.2, pooling savings-linked relationship lending is characterized by more lending compared to the arm's-length solution and, moreover, it also leads to more lending compared to the first best solution. While adverse selection only allows lending to high quality individuals in the first best world, pooling savings-linked relationship contracts additionally allow lending to low quality individuals who accomplish to save consistently. This equilibrium overinvestment can be socially or politically desirable for housing finance in order to promote home ownership.

4.2 State Subsidy

It appears to be a traditional concern of governments to promote home ownership. This is *inter alia* done by subsidizing products of housing finance. State subsidy for CSH exists and is common in economies where CSH is found. This section introduces state subsidy to the model framework to analyze its general effects on savings-linked relationship lending. My partial equilibrium model is, however, not particularly designed and suited to analyze state subsidy and can therefore not accomplish the derivation of detailed conditions for optimal state subsidy or the analysis of comprehensive welfare outcomes.

It has previously been shown that the possibility of learning about individuals' borrower quality is a crucial element of savings-linked relationship lending, even if information production is costly. Hence, state subsidy for savings-linked relationship lending in general, or for contractual saving for housing in particular, should increase the attractiveness of such contracts without diluting their informational benefits. That means that the subsidies should be perfectly distinguishable from individuals' savings to maintain banks' abilities to produce information from individuals' saving behavior. We call such state subsidy information-neutral.

Actual state subsidy for contractual saving for housing, as in Austria and Germany, can be considered information-neutral in this sense. In Germany, for instance, the direct state subsidies (*Arbeitnehmersparzulage* and *Wohnungsbauprämie*) are basically add-on payments to individuals' contractual savings in the saving stage of a CSH and they are transferred directly to CSH saving accounts of subsidized individuals. The same is true for the Austrian direct state subsidy of CSH (*Bausparprämie*). Hence, banks can perfectly distinguish between savings and subsidies and information production from individuals' saving behavior is not diluted.

To incorporate state subsidy for savings-linked relationship lending into the model, let us define $\lambda(t) > 0$ as the total state subsidy that every individual receives with the relationship loan at the time, t , the loan is made. To ensure that state subsidy is information-neutral, the subsidy payments are transferred directly to individuals' saving accounts. If individuals do not obtain the relationship loan, the subsidy is not provided, which is consistent with actual subsidy of CSH (for instance in Germany) and is in line with the underlying objective to promote home ownership. In my model such state subsidy enables higher consumption quality (which means higher quality housing in the context of housing finance) for individuals with a successful saving stage. But as individuals' future debt service potential remains unchanged, the subsidy does not affect the general conditions of pooling relationship lending. Thus, with subsidy, the utility functions for savings-linked pooling relationship lending merely differ by the expected discounted subsidy. Using a breve accent to mark that subsidy is considered, we obtain

$$\check{U}_H^{\text{PRL}} = U_H^{\text{PRL}} + \delta^t \lambda(t), \quad (31)$$

$$\check{U}_L^{\text{PRL}} = U_L^{\text{PRL}} + p^t \delta^t \lambda(t). \quad (32)$$

The optimal pooling relationship loan, $\check{\ell}_{t^{\text{PRL}}}^{\text{PRL}}$, for type H individuals usually differs with state subsidy compared to a market without subsidy. Actually,

\check{U}_H^{PRL} and U_H^{PRL} only share the same maximum if the subsidy function is of the form $\lambda(t) = c\delta^{-t}$, where c is a constant.

Since state subsidy obviously improves savings-linked relationship lending for low quality individuals in case of a successful saving stage, while their lower limit of utility (which they expect to receive when they fail in the saving stage) remains unaffected, the constraint for not attracting low quality individuals with a separating relationship contract, as originally stated by Eq. (22), changes with state subsidy $\lambda(t)$ to

$$\check{\ell}_t^{\text{SRL}|s} \leq \check{\ell}_t^{\text{sep}|s} := \frac{Tp - U_L^{\text{ALSL}}(t)}{p^t\delta^t} - t - \lambda(t) = \ell_t^{\text{sep}|s} - \lambda(t). \quad (33)$$

Banks' participation constraint for separating contracts remains unaffected. The first separating condition for SRL contracts with state subsidy therefore requires $\check{\ell}_t^{\text{SRL}|s} = \min(\check{\ell}_t^{\text{sep}|s}, \ell_t^{\text{HSL}})$. Obviously, it is $\check{\ell}_t^{\text{sep}|s} < \ell_t^{\text{sep}|s}$. But since type H individuals receive the state subsidy $\lambda(t)$ at time t with probability of 1 in subsidized relationship lending, the subsidy fully compensates the smaller loan volume $\check{\ell}_t^{\text{sep}|s}$ and the corresponding utility is the same as without state subsidy, $\check{U}_H(\check{\ell}_t^{\text{sep}|s}) = U_H(\ell_t^{\text{sep}|s}) \forall t$. Due to the subsidy, however, the utility that type H individuals obtain from loan terms ℓ_t^{HSL} increases, $\check{U}_H(\ell_t^{\text{HSL}}) > U_H(\ell_t^{\text{HSL}}) \forall t \in (0, T)$. Recalling the properties of $U_H(\ell_t^{\text{sep}|s})$ and $U_H(\ell_t^{\text{HSL}})$ —the former is monotonic increasing in t , the latter is monotonic decreasing in t , and they intersect once at t_{SRL} in the domain $t \in (0, T)$ —, we see that type H individuals' utility maximizing SRL contract in case of state subsidy is consequently characterized by declined loan terms (smaller loan at a later time) but increased utility in comparison to the equilibrium SRL contract in a market without state subsidy. This result is illustrated by Fig. 5.

To conclude, state subsidy for savings-linked relationship lending, of course, causes relationship lending to be more attractive for individuals. But for separating relationship contracts and—depending on the subsidy function—mostly also for pooling relationship contracts, state subsidy induces that individuals

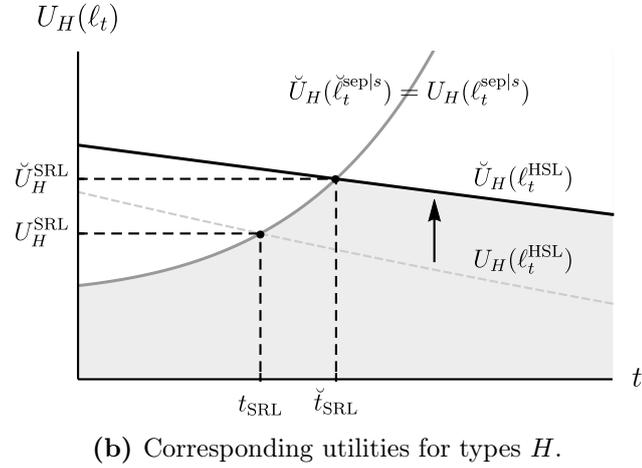
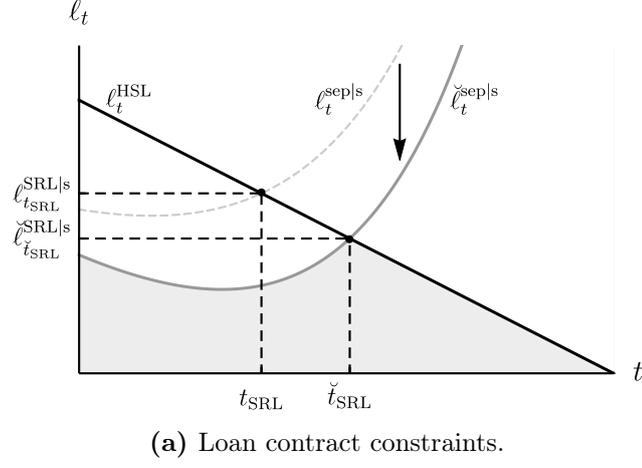


Figure 5: The effect of state subsidy on the constraints of separating savings-linked relationship loan contracts.

The gray filling shows the space (including borders) where the first separating condition is satisfied.

choose inferior loan terms. That means that only a fraction of the subsidy's value is converted into utility, and the other part is just compensating inferior loan terms. Thus, we encounter a deadweight loss due to state subsidy. This loss may be acceptable if the subsidy serves another objective that is supposed to yield positive social effects that are not captured by my partial equilibrium model. Such an objective could be the promotion of home ownership. In the basic model, where every individual can always receive financing, subsidy can clearly not improve home ownership rates. For that matter let us in the follow-

ing discuss the modified model market that entails adverse selection problems due to the modifications of Section 3.2.

With all or some of the constraints of Section 3.2, state subsidy for savings-linked relationship lending can overcome adverse selection and effect a transition from a non-financing equilibrium to a financing equilibrium and therefore accomplish the objective to promote home ownership.²⁷ Home ownership rates also rise if introducing state subsidy causes a transition from a separating equilibrium to a pooling financing equilibrium (for instance, from separating relationship lending to pooling relationship lending). But state subsidy for savings-linked relationship lending can also have inverse effects and reduce home ownership. This is the case if the state subsidy results in a transition from pooling financing to a separating solution.²⁸ Such an undesired transition can, for instance, happen over time if the market environment (e.g. the average/relative borrower quality or the time preference) changes, while the subsidy is not adjusted, since a state subsidy that leads to increasing home ownership rates in one market environment (through desired equilibrium transition) can reduce them in another (through undesired equilibrium transition).

5 Conclusion

Producing private information about households through repeated lending is often not a very promising approach since they prevalently possess very thin credit track records or no credit track records at all when they demand financing. Instead, as I argued, relationship lenders can gather credit-relevant information about households in saving relationships prior to lending. I modeled and analyzed such savings-linked relationship lending to private households in

²⁷This can particularly be the case in the parameter space in which pooling arm's-length lending is found in the basic model's equilibrium because an effective reservation utility of types H leads to non-financing in that space, as argued in Section 3.2.

²⁸With the constraints of Section 3.2, this is the case for a transition from PRL (where all high quality individuals and some type L individuals purchase housing) to SRL (where only type H individuals purchase housing).

a multi-period context. I identified market environments where savings-linked relationship lending is found in equilibrium. These environments are generally characterized by low time preference or low average borrower quality, and they show improvements in market efficiency in comparison to arm's-length lending. Savings-linked relationship lending can also overcome financing market failure due to adverse selection and enable financing when arm's-length lending is not viable in equilibrium, which is particularly true for financing volumes that are large in comparison to households' periodic savings or incomes. This characteristic makes savings-linked relationship lending particularly well suited for housing finance of private households.

Contractual saving for housing, an important product of housing finance in Continental Europe, shares major characteristics with savings-linked relationship lending. My analysis is therefore capable of giving a theoretical relationship lending explanation for CSH, which has to my best knowledge not been done before in the literature. This helps to generate a better understanding of CSH, which is important for two reasons. First, it allows to rethink and adapt CSH in changing market environments without destroying or diluting the integral value of the concept that accounts for a substantial share of housing finance in several European economies. Plus, policy makers are able to effectively plan and assess the introduction of CSH-like concepts and state subsidies for CSH. Second, centralized financial regulation is prone to overlook and violate specific features of regional financial products and concepts, especially if these are not fully understood. In order to enable regulators to make differentiated decisions, the economic benefits and shortcomings of financial concepts like CSH need to be elaborated. My work is one step further in that direction.

Future empirical research could address the validity of my results by applying the model to European transition economies that implemented contractual saving for housing several years ago. The model could help to disclose why the implementation was a success for some of the economies, while it was not for

others (as reviewed by Diamond, 1998).

Savings-linked relationship lending could also be an economically beneficial concept for another reason that is not considered and studied in this work but is worth future research. A saving relationship that precedes lending may not just convey information about households' borrower qualities to banks. It could also teach financial discipline to individuals. The remuneration of lending when not failing to save could incentivise agents to regularly save in a saving relationship and thereby sustainably increase their borrower quality.

Appendix A. Proofs

Proof of Lemma I.3.

The condition $U_H(\ell_{t_{\text{SAL}}}^{\text{SAL}}) \leq U_H(\ell_{t_{\text{PAL}}}^{\text{PAL}})$ is given by

$$T\delta^{\frac{T}{1-p} + \ln(\delta)^{-1}W\left(\frac{Tp\delta^{T/(p-1)}\ln(\delta)}{p-1}\right)} \leq \frac{(1-\theta)(1-p)\delta^{-\frac{T(\theta+p-\theta p)}{(1-\theta)(1-p)}}}{-e\ln(\delta)}. \quad (34)$$

The PAL at t_{PAL} is only relevant if $\delta \in (\delta_{\text{PAL:lo}}, \delta_{\text{PAL:hi}})$. Thus, let us substitute

$$\delta = \exp\left(\frac{(1-\theta)(1-p)[y(1-\theta)(1-p) - 1]}{T(p+\theta-\theta p)}\right) \quad (35)$$

which gives $\delta \in (\delta_{\text{PAL:lo}}, \delta_{\text{PAL:hi}})$ if $y \in (0, 1)$. We can now transform Eq. (34) to

$$\frac{pe^{y(1-\theta)(1-p)}[y(1-\theta)(1-p) - 1]^2 - (\theta+p-\theta p)[py(\theta+p-\theta p) - y + 1]}{(1-\theta)^{-1}(\theta+p-\theta p)^2} - \ln\left(\frac{\theta+p-\theta p}{1-y(1-\theta)(1-p)}\right) \leq 0. \quad (36)$$

This inequation is never true for $y \in (0, 1)$ if $p \geq 1/3$ and therefore clearly never true under the assumption $p \geq e^{-1}$. Figure 6 illustrates in gray the parameter space $(\theta \times p)$ where the inequation can be satisfied for $y \in (0, 1)$. *Q.E.D.*

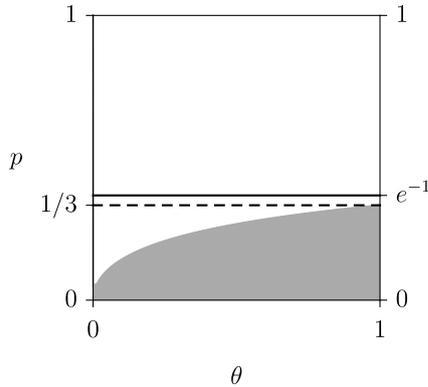


Figure 6: Space in $(\theta \times p)$ where $U_H(\ell_{t_{\text{PAL}}}^{\text{PAL}}) \geq U_H(\ell_{t_{\text{SAL}}}^{\text{SAL}})$ is possible (*gray*).

Appendix B. Parametric Illustration of Savings-linked Relationship Lending Equilibria for Different T

Figure 7 illustrates the savings-linked relationship lending equilibrium, similar to Fig. 2b, for a range of different T . Lines and fillings are used similar to Fig. 2b. Apparently the results do not change substantially for $T \geq 25$.

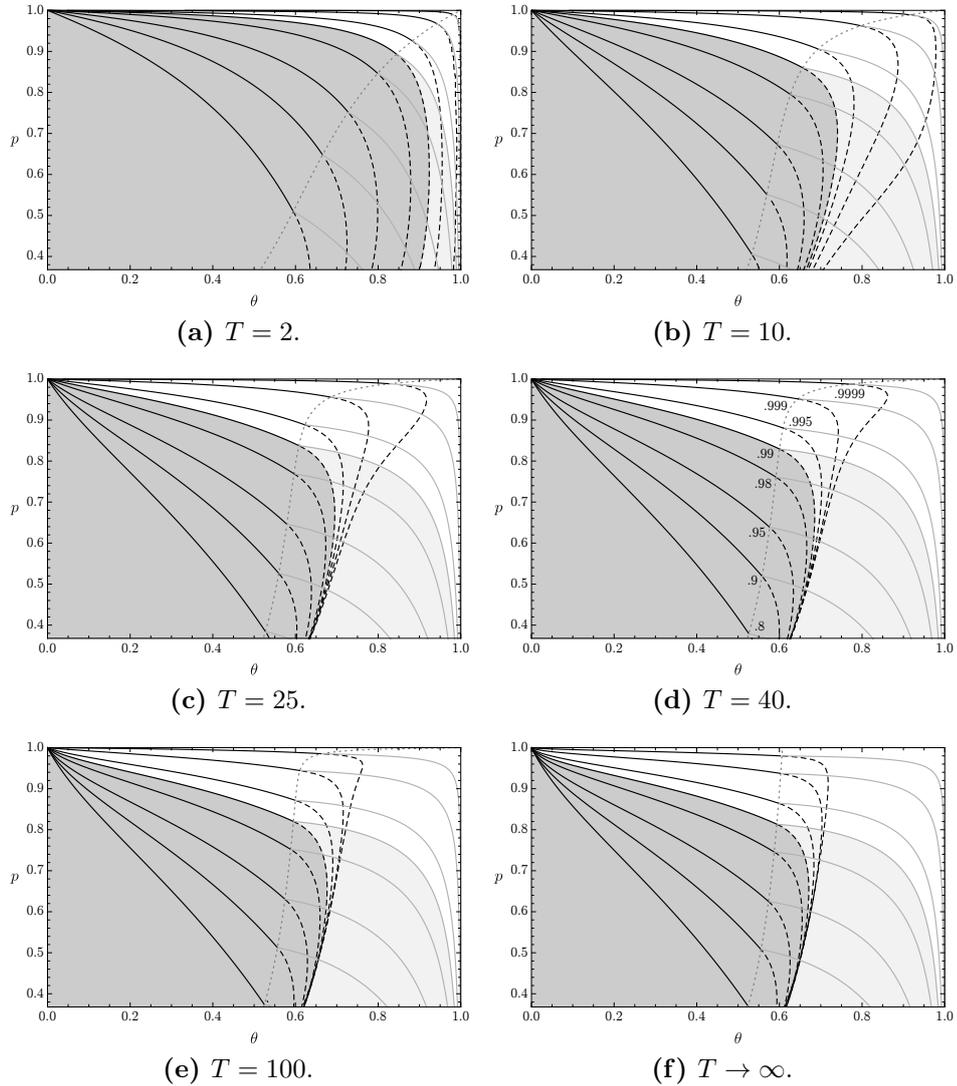


Figure 7: Savings-linked relationship lending equilibrium for different T .

Relevant parts of the isoquant maps of δ_S (solid black), δ_P (solid gray), and δ_{RL} (dashed black and solid black). Fillings conform to $\delta = 0.99$ (darkgray: SRL, lightgray: PRL, white: PAL).

Appendix C. Proof of the General Model

This section shows that the model's main results do not depend on the lower bound that have been presumed for type L individuals' probability of success in the model. Let us therefore relax the assumption $e^{-1} \leq p < 1$ and assume $0 < p < 1$ instead.

First, the equilibrium in case of arm's-length lending has to be revised. When determining the optimal separating arm's-length loan, Lemma I.2 proves that $t_{\text{SAL}} \leq t_A$ always holds if $p \geq e^{-1}$. Thus, under the new assumption $0 < p < 1$, $\ell_{t_A}^{\text{SAL}}$ can now be preferred over $\ell_{t_{\text{SAL}}}^{\text{SAL}}$ by type H individuals. The refined preference relation is given by

$$\ell_{t_A}^{\text{SAL}} \stackrel{H}{\succ} \ell_{t_{\text{SAL}}}^{\text{SAL}} \quad \text{iff } p \geq e^{-1} \vee \left(p < e^{-1} \wedge \delta \geq \delta_{\text{A,SAL}} := \exp\left[\frac{1-p}{Tpe-T}\right] \right). \quad (37)$$

This is obtained by continuing from the proof of Lemma I.2 which reduces $t_{\text{SAL}} \leq t_A$ to $1 - p + T \ln(\delta) (1 - pe) \geq 0$. Solving for δ gives $\delta \geq \delta_{\text{A,SAL}}$ if $p < e^{-1}$. $\delta_{\text{A,SAL}}$ is strictly monotonic decreasing in p as $\partial \delta_{\text{A,SAL}} / \partial p = \delta_{\text{A,SAL}} (ep - 1)^{-2} (1 - e) / T < 0$. Since $\lim_{p \rightarrow 0} \delta_{\text{A,SAL}} = e^{-1/T} < 1$ and $\lim_{p \rightarrow (e^{-1})^-} \delta_{\text{A,SAL}} = 0$, $\exists \delta \in (0, 1) (\delta \geq \delta_{\text{A,SAL}})$ if $p < e^{-1}$.

The loan $\ell_{t_{\text{SAL}}}^{\text{SAL}}$ is still preferred over ℓ_0^{PAL} by type H individuals if $\delta > \delta_l$. But since the loan contract $\ell_{t_A}^{\text{SAL}}$ is now relevant, we also need to compare this one to a pooling arm's-length loan that is made at $t = 0$. Using straightforward calculus we obtain

$$\ell_0^{\text{PAL}} \stackrel{H}{\succ} \ell_{t_A}^{\text{SAL}} \quad \text{iff } \delta > \delta_{i2} := \exp[-(T\theta e)^{-1}]. \quad (38)$$

We have received $\delta_{\text{PRL:lo}}$, $\delta_{\text{PRL:hi}}$, $\delta_{\text{A,SAL}}$, δ_l , and δ_{i2} from comparing contracts one to another. To be able to compare all of them at once, we can define static sets in the parameter space $(\theta \times p)$ based on these deltas. This is done in Definition I.2. These sets are labeled static because they do not depend on

other parameters than θ and p . The subsequently following Remark I.1 provides the rationale for the set building conditions in Definition I.2 (except for the set Φ which is explained in Remark I.2).

Definition I.2 (Static subsets of Ψ). The set that contains the full parameter space ($\theta \times p$) is given by

$$\Psi := \{(\theta, p) \mid \theta, p \in (0, 1)\}. \quad (39)$$

Consider the static subsets $\Lambda, \Gamma, \Pi, X, \Phi, \Delta, \Omega, \mathcal{U} \subseteq \Psi$. Let us define

$$\Lambda := \left\{ (\theta, p) \mid p \leq \frac{e^{-1} - \theta}{1 - \theta} \right\}, \quad (40)$$

$$\Gamma := \left\{ (\theta, p) \mid -\ln[\theta + (1 - \theta)p] \leq \frac{\theta(1 - \theta)(1 - p)}{[\theta + (1 - \theta)p]^2} \right\}, \quad (41)$$

$$\Pi := \left\{ (\theta, p) \mid p \leq \frac{\theta(1 - \theta)e - \theta}{(1 - \theta)(1 + \theta e)} \right\}, \quad (42)$$

$$X := \left\{ (\theta, p) \mid p \leq \frac{1 - 2\theta}{(1 - \theta)(1 + e)} \right\}, \quad (43)$$

$$\Phi := \left\{ (\theta, p) \mid \exp\left[\frac{\theta + (1 - \theta)p}{(1 - \theta)(1 - pe)}\right] < \frac{1}{(1 - \theta)(1 - pe)} \right\}. \quad (44)$$

As Ψ contains the full parameter space, it is $\neg\text{SUBSET} = \Psi \setminus \text{SUBSET}$. Let us further define

$$\Delta := \Lambda \cap \Gamma, \quad (45)$$

$$\mathcal{U} := \Lambda \cup \Gamma, \quad (46)$$

$$\Omega := \Psi \setminus \mathcal{U} = \neg\Lambda \cap \neg\Gamma. \quad (47)$$

Remark I.1.

Γ consists of all the points (θ, p) that satisfy $\delta_{\text{PAL:lo}} \leq \delta_l$.

Λ consists of all the points (θ, p) that satisfy $\delta_l, \delta_{l2} \leq \delta_{\text{A,SAL}}$ if $p < e^{-1}$ and $\delta_l, \delta_{l2} \geq \delta_{\text{A,SAL}}$ if $p \geq e^{-1}$. As the latter condition is never true, we obtain $(\theta, (p \geq e^{-1})) \in \neg\Lambda$. $\delta_l \leq \delta_{\text{A,SAL}}$ and $\delta_{l2} \leq \delta_{\text{A,SAL}}$ give the same set building condition of Λ .

Π consists of all the points (θ, p) that satisfy $\delta_{\text{PAL:lo}} \leq \delta_{l2}$. As $\delta_{l2} \leq \delta_l$ is always true, the set Π is a subset of Γ .

\mathbf{X} consists of all the points (θ, p) that satisfy $\delta_{\text{PAL:lo}} \leq \delta_{\text{A,SAL}}$ if $p < e^{-1}$, and $\delta_{\text{PAL:lo}} \geq \delta_{\text{A,SAL}}$ if $p \geq e^{-1}$. As the latter condition is never true, we obtain $(\theta, (p \geq e^{-1})) \in \neg\mathbf{X}$.

Ψ consists of the full parameter space of points (θ, p) and all of them satisfy $\delta_{\text{PAL:lo}}, \delta_{l2}, \delta_l < \delta_{\text{PAL:hi}}$ and $\delta_{l2} \leq \delta_l$.

Sublemma I.1. $(\theta, p) = \left(\frac{1}{e^2-e}, \frac{e-2}{e(e-1)-1}\right)$ is the unique intersection point of the frontier line of Λ or of Φ with the frontier lines of Γ , Π , and \mathbf{X} , as defined in Definition I.2.

Figure 8 illustrates the different static subsets of Ψ as defined in Definition I.2. The set Λ (Γ , Π , \mathbf{X}) contains all the points (θ, p) below and on the solid gray (solid black, dashed black, dotted black) curve, the set Φ all the points left of (but not on) the dashed gray curve. Obviously, the model under the original assumption $e^{-1} \leq p < 1$ is entirely covered by the set Ω . According to Fig. 8 and Sublemma I.1, Π is apparently a subset of Γ , Φ is a subset of Λ , and \mathbf{X} is a subset of $\Pi \cup \Phi$. Thus, $\Lambda, \Gamma, \Pi, \Phi, \mathbf{X} \subset \bar{\Omega}$.

Lemma I.3 proves that, if $p \geq 1/3$, the separating loan $\ell_{t_{\text{SAL}}}^{\text{SAL}}$ is preferred by types H over the pooling loan $\ell_{t_{\text{PAL}}}^{\text{PAL}}$ whenever the latter is relevant. Under the assumption $0 < p < 1$ we therefore need to refine this statement. And we also need to compare the separating loan $\ell_{t_{\text{A}}}^{\text{SAL}}$ with the same pooling loan. On the basis on this comparison we can define two dynamic sets in Ψ , which is done in Definition I.3. Different from the static sets in Definition I.2, the dynamic sets change with the parameter values of δ and T . The rationale for the set building conditions are provided in the following Remark I.2.

Definition I.3 (Dynamic subsets of Ψ). Consider the following two dynamic subsets $\pi(\delta_x, T), \gamma(\delta_x, T) \subseteq \Psi$, where $\delta_x \in [\delta_{\text{PAL:lo}}, \delta_{\text{PAL:hi}}]$. Let us define

$$\pi(\delta_x, T) := \left\{ (\theta, p) \left| Tp - \frac{(1-p) \left[1 - (1-\theta)\delta_x \uparrow \left(T - \frac{T}{(1-\theta)(1-p)} \right) \right]}{e \ln(\delta_x)} \leq 0 \right. \right\}, \quad (48)$$

$$\gamma(\delta_x, T) := \left\{ (\theta, p) \left| T\delta_x \uparrow \left(\frac{T}{1-p} + W \left[\frac{Tp\delta_x^{T/(p-1)} \ln(\delta_x)}{p-1} \right] \ln(\delta_x)^{-1} \right) + \frac{(1-\theta)(1-p)\delta_x \uparrow \left(T - \frac{T}{(1-\theta)(1-p)} \right)}{e \ln(\delta_x)} \leq 0 \right. \right\} \quad (49)$$

Remark I.2.

$\pi(\boldsymbol{\delta}_x, \mathbf{T})$ consists of all the points (θ, p) that satisfy $U_H(\ell_{t_A}^{\text{SAL}}) \leq U_H(\ell_{t_{\text{PAL}}}^{\text{PAL}})$ for a given T and $\delta = \delta_x \in [\delta_{\text{PAL:lo}}, \delta_{\text{PAL:hi}}]$. As $\ell_{t_{\text{PAL}}}^{\text{PAL}}$ is only the best PAL for $\delta = \delta_x$, this is the relevant case to consider. With the tiebreaker rule in Definition I.1, we get $\ell_{t_A}^{\text{SAL}} \prec \ell_{t_{\text{PAL}}}^{\text{PAL}}$ in $\pi(\delta_x, T)$ and $\ell_{t_A}^{\text{SAL}} \succ \ell_{t_{\text{PAL}}}^{\text{PAL}}$ in $\neg\pi(\delta_x, T)$.

$\gamma(\boldsymbol{\delta}_x, \mathbf{T})$ consists of all the points (θ, p) that satisfy $U_H(\ell_{t_{\text{SAL}}}^{\text{SAL}}) \leq U_H(\ell_{t_{\text{PAL}}}^{\text{PAL}})$ for a given T and $\delta = \delta_x \in [\delta_{\text{PAL:lo}}, \delta_{\text{PAL:hi}}]$, analogously to $\pi(\delta_x, T)$. Considering the tiebreaker rule in Definition I.1, it is $\ell_{t_{\text{SAL}}}^{\text{SAL}} \prec \ell_{t_{\text{PAL}}}^{\text{PAL}}$ in $\gamma(\delta_x, T)$ and $\ell_{t_{\text{SAL}}}^{\text{SAL}} \succ \ell_{t_{\text{PAL}}}^{\text{PAL}}$ in $\neg\gamma(\delta_x, T)$.

For Φ , the set building condition is obtained by reducing $U_H(\ell_{t_A}^{\text{SAL}}) > U_H(\ell_{t_{\text{PAL}}}^{\text{PAL}})$ or $U_H(\ell_{t_{\text{SAL}}}^{\text{SAL}}) > U_H(\ell_{t_{\text{PAL}}}^{\text{PAL}})$ at $\delta = \delta_{\text{A,SAL}}$. These conditions do not contain a restriction regarding $\delta_{\text{A,SAL}}$, thus, the cases $\delta_{\text{A,SAL}} < \delta_{\text{PAL:lo}}$ are also covered by Φ . But as $\ell_{t_{\text{PAL}}}^{\text{PAL}}$ is only relevant for $\delta \geq \delta_{\text{PAL:lo}}$, Φ and $\neg\Phi$ are only meaningful where they are intersecting the set X. This being said, it follows that points $(\theta, p) \in \Phi \cap X$ are in $\neg\pi(\delta_x, T)$ and in $\neg\gamma(\delta_x, T)$ for $\delta_x = \delta_{\text{A,SAL}}$. And, analogously, points $(\theta, p) \in \neg\Phi \cap X$ are in $\pi(\delta_x, T)$ and in $\gamma(\delta_x, T)$ for $\delta_x = \delta_{\text{A,SAL}}$.

Let us now analyze the dynamic sets $\pi(\delta_x, T)$ and $\gamma(\delta_x, T)$ to see how they change for different δ_x and T . Sublemma I.2 proofs that both sets dissolve if δ_x approaches its upper bound $\delta_{\text{PAL:hi}}$. While the dynamic set $\pi(\delta_x, T)$ is a subset of the static set Π (see Lemma I.5), the dynamic set $\gamma(\delta_x, T)$ is a subset of the static set $\Gamma \cup X$ (see Lemma I.6).

Sublemma I.2. $\pi(\delta_{\text{PAL:hi}}, T), \gamma(\delta_{\text{PAL:hi}}, T) = \emptyset$.

Proof. For $\delta_x = \delta_{\text{PAL:hi}}$, the set building conditions of $\pi(\delta_x, T)$ and $\gamma(\delta_x, T)$, as stated in Definition I.3, can be reduced to $\exp[p\theta](1 + e(p - p\theta)) - \exp[p + \theta](1 - \theta) \leq 0$ and to $p(\theta - 1) + W[e^{1-\theta}p(1 - \theta)] \leq 0$. Both are never true because of $\theta, p \in (0, 1)$. Thus, $\pi(\delta_{\text{PAL:hi}}, T) = \gamma(\delta_{\text{PAL:hi}}, T) = \emptyset$. \blacksquare

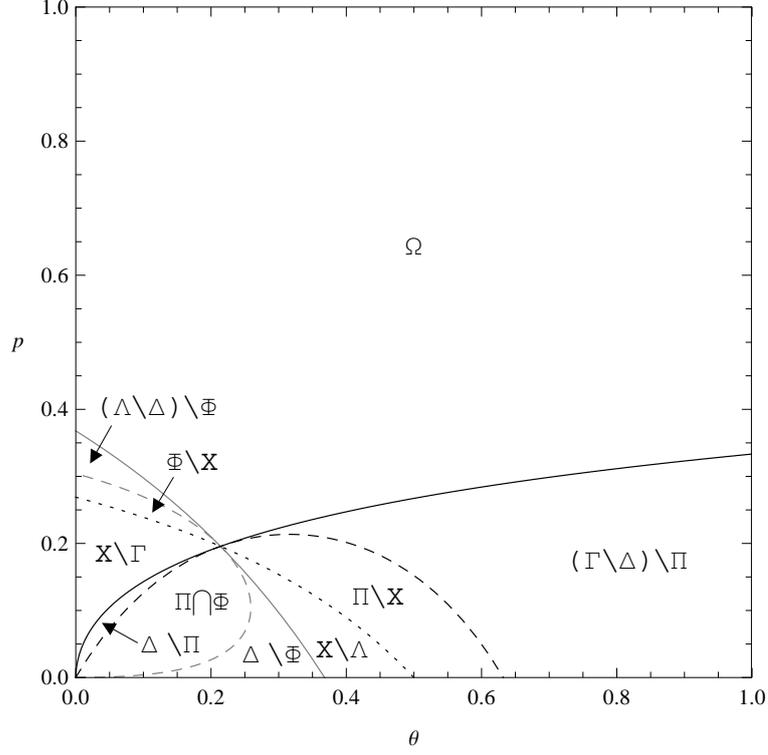


Figure 8: Static subsets of Ψ .

Lemma I.5. $\pi(\delta_x, T) \subseteq \Pi$.

Proof. For $\delta_x = \delta_{\text{PAL:lo}}$, the set building condition of $\pi(\delta_x, T)$ simplifies to

$$T[(e - \theta e)^{-1} - (1 - p)e^{-1} - (1 - p)\theta] \leq 0. \quad (50)$$

Solving for p gives

$$p \leq \frac{\theta(1 - \theta)e - \theta}{(1 - \theta)(1 + \theta e)} \quad (51)$$

which is the set building condition of Π . Thus, $\pi(\delta_{\text{PAL:lo}}, T) = \Pi$.

The first derivative of $U_H(\ell_{t_A}^{\text{SAL}}) - U_H(\ell_{t_{\text{PAL}}}^{\text{PAL}})$ (which is the left-hand side of the set building condition of $\pi(\delta_x, T)$ in Eq. (48)) with respect to δ_x equals

$$\frac{1 - p + [-(1 - \theta)(1 - p) - T \ln(\delta_x)(\theta + p - \theta p)] \delta_x^{T - \frac{T}{(1 - \theta)(1 - p)}}}{\delta \ln(\delta_x)^2 e} \quad (52)$$

which is always {positive, negative} if the numerator is {positive, negative}. The numerator is strictly monotonic decreasing in δ_x as the first derivative of the numerator with respect to δ_x is obviously always negative:

$$\frac{T^2 \ln(\delta_x)(\theta + p - \theta p)^2 \delta_x^{T - \frac{T}{(1-\theta)(1-p)} - 1}}{(1-\theta)(1-p)} < 0. \quad (53)$$

Solving the numerator of Eq. (52) at the lower bound of δ_x , that is $\delta_x = \delta_{\text{PAL:lo}}$, gives $1-p$ which is always strictly positive since $p \in (0, 1)$. Solving the numerator at the upper bound of δ_x , that is $\delta_x = \delta_{\text{PAL:hi}}$, gives $p-1 - \exp[\theta + (1-\theta)p](1-\theta)^2(1-p)^2$ which is always strictly positive since $\theta, p \in (0, 1)$. Thus, the numerator of Eq. (52) is strictly positive for every δ_x , which eventually makes Eq. (52) entirely strictly positive in the relevant range and $U_H(\ell_{t_A}^{\text{SAL}}) - U_H(\ell_{t_{\text{PAL}}}^{\text{PAL}})$ strictly monotonic increasing in δ_x .

In conclusion, all the points (θ, p, δ_x, T) that yield positive function values for $U_H(\ell_{t_A}^{\text{SAL}}) - U_H(\ell_{t_{\text{PAL}}}^{\text{PAL}})$ yield even more positive values for greater δ_x and therefore fail the set building condition of $\pi(\delta_x, T)$ even more. Negative function values of some points (θ, p, δ_x, T) turn positive with increasing δ_x , which causes them to switch to $-\pi(\delta_x, T)$. Ergo, it is $\pi(\delta_{x,2}, T) \subset \pi(\delta_{x,1}, T) \forall \delta_{x,1} < \delta_{x,2}$ (unless it is $\pi(\delta_{x,1}, T) = \emptyset$, we then have $\pi(\delta_{x,2}, T) = \emptyset$). When recalling $\pi(\delta_{\text{PAL:lo}}, T) = \Pi$, we consequently obtain $\pi(\delta_x, T) \subseteq \Pi$. ■

Lemma I.6. $\gamma(\delta_x, T) \subset \Gamma \cup X$.

Proof. That $\gamma(\delta_x, T) \subset \Gamma \cup X$ is true is shown by numerically simulating 25 million uniformly i.i.d pseudo-random parameter sets $(\theta_i, p_i, \delta_i, T_i)$, where $\theta_i, p_i \in \mathbb{R} | (0 < \theta_i, p_i < 1) \forall i$, $\delta_i \in \mathbb{R} | (\delta_{\text{PAL:lo}}(\theta_i, p_i, T_i) \leq \delta_i \leq \delta_{\text{PAL:hi}}(\theta_i, p_i, T_i)) \forall i$, and $T_i \in \mathbb{Z} | (2 \leq T_i \leq 1000) \forall i$. 2,874,987 of the simulated parameter sets satisfy the set building condition of $\gamma(\delta_x, T)$ and the corresponding simulated points $(\theta_i, p_i) \in \gamma(\delta_i, T_i)$ are illustrated in Fig. 9. For every single point $(\theta_i, p_i) \in \gamma(\delta_i, T_i)$ in the simulation, $(\theta_i, p_i) \in \Gamma \cup X$ is also true. Figure 9 evidently shows that $(\theta_i, p_i) \in \gamma(\delta_i, T_i) \wedge (\theta_i, p_i) \notin \Gamma$ is only possible for very small θ_i, p_i and the corresponding points are clearly in X: we actually obtain $\theta < 0.0138$ and $p < 0.0563$ for these points from the simulation. ■

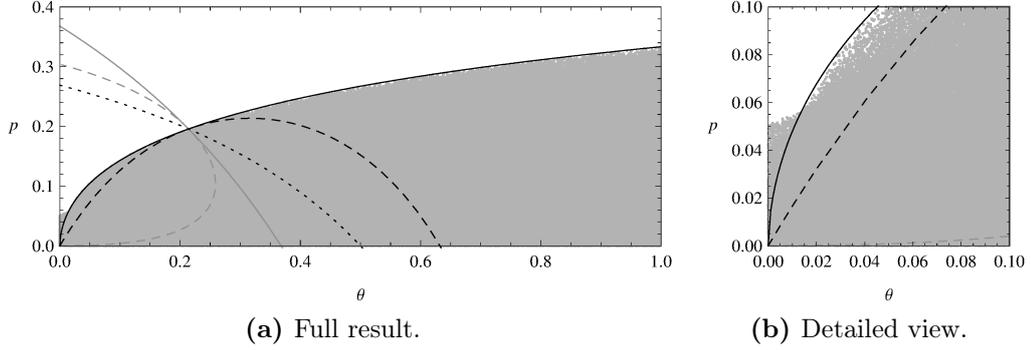


Figure 9: Simulation results to Lemma I.6: $\{(\theta_i, p_i) \mid (\theta_i, p_i) \in \gamma(\delta_i, T_i)\}$.

Now we are aware of the bounds of the dynamic sets $\pi(\delta_x, T)$ and $\gamma(\delta_x, T)$. But to be able to assess if a particular point (θ, p) is inside or outside the dynamic sets, we need to analyze their dynamics for changes in δ_x in detail. For this purpose let us establish Definition I.4.

Definition I.4. For a given point (θ, p, T) , let us define $\delta_\pi(\theta, p, T)$ as the globally smallest δ_x for which $(\theta, p) \in \neg\pi(\delta_x, T)$. Equivalently, let us define $\delta_\gamma(\theta, p, T)$ as the globally smallest δ_x for which $(\theta, p) \in \neg\gamma(\delta_x, T)$.

As $\delta_x \in [\delta_{\text{PLS:lo}}, \delta_{\text{PLS:hi}}]$ by definition and $\pi(\delta_{\text{PAL:hi}}, T), \gamma(\delta_{\text{PAL:hi}}, T) = \emptyset$ (see Sublemma I.2), there are always unique $\delta_\pi(\theta, p, T)$ and $\delta_\gamma(\theta, p, T)$ in the sense of Definition I.4. But there may be other (that is, greater) local minima of δ_x for which $(\theta, p) \in \neg\pi(\delta_x, T)$ or $(\theta, p) \in \neg\gamma(\delta_x, T)$. That is the case if a particular point (θ, p) switches more than once between $\pi(\delta_x, T)$ and $\neg\pi(\delta_x, T)$ or between $\gamma(\delta_x, T)$ and $\neg\gamma(\delta_x, T)$ for increasing δ_x and given T . Figure 10 shows the dynamics of $\pi(\delta_x, T)$ and $\gamma(\delta_x, T)$ for increasing δ_x , where the lighter the short-dashed (solid) frontier line of the set $\pi(\delta_x, T)$ ($\gamma(\delta_x, T)$), the greater is δ_x . While there is no indication for additional local minima in $\pi(\delta_x, T)$, Fig. 10d shows that for some points (θ, p) in the parameter space there is clearly multiple switching between $\gamma(\delta_x, T)$ and $\neg\gamma(\delta_x, T)$ for increasing δ_x . This issue is addressed by the finality property formulated in Lemma I.7. It is shown that there are indeed no local minima of δ_x different from $\delta_\pi(\theta, p, T)$ for which $(\theta, p) \in \neg\pi(\delta_x, T)$. And points of multiple switching between $\gamma(\delta_x, T)$

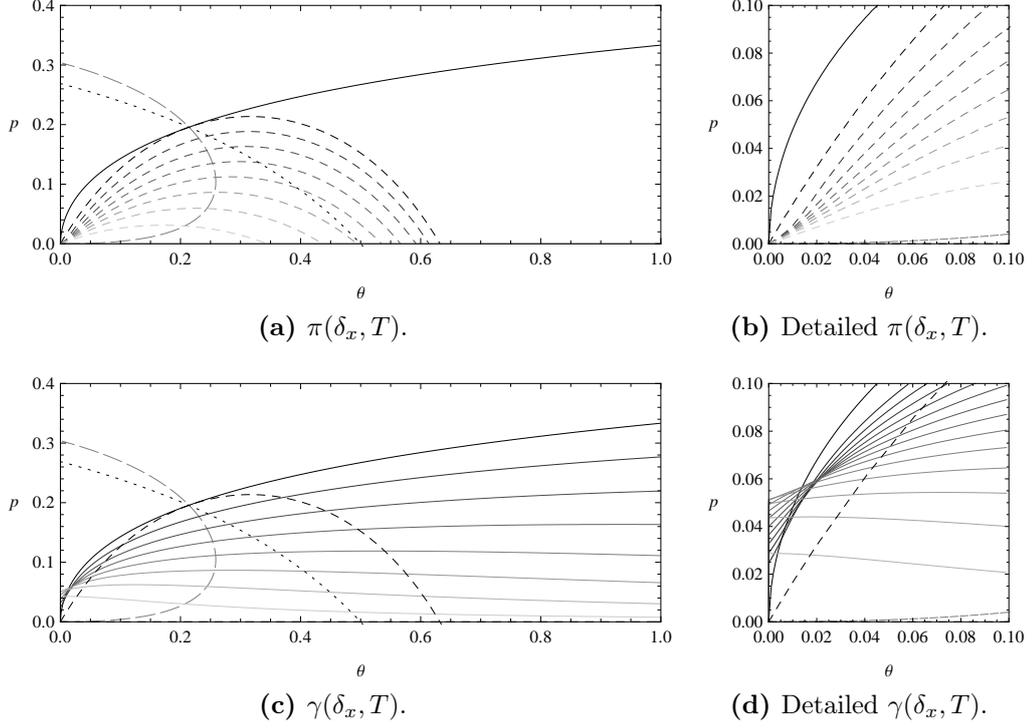


Figure 10: Dynamics of $\pi(\delta_x, T)$ and $\gamma(\delta_x, T)$ for increasing δ_x .

and $\neg\gamma(\delta_x, T)$ for increasing δ_x are solely located in a small fraction of the set $X \setminus \Pi$. To subsume, the finality property in Lemma I.7 states that if a point (θ, p) is in $\neg\pi(\delta_x, T)$ for a given $\delta_x = \delta_j$, it is also in $\neg\pi(\delta_x, T)$ for any $\delta_x \geq \delta_j$. And if a point (θ, p) is in $\neg\gamma(\delta_x, T)$ for a given $\delta_x = \delta_j$, it is also in $\neg\gamma(\delta_x, T)$ for any $\delta_x \geq \delta_j$ if $(\theta, p) \notin X \setminus \Pi$.

Lemma I.7 (Finality property).

1. $(\theta, p) \in \neg\pi(\delta_x, T) \forall \delta_x \geq \delta_\pi(\theta, p, T)$;
2. $(\theta, p) \in \neg\gamma(\delta_x, T) \forall \delta_x \geq \delta_\gamma(\theta, p, T)$ if $(\theta, p) \notin X \setminus \Pi$.

Proof.

1. Lemma I.5 states that $\pi(\delta_x, T) \subseteq \Pi$. Thus, it is $(\theta, p) \in \neg\pi(\delta_x, T) \forall \delta_x$ if $(\theta, p) \in \neg\Pi$. For the case of $(\theta, p) \in \Pi$, the proof of Lemma I.5 already contains the proof that if (θ, p) is in $\neg\pi(\delta_x, T)$ for some $\delta_x = \delta_j$, it is also in $\neg\pi(\delta_x, T)$ for every $\delta_x \geq \delta_j$. This is true as the left-hand side of the set building condition of $\pi(\delta_x, T)$ is strictly monotonic increasing in δ_x .

Ergo, there are no local minima of δ_x different from $\delta_\pi(\theta, p, T)$ for which $(\theta, p) \in \neg\pi(\delta_x, T)$.

2. The sufficient condition for switching back to $\neg\gamma(\delta_x, T)$ for increasing δ_x after once being in $\gamma(\delta_x, T)$ is that points (θ, p) that lie on the frontier line of $\gamma(\delta_x, T)$ for given T (that is, the set building condition is satisfied with equality) show a negative slope of $U_H(\ell_{t_{\text{SAL}}}^{\text{SAL}}) - U_H(\ell_{t_{\text{PAL}}}^{\text{PAL}})$ in δ_x .

Let us first narrow down the set of points that can show multiple switching by determining the points that can possess a negative slope in δ_x at all. Of the 2,874,987 simulated parameter sets $(\theta_i, p_i, \delta_i, T_i)$ in Lemma I.6 that satisfy the set building condition of $\gamma(\delta_x, T)$, 60,510 parameter sets show a negative slope of $U_H(\ell_{t_{\text{SAL}}}^{\text{SAL}}) - U_H(\ell_{t_{\text{PAL}}}^{\text{PAL}})$ in δ_x . Those parameter sets, as illustrated in Fig. 11a, are characterized by $\theta < 0.132$ and $p < 0.062$ in the simulation and, thus, we can focus on a smaller range in the further analysis. It actually does not matter if a point oscillates in the negative or positive range for increasing δ_x , but a negative slope of points that lie on the frontier line of $\gamma(\delta_x, T)$ is crucial for multiple switching. Therefore, I employ a numerical simulation of uniformly i.i.d. pseudo-random parameter sets (θ_i, p_i, T_i) , where $\theta_i \in (0, 0.15) \forall i$, $p_i \in (0, 0.1) \forall i$, and $T_i \in [2, 1000] \cap \mathbb{Z} \forall i$. For every parameter set (θ_i, p_i, T_i) I apply Newton-Raphson method to determine a real-numbered $\delta_i \in [\delta_{\text{PAL:lo}}(\theta_i, p_i, T_i), \delta_{\text{PAL:hi}}(\theta_i, p_i, T_i)]$ —if existent—that approximates $U_H(\ell_{t_{\text{SAL}}}^{\text{SAL}}) - U_H(\ell_{t_{\text{PAL}}}^{\text{PAL}}) = 0$ with accuracy 10^{-12} . To avoid systematic convergence problems or results that are induced by a systematic starting point choice, I adopt a random starting point in the interval $(\delta_{\text{PAL:lo}}(\theta_i, p_i, T_i), \delta_{\text{PAL:hi}}(\theta_i, p_i, T_i))$ for the Newton-Raphson method.

This procedure is operated until we receive 1,000,000 simulated points (θ_i, p_i) that lie on the frontier line of $\gamma(\delta_i, T_i)$ (at least approximately with accuracy 10^{-12}). 86,068 of the simulated points additionally possess a negative slope of $U_H(\ell_{t_{\text{SAL}}}^{\text{SAL}}) - U_H(\ell_{t_{\text{PAL}}}^{\text{PAL}})$ in δ_x . Figure 11b illustrates these points and the fact that every single one of them is by far in the set $X \setminus \Pi$ (which is a proper subset of $\Phi \cap X$) and therefore clearly not in $\Gamma \setminus (\Delta \setminus \Pi)$. Multiple switching between $\gamma(\delta_x, T)$ and $\neg\gamma(\delta_x, T)$ for increasing δ_x can therefore only occur in a little fraction of the set $X \setminus \Pi$. Thus, if a point (θ, p) is not in $X \setminus \Pi$, it is in $\neg\gamma(\delta_x, T)$ for every $\delta_x \geq \delta_\gamma(\theta, p, T)$. ■

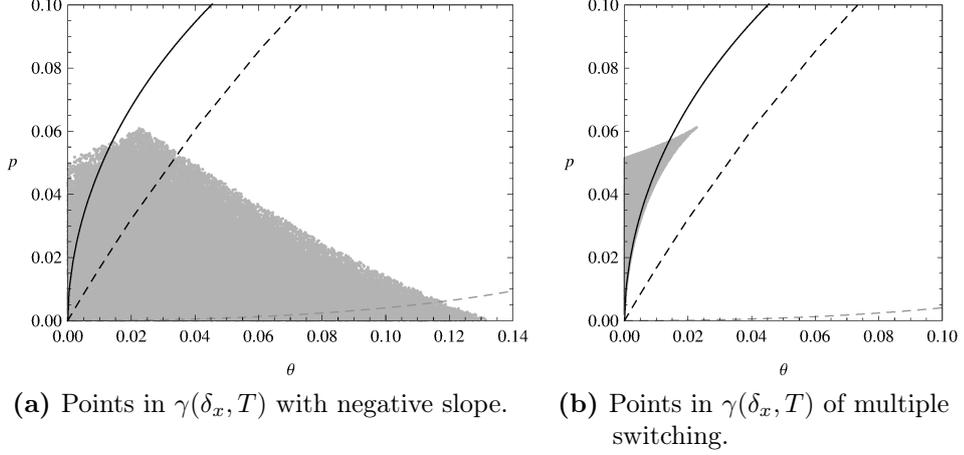


Figure 11: Simulation results to Lemma I.7.

We are now able to entirely deduce the choice pattern of type H individuals (see Lemma I.8) and, based on that, to describe the equilibrium in a market of arm's-length lending.

Lemma I.8. *Under informational asymmetries and without relationship lending, type H individuals choose the following loan:*

$$\text{in } \Omega : \begin{cases} \ell_0^{\text{PAL}} & \text{if } \delta \leq \delta_l, \\ \ell_{t_{\text{SAL}}}^{\text{SAL}} & \text{if } \delta_l < \delta, \end{cases} \quad (54)$$

$$\text{in } \Lambda \setminus \Pi : \begin{cases} \ell_0^{\text{PAL}} & \text{if } \delta \leq \delta_{l2}, \\ \ell_{t_A}^{\text{SAL}} & \text{if } \delta_{l2} < \delta < \delta_{A,\text{SAL}}, \\ \ell_{t_{\text{SAL}}}^{\text{SAL}} & \text{if } \delta_{A,\text{SAL}} \leq \delta, \end{cases} \quad (55)$$

$$\text{in } \Gamma \setminus \Phi : \begin{cases} \ell_0^{\text{PAL}} & \text{if } \delta \leq \delta_{\text{PAL:lo}}, \\ \ell_{t_{\text{PAL}}}^{\text{PAL}} & \text{if } \delta_{\text{PAL:lo}} < \delta < \delta_\gamma(\theta, p, T), \\ \ell_{t_{\text{SAL}}}^{\text{SAL}} & \text{if } \delta_\gamma(\theta, p, T) \leq \delta. \end{cases} \quad (56)$$

$$\text{in } \Pi \cap \Phi : \begin{cases} \ell_0^{\text{PAL}} & \text{if } \delta \leq \delta_{\text{PAL:lo}}, \\ \ell_{t_{\text{PAL}}}^{\text{PAL}} & \text{if } \delta_{\text{PAL:lo}} < \delta < \delta_{\pi}(\theta, p, T), \\ \ell_{t_A}^{\text{SAL}} & \text{if } \delta_{\pi}(\theta, p, T) \leq \delta < \delta_{\text{A,SAL}}, \\ \ell_{t_{\text{SAL}}}^{\text{SAL}} & \text{if } \delta_{\text{A,SAL}} \leq \delta. \end{cases} \quad (57)$$

Proof. The elements of Ω are by definition and by Sublemma I.1 in $\neg\Gamma \cap \neg\Lambda \cap \neg\Pi \cap \neg X$. Thus, we obtain from Remark I.1 that in Ω it is $\delta_{i2} \leq \delta_i < \delta_{\text{PAL:lo}} < \delta_{\text{A,SAL}}$ in case of $p \geq e^{-1}$, and $\delta_{\text{A,SAL}} < \delta_{i2} \leq \delta_i < \delta_{\text{PAL:lo}}$ in case of $p < e^{-1}$. $\ell_{t_{\text{SAL}}}^{\text{SAL}} \succ \ell_{t_A}^{\text{SAL}}$ is always true in case of $p \geq e^{-1}$. Here, it is also true for $p < e^{-1}$ because of $\delta_{\text{A,SAL}} < \delta_{i2}$: the loan $\ell_{t_A}^{\text{SAL}}$ can never be better than a PAL for types H if $\delta \leq \delta_{i2}$ but I have shown that $\ell_{t_{\text{SAL}}}^{\text{SAL}} \succ \ell_{t_A}^{\text{SAL}}$ if $\delta \geq \delta_{\text{A,SAL}}$. Since $\gamma(\delta_x, T) \subset (\Gamma \cup X) \not\subset \Omega$, according to Lemma I.6, from Remark I.2 follows that $\ell_{t_{\text{SAL}}}^{\text{SAL}} \succ \ell_{t_{\text{PAL}}}^{\text{PAL}}$ is always true in Ω whenever $\ell_{t_{\text{PAL}}}^{\text{PAL}}$ is the best PAL, that is, if $\delta > \delta_{\text{PAL:lo}}$. Hence, ℓ_0^{PAL} is the best loan contract for types H if $\delta \leq \delta_i$, and $\ell_{t_{\text{SAL}}}^{\text{SAL}}$ is the best choice if $\delta > \delta_i$.

Let us subdivide the set $\Lambda \setminus \Pi$ into three subsets $\mathcal{A} := \Lambda \setminus X$, $\mathcal{B} := (\Lambda \cap X) \setminus \Delta$, and $\mathcal{C} := \Delta \setminus \Pi$ to gather the solution. In \mathcal{A} it is $\delta_{i2} \leq \delta_i \leq \delta_{\text{A,SAL}} < \delta_{\text{PAL:lo}}$, according to Remark I.1. When imagining to increase δ beginning from the lower bound, we see that ℓ_0^{PAL} is the most preferred loan of types H until δ passes δ_{i2} . If $\delta_{i2} < \delta_{\text{A,SAL}}$, $\ell_{t_A}^{\text{SAL}}$ is type H individuals' first choice for $\delta_{i2} < \delta < \delta_{\text{A,SAL}}$. When δ reaches $\delta_{\text{A,SAL}}$ we obtain $\ell_{t_{\text{SAL}}}^{\text{SAL}} \succ \ell_{t_A}^{\text{SAL}}$, and from $\delta_i < \delta_{\text{PAL:lo}}$ follows $\ell_{t_{\text{SAL}}}^{\text{SAL}} \succ \ell_0^{\text{PAL}}$. In the special case $\delta_{i2} = \delta_i = \delta_{\text{A,SAL}}$, however, $\ell_{t_A}^{\text{SAL}}$ is never preferred and $\ell_{t_{\text{SAL}}}^{\text{SAL}}$ directly replaces ℓ_0^{PAL} as the first choice when δ passes $\delta_{i2} = \delta_i = \delta_{\text{A,SAL}}$. Further, from Lemma I.6 follows that $\mathcal{A} \subset \neg\gamma(\delta_x, T)$ since $\mathcal{A} \subset \neg\Pi$, and we therefore obtain from Remark I.2 that $\ell_{t_{\text{SAL}}}^{\text{SAL}} \succ \ell_{t_{\text{PAL}}}^{\text{PAL}}$ for $\delta > \delta_{\text{PAL:lo}}$.

In \mathcal{B} it is $\delta_{i2} \leq \delta_i < \delta_{\text{PAL:lo}} \leq \delta_{\text{A,SAL}}$ and in \mathcal{C} we obtain $\delta_{i2} < \delta_{\text{PAL:lo}} \leq \delta_i < \delta_{\text{A,SAL}}$ by Remark I.1 and Sublemma I.1.²⁹ Since $\delta_{i2} < \delta_{\text{PAL:lo}} \leq \delta_{\text{A,SAL}}$

²⁹The strict inequality $\delta_i < \delta_{\text{A,SAL}}$ in \mathcal{C} is true since the frontier line of Λ is not contained in the set (see Sublemma I.1).

and $\delta_l < \delta_{A,SAL}$ is true in both subsets, it follows that $\ell_{t_A}^{SAL} \succ \ell_{t_{SAL}}^{SAL}, \ell_0^{PAL}$ for $\delta_{l2} < \delta < \delta_{A,SAL}$ and that $\ell_{t_{SAL}}^{SAL} \succ \ell_{t_A}^{SAL}, \ell_0^{PAL}$ for $\delta \geq \delta_{A,SAL}$. As $\mathcal{B}, \mathcal{C} \subset \neg\Pi$ and $\pi(\delta_x, T) \subseteq \Pi$ (see Lemma I.5), we get $\mathcal{B}, \mathcal{C} \subset \neg\pi(\delta_x, T)$. Thus, from Remark I.2 follows that $\ell_{t_A}^{SAL} \succ \ell_{t_{PAL}}^{PAL}$ for $\delta > \delta_{PAL:l0}$. Since $\ell_{t_{SAL}}^{SAL} \succ \ell_{t_A}^{SAL}$ if $\delta \geq \delta_{A,SAL}$ and $\delta_{PAL:l0} \leq \delta_{A,SAL}$ is true in both \mathcal{B} and \mathcal{C} , we consequentially receive $\ell_{t_{SAL}}^{SAL} \succ \ell_{t_A}^{SAL} \succ \ell_{t_{PAL}}^{PAL}$ for $\delta \geq \delta_{A,SAL}$.

As the set $\Gamma \setminus \Delta$ is a proper subset of $\neg\Lambda$ and also of Γ , it is characterized by $\delta_{A,SAL} < \delta_{l2} \leq \delta_l$ and $\delta_{PAL:l0} \leq \delta_l$ (see Remark I.1). From the former inequality follows that $\ell_{t_A}^{SAL}$ is never preferred by types H when feasible. In combination with the latter inequation we see that a PAL is the first choice of types H unless $\ell_{t_{SAL}}^{SAL} \succ \ell_{t_{PAL}}^{PAL}$ is true. Since $\Gamma \setminus \Delta \subset \Gamma \setminus (\Delta \setminus \Pi)$, from the finality property in Lemma I.7 follows that $\ell_{t_{SAL}}^{SAL} \succ \ell_{t_{PAL}}^{PAL}$ is true for every $\delta \geq \delta_\gamma(\theta, p, T)$, where $\delta_\gamma(\theta, p, T) > \delta_l \geq \delta_{PAL:l0}$.³⁰

$\Delta \setminus \Phi$ is a proper subset of $\Lambda \cap \Gamma \cap \Pi \cap X$ and—as the only one of the subsets as stated in Lemma I.8—contains the general intersection point of the static subsets' frontier lines, as specified in Sublemma I.1. Thus, Remark I.1 gives the order $\delta_{PAL:l0} \leq \delta_{l2} \leq \delta_l \leq \delta_{A,SAL}$. As $\Delta \setminus \Phi \subset \neg\Phi \cap X$, Remark I.2 states that $\Delta \setminus \Phi \subset \pi(\theta, p, T)$ and $\Delta \setminus \Phi \subset \gamma(\theta, p, T)$ for $\delta = \delta_{A,SAL}$. From Lemma I.7 follows that the finality property is unconditionally valid in $\Delta \setminus \Phi$. Therefore, $\ell_{t_A}^{SAL}, \ell_{t_{SAL}}^{SAL} \prec \ell_{t_{PAL}}^{PAL}$ is not only true for $\delta = \delta_{A,SAL}$ but also for $\delta_{PAL:l0} \leq \delta \leq \delta_{A,SAL}$. I have shown that $\ell_{t_{SAL}}^{SAL} \succ \ell_{t_A}^{SAL}$ for $\delta \geq \delta_{A,SAL}$, and from the finality property follows that $\ell_{t_{SAL}}^{SAL} \succ \ell_{t_{PAL}}^{PAL}$ if $\delta \geq \delta_\gamma(\theta, p, T)$, where obviously $\delta_\gamma(\theta, p, T) > \delta_{A,SAL}$. For the general intersection point of the static subsets' frontier lines, as specified in Sublemma I.1, it is $\bar{\delta} = \delta_{PAL:l0} = \delta_{l2} = \delta_l = \delta_{A,SAL}$, and, therefore, the equilibrium choice of type H obviously reduces to ℓ_0^{PAL} if $\delta \leq \bar{\delta}$ and to $\ell_{t_{SAL}}^{SAL}$ otherwise. Thus, we finally obtain the same result for set $\Delta \setminus \Phi$ as for set $\Gamma \setminus \Delta$, which in union produce the set $\Gamma \setminus \Phi$.

³⁰ $\delta_l = \delta_{PAL:l0}$ is only true for points on the frontier line of Γ . As $\delta_\gamma(\theta, p, T)$ shares the frontier line with Γ in the subset $\Gamma \setminus \Delta$, $\delta \leq \delta_{PAL:l0}$ can never satisfy $\neg\gamma(\delta_x, T)$ for these points according to Lemma I.6. In case of $\delta_l > \delta_{PAL:l0}$, an SAL can only be better than a PAL if $\delta > \delta_l$.

The set $\Pi \cap \Phi$ differs in two aspects from the previously analyzed set $\Delta \setminus \Phi$. First, it does not contain the general intersection point specified in Sublemma I.1. Thus, we obtain the order $\delta_{\text{PAL:lo}} \leq \delta_{i2} \leq \delta_i < \delta_{\text{A,SAL}}$, according to Remark I.1. Second, $\Pi \cap \Phi$ is in $\Phi \cap X$. Hence, we have $\Delta \setminus \Phi \subset \neg\pi(\theta, p, T)$ and $\Delta \setminus \Phi \subset \neg\gamma(\theta, p, T)$ at $\delta = \delta_{\text{A,SAL}}$, according to Remark I.2. Since $\Pi \cap \Phi \subset \Gamma \setminus (\Delta \setminus \Pi)$, the finality property in Lemma I.7 is unconditionally valid in $\Pi \cap \Phi$. Therefore, $\ell_{t_A}^{\text{SAL}} \succ \ell_{t_{\text{PAL}}}^{\text{PAL}}$ is true for $\delta \geq \delta_\pi(\theta, p, T)$, where obviously $\delta_\pi(\theta, p, T) \leq \delta_{\text{A,SAL}}$. I have shown that $\ell_{t_{\text{SAL}}}^{\text{SAL}} \succ \ell_{t_A}^{\text{SAL}}$ if $\delta \geq \delta_{\text{A,SAL}}$ and, thus, it is also $\ell_{t_{\text{SAL}}}^{\text{SAL}} \succ \ell_{t_{\text{PAL}}}^{\text{PAL}}$ in case of $\delta \geq \delta_{\text{A,SAL}}$ because of $\delta_\pi(\theta, p, T) < \delta_{\text{A,SAL}}$, and the finality property holds. ■

If a PAL is preferred by types H (which cases are derived in Lemma I.8), type L individuals also prefer it if $t < t_\alpha$ as, in that case, it is also the utility maximizing contract for types L according to Lemma I.1. As shown before, types H never choose a PAL with $t \geq t_\alpha$ because, in that case, there is always a feasible SAL that provides greater utility. If an SAL is chosen by types H , types L choose their best alternative which is ℓ_0^{LSL} , as follows from Lemma I.1. Type L individuals can at best receive the type-adequate selective loan terms since an SAL is perfectly separating and, thus, type L individuals' choice of a different loan is perfectly type-revealing.

Now that we have derived the equilibrium in a market of arm's-length lending for $0 < p < 1$, we need to take savings-linked relationship lending into account to finally arrive at the relationship lending equilibrium. Since the arm's-length lending equilibrium now additionally incorporates the loan contracts $\ell_{t_A}^{\text{SAL}}$ and $\ell_{t_{\text{PAL}}}^{\text{PAL}}$, we owe the comparison of those two contracts with the best savings-linked relationship loan contracts.

The comparison of relationship lending with the separating arm's-length loan $\ell_{t_A}^{\text{SAL}}$ can be reduced to a few cases. From Lemma I.8 follows that $\ell_{t_A}^{\text{SAL}}$ can only be an equilibrium contract for a defined range of δ in the subsets $\Lambda \setminus \Pi$ and $\Pi \cap \Phi$ of the parameter space $(\theta \times p)$, which in conjunction produce the set

$\Lambda \setminus (\Delta \setminus \Phi)$. Lemma I.9 shows that the separating arm's-length loan contract $\ell_{t_A}^{\text{SAL}}$ is dominated by the optimal separating relationship loan contract whenever the former is relevant. Since I have shown before by Lemma I.4 that the separating arm's-length loan $\ell_{t_{\text{SAL}}}^{\text{SAL}}$ is also dominated by the best separating relationship loan, we can finally conclude that there is always a separating relationship loan that is preferred over a separating arm's-length loan by type H individuals.

Lemma I.9. $\ell_{t_{\text{SRL}}}^{\text{SRL}} \succ^H \ell_{t_A}^{\text{SAL}}$ if $\delta \in (\min[\delta_{t_2}, \delta_{\text{PAL:lo}}], \delta_{\text{A,SAL}})$ and $(\theta, p) \in \Lambda \setminus (\Delta \setminus \Phi)$.

Proof. I apply a numerical simulation of 5 million uniformly i.i.d. pseudo-random parameter sets $(\theta_i, p_i, \delta_i, T_i)$, where $\theta_i, p_i \in \Lambda \setminus (\Delta \setminus \Phi) \forall i$, $T_i \in [2, 1000] \cap \mathbb{Z} \forall i$, and $\delta_i \in (\min\{\delta_{t_2}(\theta_i, T_i), \delta_{\text{PAL:lo}}(\theta_i, p_i, T_i)\}, \delta_{\text{A,SAL}}(p_i, T_i)) \forall i$. The interval endpoints of δ_i are due to the relevant values of δ for which $\ell_{t_A}^{\text{SAL}}$ can be an equilibrium contract, as specified in Lemma I.8. I employ the secant method with initial starting values 0 and T_i to approximate a real-numbered $t_{\text{SC},i} \in (0, T_i)$ for every parameter set i with minimum precision of 1×10^{-9} .³¹ Every single simulated parameter set i satisfies $U_H(\ell_{t_{\text{SRL}}}^{\text{SRL}}) > U_H(\ell_{t_A}^{\text{SAL}})$. ■

Lemma I.10. $\ell_t^{\text{PRL}} \succ^H \ell_t^{\text{PAL}} \forall t \in (0, T)$.

Proof. The difference of the particular general utilities, $U_H^{\text{PCLC}} - U_H^{\text{PAL}}$, easily simplifies to

$$\frac{\delta^t \theta (1 - \theta) (1 - p) (1 - p^t) (T - t)}{\theta + p^t (1 - \theta)}, \quad (58)$$

which is obviously always strictly positive if $0 < t < T$, and zero if $t = 0$. Hence, for each $t \in (0, T)$, it is $U_H^{\text{PCLC}}(t) > U_H^{\text{PAL}}(t)$ and thus $\ell_t^{\text{PCLC}} \succ \ell_t^{\text{PAL}}$ is true for type H individuals. ■

From Lemma I.10 follows that there is always a pooling relationship loan that types H prefer over a pooling arm's-length loan at $t > 0$. Thus, ℓ_0^{PAL} is the only PAL that can be preferred by types H over the best relationship loan contract

³¹The average approximation accuracy appears to be about 1×10^{-15} in my simulation.

and is therefore the only non-relationship loan contract that is relevant in the model market. Hence, the remaining loans to consider in the model market with relationship lending and asymmetric information reduce to ℓ_0^{PAL} , $\ell_{t_{\text{SRL}}}^{\text{SRL}}$, and $\ell_{t_{\text{PRL}}}^{\text{PRL}}$. These are the same contracts that remained in equilibrium under the assumption $e^{-1} \leq p < 1$, and the same final equilibrium conditions emerge.

Q.E.D.

Part II

Credit Information Sharing and Borrower Discipline Revisited

Abstract

Credit information sharing between lenders can have a disciplinary effect on borrowers because defaulting with one lender ruins the reputation with every other lender (Vercammen, 1995; Padilla and Pagano, 2000). This reputation effect, however, diminishes and finally disappears the more comprehensive credit registries become. I show in a multi-period model of repeated lending that credit information sharing can induce borrower discipline beyond “passive” reputation effects if banks apply classical disciplining, that is, if failure to pay inevitably provokes consequences. I find that such disciplining can Pareto improve the efficiency of the financing market and reduce defaults by overcoming market failure and mitigating underinvestment in projects and in effort, even for comprehensive and unrestricted credit information sharing. I further show that disciplining borrowers by pro rata rationing credit after default is more promising than tightening credit rates. Hence, my model provides a rare case of efficient equilibrium credit rationing: disciplining by credit rationing enhances the efficiency of the market while constituting aggregate equilibrium credit rationing in the sense of Stiglitz and Weiss (1981). Contrary to the previous literature that suggests to restrict and randomize credit reporting in order to prevent diminishing reputation effects, the policy implications following from my work are, first, to rather restrict access to credit registries than their content and, second, to enhance transparency of information sharing.

1 Introduction

Asymmetric information between lenders and borrowers about the quality of borrowers is uncontroversially considered to be the major cause of inefficiencies and failures of the credit market. To reduce these problems, lenders can produce information about borrowers' quality by monitoring or screening. As an alternative to producing information about every borrower themselves, lenders can share information about borrowers with other lenders. Credit information sharing among banks is actually a widespread practice in most countries by using publicly regulated or private credit registries (Jappelli and Pagano, 2002; Djankov et al., 2007). Theoretical research identified three basic effects and explanations of such information sharing. First, credit information sharing allows lenders to offer more accurate loan terms and thereby reduces adverse selection problems (Pagano and Jappelli, 1993). Second, sharing information about borrowers prevents banks from extracting rents from borrowers, which entails more competitive credit terms and increases borrowers' incentives to perform (Padilla and Pagano, 1997). Third, information sharing can discipline borrowers and reduce moral hazard (Vercammen, 1995; Padilla and Pagano, 2000). My work directly relates to the third approach.

The works of Vercammen (1995) and Padilla and Pagano (2000) are based on reputation effects of information sharing which are in line with the definition of Diamond (1989): "Reputation effects on decisions arise when an agent adjusts his or her behavior to influence data others use in learning about him". Hence, defaulting borrowers may lose their reputation with every lender and not just the current one if lenders share credit information. This can induce incentives to perform. But, by this definition, the reputation effects should decline if there is less to learn about agents. That is exactly what Vercammen (1995) and Padilla and Pagano (2000) find in their theoretical work and Brown and Zehnder (2007) confirm by studying a laboratory credit market: the more comprehensive the credit registry, the weaker the reputation effects

of information sharing. The disciplinary reputation effects can be maintained in the two-period model framework of Padilla and Pagano (2000) if lenders solely share default information instead of sharing comprehensive information about borrowers' qualities. But this result does not persist in a multi-period setting. When considering many periods, the informational difference between sharing default information and sharing full quality information diminishes over time and finally disappears as the credit reporting system matures and thereby becomes more comprehensive, like in Vercammen (1995) and, in a comparable setting, in Holmström (1982).³² To solve the problem of diminishing reputation effects, Vercammen (1995) suggests to restrict credit information sharing by, for instance, partially preventing access to credit histories. Padilla and Pagano (2000) suggest a policy to randomize credit information sharing in order to control the informativeness of the registries.³³

Instead of building on “passive” reputation effects that are due to credit information sharing and trying to fix their diminishing nature for maturing credit registries by artificially reducing their informativeness, I suggest a different approach to discipline borrowers on the basis of credit information sharing: classical disciplining. Disciplining generally requires that undesirable behavior inevitably provokes unfavorable consequences. If behavior is not directly observable, the undesirable outcome that is due to the undesirable behavior can be used as proxy. The outcome that is undesirable for creditors in lending relationships is borrowers' failure to pay.

Reputation effects do not qualify as classical disciplining, as illustrated in the following. The only direct and inevitable consequence of a borrower's default in

³²Diamond (1989), in contrast, finds that reputation effects strengthen with time. This result is due to a different modeling. Default is modeled to be a perfectly revealing signal of high-risk project choice. With time, pools of high-risk and low-risk borrowers get more refined and the differences in credit terms between these pools widen. Thus, costs of reputation (that is, costs of being assigned to the high-risk pool) increase.

³³Actually, Padilla and Pagano (2000) suggest to randomly share combinations of plain default information and information about borrowers' quality. But, again, this approach does not seem promising in a multi-period context that takes maturing credit reporting systems into account, as follows from the previous argumentation.

Vercammen (1995) and in Padilla and Pagano (2000) is that the information that the borrower defaulted is shared. This direct consequence of default is not unfavorable for a borrower *per se* but it can constitute unfavorable indirect consequences: when banks use the default information shared in their assessment of the borrower's quality (or reputation), future credit terms offered to that borrower can be unfavorably affected. But, as discussed above, the existence and the extent of these indirect consequences of default heavily depend on and vary with the particular information set that lenders possess about the particular borrower. A new piece of default information has only little or no effect on lenders' assessment of the borrower's quality and therefore does not affect her reputation if lenders possess extensive or even comprehensive information about her. My work sets in at this point where reputation effects break down and fail to induce borrower discipline.

I develop a multi-period model of repeated bank lending to entrepreneurs in a competitive financing market with a comprehensive credit registry in place. While comprehensive information sharing solves the problems of adverse selection, the problems of moral hazard (related to entrepreneurs' non-contractible effort choice that is private information) remain present. To discipline borrowers and induce incentives to perform, the information shared in the registry can be used by banks to establish direct unfavorable consequences in the next financing after a borrower's default. While consequences of default can be implemented in different ways, I argue that punishing borrowers that fail to repay a credit with *pro rata* credit rationing is a promising approach.³⁴ I show that disciplining by tightening the credit rate after a default has serious limitations in comparison to disciplining by credit rationing.³⁵ First, punishing defaulters in terms of a

³⁴In related work, Allen (1983) and Stiglitz and Weiss (1983) find total exclusion from the credit market to be an effective disciplinary device.

³⁵Increasing collateral requirements would be another way to implement disciplining but is not studied in this paper. Though, empirical research finds collateral to be increasing with credit information sharing (Doblas-Madrid and Minetti, 2013), which could be explained by disciplining. Another explanation in terms of reducing adverse selection problems is given by Karapetyan and Stacescu (2014).

higher credit rate reduces their incentives to perform after a default and they become more likely to default again, which is not the case for disciplining by credit rationing. Second, disciplining by credit rate tightening breaks down in more cases than disciplining by credit rationing since it can lead to punishment excesses where every punishment requires an even greater next punishment. Hence, my model adds a novel explanation for the empirical observation that banks rather apply changes to the availability of credit than to the price of credit when their costs of lending shift (Petersen and Rajan, 1994).

The model shows that, in spite of comprehensive and unrestricted credit registries, classical disciplining on the basis of credit information sharing is feasible and can increase borrowers' incentives to perform. In particular disciplining by credit rationing can Pareto improve the efficiency of the financing market and reduce default probabilities. It can overcome market failure and mitigate underinvestment in projects and in effort. Hence, my work demonstrates that credit information sharing can induce borrower discipline beyond "passive" reputation effects by actively using the information shared. For classical disciplining, contrary to reputation effects, it is in general not important how detailed the information shared is, as long as the relevant information for disciplining is included.

The use of disciplining can, of course, not lead to a perfectly efficient allocation. The positive effects of disciplining by credit rationing are achieved at the price of equilibrium credit rationing in the sense of Stiglitz and Weiss (1981). But since the efficiency enhancement of the market is intrinsically tied to and induced by credit rationing, the equilibrium credit rationing occurring in my model is efficient and not a case for government intervention (as also in de Mesa and Webb, 1992), contrary to a substantial previous literature on credit rationing (Jaffee and Russell, 1976; Keeton, 1979; Stiglitz and Weiss, 1981, 1983; Williamson, 1986). My model therefore provides a rare case of efficient equilibrium credit rationing.

My work also gives a novel explanation why lenders do not only share default information but also contract-specific information like credit volumes and credit rates. Bennardo et al. (2015) provide an understanding why banks share such data and how this affects credit market performance. They show that the possibility of multi-bank lending can produce incentives to overborrow which can be mitigated by sharing contract-specific information about past debts. My model offers a different and very direct reason to share contract-specific data that does not rely on the effects of multi-bank lending: contract-specific data show to be required to set up and maintain disciplining effectively.

The remainder of this part is organized as follows. Section 2 develops the basic model featuring comprehensive credit information sharing. I solve the stage game before I generally analyze the repeated game for a market without and with disciplining. Section 3 analyzes disciplining by credit rationing and evaluates its efficiency. Section 4 shows the limitations of disciplining by credit rate tightening, before Section 5 discusses policy implications for credit reporting systems. I conclude in Section 6.

2 The Basic Model

Consider many externally heterogeneous and internally homogeneous groups of risk-neutral entrepreneurs in the market. Entrepreneurs of the same group j share the identical entrepreneurial quality, entrepreneurs of different groups differ in quality. Every entrepreneur of group j can operate a risky one-period project every period. Entrepreneurs are assumed to have no endowment and to entirely and immediately consume all the residual income they receive. Thus, entrepreneurs' projects are completely debt-financed. The financing market be competitive and features established credit information sharing between lenders. The credit registry provides data about entrepreneurs' qualities, their credit histories, and details about past contracts (like credit volumes and credit rates). The credit reporting system therefore constitutes a fully comprehensive credit

registry that perfectly reveals every borrower's quality to every lender. The fact of information sharing between lenders and the informativeness of the credit registry are common knowledge. I regard as given that information is collected and shared truthfully in the credit registry without explicitly modeling and exploring banks' incentives to do so.

Although the quality of every entrepreneur of the same group j is identical, they can still differ in their credit history. That means that the track record of successes and defaults is individual even for entrepreneurs of the same quality group since project operation is assumed to be stochastic and defaults are not perfectly correlated. Entrepreneurs are therefore fully characterized by their quality group j and their credit history h . Let $d_i \in \{s, f, 0\}$ label if there is a success (s), a failure (f), or no project operation (0) in period i . Then a credit history h after period t is defined as the ordered sequence $h = \langle d_i \rangle_{i \leq t}$. Let the history h^+ be a direct successor of history h . That means, if h is a history after period t , it is $h^+ = h \| d_{t+1}$, where $\|$ denotes sequence concatenation. Because of the binomial character of the success probability distribution, we have $h^+ \in \{h^s, h^f, h^0\}$, where history h^s marks the direct successor of history h that is characterized by success following on history h , history h^f analogously marks the direct successor of history h that is characterized by failure in the project following on history h , and h^0 marks that there is no project operation after history h . Given a history h after period t , let us generally write $h^{d_{t+1}d_{t+2}\dots d_{t+m}} = h \| d_{t+1} \| d_{t+2} \| \dots \| d_{t+m}$ to specify the m next project outcomes following on history h . Further, let the history h^\rightarrow be an unspecified possible successor of history h .

Comprehensive credit registries allow banks to offer different loan terms to different quality groups of entrepreneurs, but they generally also allow lenders to base credit terms on individual credit histories. After every history h , risk-neutral lenders can offer one-period standard loan contracts $\theta_j(h) = (\lambda_j(h), r_j(h))$ of volume $\lambda_j(h) \geq 0$ and with credit rate $r_j(h) \geq 0$ to entrepreneurs of group j .

The entrepreneurial quality of entrepreneurs in the same group j is determined by $\pi_j := \{R_j, e_j(p), \bar{p}_j\}$. An entrepreneur's project returns $\lambda_j(h) R_j > \lambda_j(h)$ with probability $p_j(h)$ and it returns 0 otherwise for an investment of $\lambda_j(h)$ monetary units after history h . Entrepreneurs choose the success probability $p_j(h) \in [0, \bar{p}_j]$ for every project independently,³⁶ where the maximum success probability, \bar{p}_j , is characterized by $0 < \bar{p}_j < 1$. Entrepreneurs face costs of effort of $\lambda_j(h) e_j[p_j(h)]$ for an investment of $\lambda_j(h)$ monetary units. About the effort function I assume $e'_j(p), e''_j(p) > 0 \forall p > 0$, and $e_j(0) = e'_j(0) = 0$. Let us further assume that $e'_j(\bar{p}_j) > R_j - 1/\bar{p}_j$ to obtain an interior solution of the stage game.

While the success probability $p_j(h)$ is private information of an entrepreneur and not contractible, the actual return of the project is observable, verifiable, and contractible by the current lender. In this way I assume that legal forces ensure that entrepreneurs repay the loan whenever they can. Standardizing the discount factor to one, an entrepreneur's expected utility of a one-period investment project that is operated after history h with credit terms $\theta_j(h)$ is given by

$$U_j[\theta_j(h), p_j(h)] = \lambda_j(h) [p_j(h) (R_j - 1 - r_j(h)) - e_j[p_j(h)]]. \quad (59)$$

Since preferences of entrepreneurs are assumed to be monotonic in the credit volume, we require an upper limit for financing. Let therefore every single project be scalable up to an investment of one monetary unit, that is, $\lambda_j(h) \leq 1 \forall h, j$.

Borrowers of different quality exist in the model market but lenders can perfectly distinguish between different quality types because of comprehensive credit registries. So lenders can address different credit terms to different quality groups j of entrepreneurs. Information sharing has therefore resolved the problem of adverse selection (see Pagano and Jappelli, 1993) but moral hazard problems remain present in the strategic interaction between lenders and borrowers.

³⁶Adding a stochastic relationship between effort choice and probability of success makes the analysis much more complex but does not change the main results.

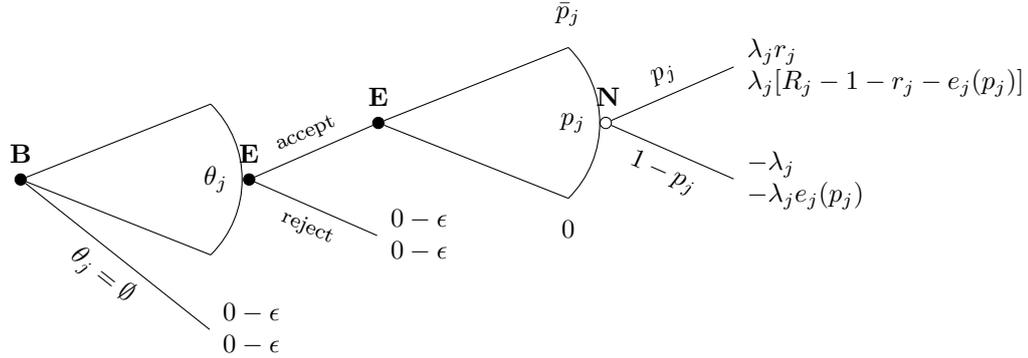


Figure 12: Extensive form representation of the model's stage game between bank B and entrepreneur E of quality group j .

We consider sequential interaction of banks and entrepreneurs in every period. Figure 12 illustrates the model's stage game in extensive form representation. First, banks can offer standard loan contracts with terms $\theta_j(h)$ to entrepreneurs of quality group j after a history h ; an empty contract set, $\theta_j(h) = \emptyset$, represents that a bank does not provide a loan contract to entrepreneurs of group j after history h . Then entrepreneurs either reject the loan and do not operate the project, or they accept a loan contract and choose the success probability, $p_j(h)$, for their current project operation before nature decides about success or failure of the project. In contrast to Vercammen (1995) and Padilla and Pagano (2000), entrepreneurs do not behave as price takers choosing their effort level along the supply curve of the bank. This structural difference ensures that my model features a more severe problem of moral hazard that disciplining needs to resolve.

The financing market is assumed to be competitive. Banks and entrepreneurs possess a general preference to be in business. Thus, banks prefer zero-profit lending over no lending, and entrepreneurs prefer to operate a project that yields zero utility over gaining zero utility from omitting the project (for which I account in Fig. 12 by subtracting a marginal amount ϵ from the payoff in case of non-financing).

2.1 Stage Game Analysis

In the one-period stage game, future consequences of default can naturally not be implemented and disciplining is not possible. The result of the stage game analysis is therefore the market solution without disciplining effects. Since credit histories are not relevant in the stage game, entrepreneurs are sufficiently characterized by their quality π_j .

A bank's expected repayment of the loan contract $\theta_j = (\lambda_j, r_j)$ made to an entrepreneur of quality group j in the stage game is $p_j \lambda_j (1 + r_j)$. Banks' break-even loan rate is therefore given by $r_B(p_j) := 1/p_j - 1$, where $1/\bar{p}_j - 1 \leq r_B \leq R_j - 1$. Or, *vice versa*, if a bank offers the credit rate r_j to an entrepreneur of group j , the bank requires the entrepreneur to choose the success probability $p_j = p_B(r_j) := 1/(1 + r_j)$ to break even. The bounds for the break-even rate unfold as follows. As investment projects never return more than $\lambda_j R_j$, the total repayment of a loan, $\lambda_j (1 + r_j)$, can never exceed $\lambda_j R_j$. This induces the upper bound, $R_j - 1$, for the break-even loan rate and, correspondingly, the lowest possible success probability for which financing can be possible to be R_j^{-1} . The break-even rate's lower bound corresponds to the maximum success probability, \bar{p}_j , that entrepreneurs can choose for project operation. The solution space for financing equilibria is therefore given by

$$\Omega_j := \omega_j^p \times \omega_j^r, \quad (60)$$

$$\text{where } \omega_j^p := \{p_j \mid R_j^{-1} \leq p_j \leq \bar{p}_j\}, \quad (61)$$

$$\omega_j^r := \{r_j \mid 1/\bar{p}_j - 1 \leq r_j \leq R_j - 1\}.$$

Subsequent to banks' loan contract offers, entrepreneurs enter a contract and choose the success probability of their project with their effort choice. Given the loan contract (λ_j, r_j) , an entrepreneur's optimal choice of p_j follows from the first-order condition $\partial U_j(\lambda_j, r_j)/\partial p_j = 0$.³⁷ We obtain the best response

³⁷The second-order condition is always satisfied in case of financing since we have $\partial^2 U_j/\partial p_j^2 \equiv$

function, \hat{r}_j^0 , of entrepreneur j that describes optimal combinations of the credit rate and the success probability choice in the stage game,

$$\hat{r}_j^0 := R_j - 1 - e'_j(p_j). \quad (62)$$

Due to the characteristics of the effort function $e_j(p)$, there is a unique optimal p_j for every loan rate offered. Entrepreneurs' optimal choice is, as well as banks' break-even constraint, obviously independent from the credit volume λ_j . But if project operation has a positive expected utility for entrepreneurs, greater credit volume provides greater utility. Hence, the maximum credit volume $\lambda_j = 1$ is offered in the competitive stage game if a loan is offered at all.

A financing Nash equilibrium of the stage game generally requires that the best response functions of banks and entrepreneurs agree in the solution space Ω_j .³⁸ Competition in the financing market entails that the best response of a bank to the strategy p_j of an entrepreneur is the break-even loan rate $r_B(p_j)$. Thus, a Nash equilibrium of the stage game requires $\hat{r}_j^0 = r_B$ in the solution space Ω_j , which forms the equilibrium condition

$$e'_j(p_j) = R_j - 1/p_j. \quad (63)$$

Depending on entrepreneurs' quality π_j , Eq. (63) has no, one, or two solutions in the space Ω_j , as stated and shown by Lemma II.1.

Lemma II.1. *Dependent on an entrepreneur's quality π_j , the best response functions do either not intersect, or intersect tangentially exactly once, or intersect transversely exactly twice in the solution space Ω_j .*

Proof. r_B is a monotonic decreasing strictly convex function in p_j , while \hat{r}_j^0 is a monotonic decreasing strictly concave function in p_j , since $\partial r_B / \partial p_j < 0$,

$-\lambda_j e''_j(p_j) < 0$.

³⁸Banks would not provide financing if $\hat{r}_j^0 < r_B$ since they would face losses. If $\hat{r}_j^0 > r_B$, banks would earn strictly positive profits from financing and other banks would be able to offer more attractive loan terms to win borrowers over.

$\partial r_B / \partial^2 p_j > 0$, and $\partial \hat{r}_j^0 / \partial p_j, \partial \hat{r}_j^0 / \partial^2 p_j < 0$. For the smallest $p_j \in \omega_j^p$, which is given by R_j^{-1} , we obtain $r_B(R_j^{-1}) = R_j - 1 > \hat{r}_j(R_j^{-1}) = R_j - 1 - e'_j(p_j)$ because of $e'_j(p) > 0$. For the greatest $p_j \in \omega_j^p$, which is given by \bar{p}_j , we have $r_B(\bar{p}_j) > \hat{r}_j^0(\bar{p}_j)$ if $e'_j(\bar{p}_j) > R_j - 1/\bar{p}_j$ which is true by assumption. Thus, depending on entrepreneurs' quality π_j , $r_B = \hat{r}_j^0$ has either no, one, or two solutions in the solution space Ω_j . If there is only one solution, it necessarily is the point of tangency of the best response functions. ■

With Definition II.1 I aggregate entrepreneurs of quality groups j to generic groups G_ι , depending on the number of intersections, ι , that their best response functions and banks' best response function show in the solution space according to Lemma II.1. These definitions are sufficient and complete under the global assumptions of the model, as follows from Lemma II.1.

Definition II.1 (Generic Groups).

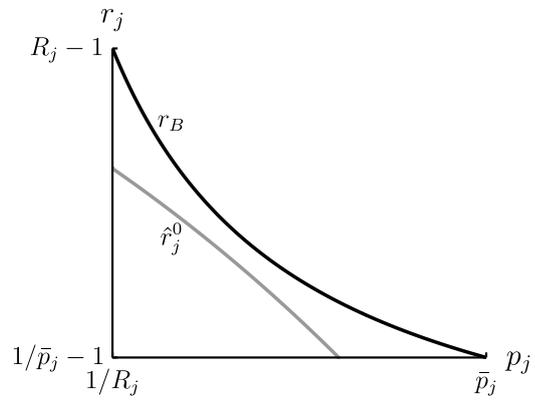
$$G_\emptyset := \{j \mid \Omega_j = \emptyset\}, \quad (64)$$

$$G_0 := \{j \mid \Omega_j \neq \emptyset \wedge \forall p_j \in \omega_j^p : e'_j(p_j) > R_j - 1/p_j\}, \quad (65)$$

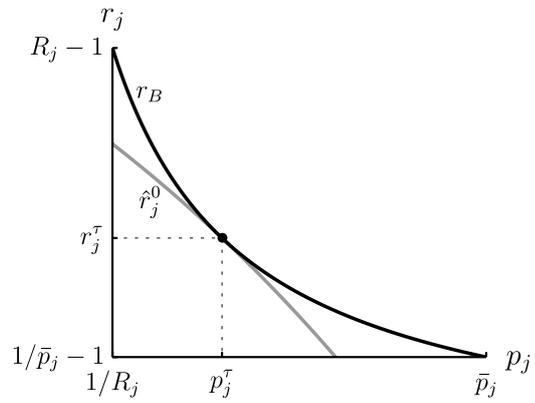
$$G_1 := \{j \mid j \notin G_\emptyset \cup G_0 \wedge \nexists p_j \in \omega_j^p : e'_j(p_j) < R_j - 1/p_j\}, \quad (66)$$

$$G_2 := \{j \mid \exists p_j \in \omega_j^p : e'_j(p_j) < R_j - 1/p_j\}. \quad (67)$$

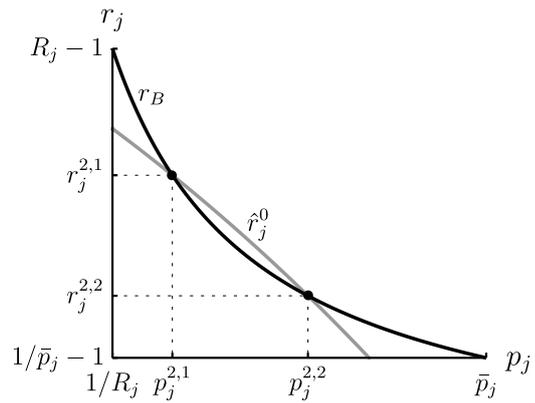
Figure 13 illustrates the best response functions and the financing Nash equilibria for entrepreneurs of different generic groups. Entrepreneurs of a group $j \in G_\emptyset$ are characterized by $\bar{p}_j < R_j^{-1}$, that is, by an empty solution space, Ω_j , for financing equilibria. Thus, there is no feasible combination of credit rate and success probability for which financing could be possible at all and an illustration is pointless. For entrepreneurs of generic group G_0 there is no solution to Eq. (63) in the non-empty solution space Ω_j , that is, the best response functions do not intersect. Moral hazard prevents banks to break even when offering financing in this case. For entrepreneurs of a group $j \in G_1$ there is exactly one solution to $\hat{r}_j^0 = r_B$ in Ω_j , which is the best response functions' point of tangency (p_j^T, r_j^T) , where p_j^T satisfies $\partial \hat{r}_j^0 / \partial p_j = \partial r_B / \partial p_j$ which reduces to $e''_j(p_j) = 1/p_j^2$. This forms the unique financing Nash equilibrium of the stage



(a) $j \in G_0$.



(b) $j \in G_1$.



(c) $j \in G_2$.

Figure 13: Best response functions and financing equilibria in the stage game for entrepreneurs of different generic groups G .

game for $j \in G_1$. For entrepreneurs of generic group G_2 there are exactly two intersection points of the best response functions in Ω_j , which are $(p_j^{2,1}, r_j^{2,1})$ and $(p_j^{2,2}, r_j^{2,2})$, where $r_j^{2,1} > r_j^{2,2}$ and $p_j^{2,1} < p_j^{2,2}$. From Proposition II.1, the second solution, $(p_j^{2,2}, r_j^{2,2})$, dominates the first one, $(p_j^{2,1}, r_j^{2,1})$, and is therefore the only one that occurs under competition. Building on Lemma II.2, Proposition II.1 subsumes the competitive subgame perfect stage game equilibrium.

Lemma II.2. *In the stage game, entrepreneur's participation constraints are always satisfied at intersection points of the best response functions.*

Proof. At an intersection point (p_j, r_j) of the best response functions we have $r_j = 1/p_j - 1$ and $R_j = e'_j(p_j) + 1/p_j$, as has been shown before. Therefore, the participation constraint of an entrepreneur of group j , which is $U_j(r_j, p_j) \geq 0$, mathematically reduces to $e'_j(p_j) \geq e_j(p_j)/p_j$ at the intersection point. This constraint is always satisfied under the model's assumption, which is shown in the following. Since $e_j(0) = 0$ by assumption, we can subtract $e_j(0)/p_j$ from the right-hand side of the inequation $e'_j(p) \geq e_j(p)/p_j$ to obtain, at some $p'_j > 0$,

$$e'_j(p'_j) \geq \frac{e_j(p'_j) - e_j(0)}{p'_j}. \quad (68)$$

Because of the assumption $e''_j(p) > 0$, the effort function $e_j(p)$ is strictly convex in p . That means that the mean slope of the line segment of $e_j(p_j)$ between $p_j = 0$ and $p_j = p'_j$, as found on the right-hand side of Eq. (68), is clearly smaller than the slope of $e_j(p_j)$ at $p_j = p'_j$, as found on the left-hand side. Hence, Eq. (68) holds if the model's assumptions about the effort function hold, and thus $e'_j(p_j) > e_j(p_j)/p_j$ is satisfied for every $p_j \in \omega_j^p$. ■

Proposition II.1 (Stage Game Equilibrium). *The competitive subgame perfect Nash equilibrium of the stage game is characterized by non-financing for entrepreneurs of groups $j \in G_\emptyset \cup G_0$, the financing equilibrium is given by*

$$(\theta_j^*, p_j^*) = \begin{cases} ((1, r_j^\tau), p_j^\tau) & \text{if } j \in G_1, \\ ((1, r_j^{2,2}), p_j^{2,2}) & \text{if } j \in G_2. \end{cases} \quad (69)$$

Proof. The proof of the non-financing equilibrium and the equilibrium for $j \in G_1$

follow straightforward from Lemmas II.1 and II.2. The competitive equilibrium for $j \in G_2$ is determined as follows. Moving along banks' offer curve $r_B(p_j)$, an entrepreneur's utility increases with increasing p_j if $\partial U_j(r_B(p_j))/\partial p_j \equiv \lambda_j[R_j - e'_j(p_j)] > 0$. This is obviously true in the stage game if $e'_j(p_j) < R_j$ which is certainly the case if $j \in G_2 \wedge p_j \in [p_j^{2,1}, p_j^{2,2}]$ according to Lemma II.1 and the set building condition of G_2 in Definition II.1. Hence, $U_j(r_j^{2,2}, p_j^{2,2}) > U_j(r_j^{2,1}, p_j^{2,1})$. As both possible financing Nash equilibria for generic group G_2 satisfy the break-even constraint of banks with equality, competition rules out the one that is inferior for entrepreneurs. ■

To assess the efficiency of the stage game equilibrium, I compare it with the first best solution. The latter is given by the stage game's equilibrium in absence of agency problems. As we regard a situation of established information sharing that has created comprehensive credit registries, problems of adverse selection do not arise. Moral hazard would be eliminated from my model if entrepreneurs chose their success probability along the break-even supply curve, $r_B(p_j)$, of banks. The first-order condition, $\partial U_j[r_B(p_j)]/\partial p_j = 0$, then reduces to $R_j = e'_j(p_j)$. Since R_j is static and greater than zero, and $e'_j(p) > 0$ as well as $e'_j(0) = 0$ by assumption, there is a unique solution $p_j^{\text{fb}} > 0$ to the first-order condition. If $p_j^{\text{fb}} > \bar{p}_j$, the utility maximizing $p_j \in \omega_j^p$ is found at the success probability's upper bound, which follows from the uniqueness of the extremum. When considering entrepreneurs' optimal choice of p_j along banks' offer curve, entrepreneurs' participation constraint reduces to $e'_j(p_j) \geq 1/p_j + e_j(p_j)/p_j$. Proposition II.2 subsumes the first best solution.

Proposition II.2 (First Best Solution). *In the first best world there is financing in equilibrium with loan terms $(\lambda_j^{\text{1st}}, r_j^{\text{1st}}) = (1, r_B[p_j^{\text{1st}}])$, where $p_j^{\text{1st}} = \min[p_j^{\text{fb}}, \bar{p}_j]$, if $e'_j(p_j^{\text{1st}}) \geq 1/p_j^{\text{1st}} + e_j(p_j^{\text{1st}})/p_j^{\text{1st}}$.*

The participation constraint of entrepreneurs of generic group G_0 (that is, for $\bar{p}_j < R_j^{-1}$) is never satisfied in the first best world. Like in the stage game solution we do not see financing in this case. For entrepreneurs of groups $j \in G_0$, however, financing can be possible in the first best world, whereas moral hazard

prevents financing to happen in the stage game equilibrium. Hence, agency problems lead to underinvestment in projects in the stage game equilibrium. Whenever we find a financing equilibrium in the stage game (which is always the case for entrepreneurs of generic groups G_1 and G_2 , according to Lemma II.2), financing is also found in the first best solution. But the first best financing equilibrium is characterized by lower credit rates and higher effort choices than the stage game equilibrium. Thus, moral hazard leads to underinvestment in effort. Proposition II.3 recapitulates and proves the efficiency evaluation of the stage game equilibrium.

Proposition II.3 (Efficiency of the Stage Game Equilibrium). *The stage game equilibrium shows inefficient underinvestment in projects (for $j \in G_0$) and in effort (for $j \in G_1 \cup G_2$).*

Proof. For entrepreneurs of a group $j \in G_0$, it is $e'_j(p) > R_j - 1/p \forall p \in \omega_j^p$ according to Definition II.1. Entrepreneurs' participation constraint is satisfied in the first best world if $e'_j(p) \geq 1/p + e_j(p)/p$ for the utility maximizing p , according to Proposition II.2. With $R_j = e'_j(p_j^{1st})$, this constraint can be written as $R_j - 1/p_j^{1st} \geq e_j(p_j^{1st})/p_j^{1st}$. Since it is $e'_j(p) > e_j(p)/p \forall p \in \omega_j^p$ according to the proof of Lemma II.2, we see that groups $j \in G_0$ exist that satisfy entrepreneurs' participation constraint and allow financing to occur in the first best solution. As there is never financing for $j \in G_0$ in the stage game equilibrium, it is characterized by underinvestment.

The first best solution is the utility maximizing combination out of all the combinations of credit rate and success probability that lie on banks' offer curve, $r_B(p)$. In a financing equilibrium of the stage game, banks' break-even constraint is binding due to competition, and thus the stage game equilibrium (r_j^*, p_j^*) also lies on $r_B(p)$ but differs from the first best solution. Hence, financing is always a feasible equilibrium outcome in the first best world whenever it is a feasible equilibrium outcome in the stage game (and the latter is always the case for entrepreneurs of generic groups G_1 and G_2 , according to Lemma II.2). The equilibrium success probability of the stage game is characterized by $p_j^* < \bar{p}_j$ and solves the condition $e'_j(p_j^*) = R_j - 1/p_j^*$, while the first best success probability is given by $p_j^{1st} = \min(\bar{p}_j, p_j^{fb})$, where $e'_j(p_j^{fb}) = R_j$. Therefore, it is necessarily

$e'_j(p_j^*) < e'_j(p_j^{1st})$. From $e_j(p), e'_j(p), e''_j(p) > 0 \forall p \in \omega_j^p$ follows that $p_j^* < p_j^{1st}$ and finally $e_j(p_j^*) < e_j(p_j^{1st})$. ■

2.2 The Repeated Game and Disciplining

We now consider the stage game to be played repeatedly over an indefinite time horizon. That means, at every endpoint of the stage game in the extensive-form representation in Fig. 12, the stage game is played again with probability δ , and so forth. The probability of continuation, δ , is assumed to be constant and uniform for every entrepreneur after every history h . Let us regard δ to be less than 1 but very large and approaching 1. The probability of continuation therefore does not affect nearby project payoffs but prevents dealing with infinitely large total payoff streams.³⁹ In this setting, disciplining borrowers becomes conceptually possible since after every period it is highly likely that there is a next period where consequences of default can carry into effect. Comprehensive information sharing does now allow banks to not only offer different terms to entrepreneurs of different qualities but also to entrepreneurs with different credit histories. Thus, banks can employ the information shared to establish disciplining by implementing unfavorable consequences subsequent to entrepreneurs' defaults. To prevent the possibility of splitting and postponing consequences of default over the indefinite number of future periods, let us assume that disciplining is only effective as far as it is implemented in the very next period after default, and this fact is common knowledge. This assumption is basically in line with and well-founded in the literature on behavior analysis where delays in punishment are found to severely and increasingly decrease disciplining effectiveness (Banks and Vogel-Sprott, 1965; Trenholme and Baron, 1975; Goodall, 1984).

The project utility $U_j(h)$ is an entrepreneur's expected utility of operating a one-period project after history h , as stated in Eq. (59). The expected total

³⁹This is true since the assumptions of the model, especially the bounded solution space, ensure that the magnitude of the project utility is limited at least on the equilibrium path.

utility of the entrepreneur after history h is generally given by

$$TU_j(h) = U_j(h) + FU_j(h) \quad (70)$$

$$\begin{aligned} &= U_j(h) + \delta p_j(h) [U_j(h^s) + FU_j(h^s)] \\ &\quad + \delta (1 - p_j(h)) [U_j(h^f) + FU_j(h^f)], \end{aligned} \quad (71)$$

where $FU_j(h)$ is defined to be an entrepreneur's expected utility after history h of all future periods beyond the current period. More precisely, $FU_j(h)$ is the sum of all future project utilities, excluding $U_i(h)$, for every possible history following on h , weighted with the probabilities of occurrence conditional on being after history h . Let $\text{prob}_j(h^\rightarrow|h)$ denote the conditional probability that history h^\rightarrow occurs given history h , where it captures the probability of the particular credit history (that is, the probability of successes and defaults on the path from h to h^\rightarrow) as well as the probability of continuation. We obtain

$$FU_j(h) = \sum_{h^\rightarrow} \text{prob}_j(h^\rightarrow|h) U_j[\theta_j(h^\rightarrow), p_j(h^\rightarrow)]. \quad (72)$$

Since entrepreneurs can choose the success probability for every single project independently from their past choices, only the utility of the current project, $U_j[\theta_j(h), p_j(h)]$, directly depends on $p_j(h)$ while the future project utilities, $U_j[\theta_j(h^\rightarrow), p_j(h^\rightarrow)]$, depend on entrepreneurs' independent future choices of $p_j(h^\rightarrow)$ and on the particular future credit terms. Thus, the first-order condition to maximize the total utility after history h with respect to $p_j(h)$ is given by

$$\frac{\partial U_j(h)}{\partial p_j(h)} + \delta \left(\underbrace{U_j(h^s) - U_j(h^f)}_{c_j(h)} + \underbrace{FU_j(h^s) - FU_j(h^f)}_{\zeta_j(h)} \right) = 0. \quad (73)$$

In the repeated game, consequences of defaulting in the current project can generally be quantified as the difference between the expected utility of the future if not defaulting in the current project and the expected utility of the

future given a default in the current project. Then $c_j(h)$ marks the consequences of defaulting after history h that an entrepreneur of group j expects to face in the very next period, that is, after history h^f . $\zeta_j(h)$ marks the expected consequences of defaulting after history h that occur in future periods two or more periods after the default, that is, after histories $h^{f\rightarrow}$. With the assumption that consequences that are not implemented in the very next period after default are ineffective for disciplining borrowers, entrepreneurs are implicitly assumed to consider $\zeta_j(h) = 0$ for their effort choice. Thus, with δ approaching 1, the best response of an entrepreneur to credit terms $\theta_j(h) = (\lambda_j(h), r_j(h))$ after history h satisfies

$$\hat{r}_j(h) = R_j - 1 - e'_j[p_j(h)] + c_j(h)/\lambda_j(h). \quad (74)$$

In comparison to the stage game, the best response function of entrepreneurs additionally depends on the expected consequences of default relative to the credit volume of the current financing. Strictly positive expected consequences of default result in an upward shift of entrepreneurs' best response functions in comparison to the stage game. If, however, no disciplining is applied, that means that $c_j(h) = 0 \forall h$, we have $\hat{r}_j(h) = \hat{r}_j^0 \forall h$ and the repeated stage game equilibrium forms a feasible Nash equilibrium of the iterated game. The total utility that entrepreneurs of quality group j obtain from the repeated stage game equilibrium is given by

$$TU_j^* = \lim_{T \rightarrow \infty} \sum_{t=0}^T \delta^t U_j(\theta_j^*, p_j^*) = \frac{U_j(\theta_j^*, p_j^*)}{1 - \delta}. \quad (75)$$

Since the model does not entail costs of switching the lender, since banks face perfect competition, and since comprehensive credit registries prevent endogenous informational advantages of particular inside banks, the iterated game can essentially be viewed as a sequence of one-shot games from a bank's perspective. Hence, banks need to break even on every loan and their break-even

offer curve in case of disciplining remains unchanged in comparison to the stage game analysis.

Under the previously stated presupposition that punishment is only effective if it occurs without delay, I conceptionally follow the equilibrium notion of sequential equilibrium (Kreps and Wilson, 1982) in the repeated game. That generally means that banks and entrepreneurs form beliefs about each other's future strategy choices at every information set. Specifically, for a given credit contract after a history h , entrepreneurs of group j form rational beliefs about future credit terms that banks offer after possible successors of history h . This is essentially equivalent to saying that entrepreneurs form beliefs about consequences of default. If entrepreneurs' *a priori* beliefs about consequences of default are not met after a default (that is, actual consequences differ from expected consequences), entrepreneurs integrate this deviation as new information in their belief formation about future consequences.⁴⁰

A financing equilibrium in the repeated game is still characterized by agreement of best response functions in the solution space Ω_j after every history h . Thus, from $\hat{r}_j(h) = r_B$ we obtain the equilibrium condition

$$e'_j[p_j(h)] = R_j - 1/p_j(h) + c_j(h)/\lambda_j(h) \quad \forall h. \quad (76)$$

If positive consequences can actually be implemented and are robust to competition, different equilibria than in the stage game become possible. The repeated stage game equilibrium acts as a benchmark for equilibria with disciplining to show if and under which conditions disciplining can improve the efficiency of the market. In the following section I analyze disciplining by *pro rata* credit rationing for that matter.

⁴⁰It is a general question if deviations from the equilibrium path are considered as information or as errors. By using the concept of sequential equilibrium I follow the former view which is captured by believe-updating in case of unanticipated events.

3 Disciplining by Credit Rationing

Let us now consider that banks make use of the information shared through credit registries to implement *pro rata* credit rationing as a disciplining device. That means that banks can adjust the credit volume offered to a borrower with respect to the credit history of the borrower. To analyze the pure effect of disciplining by credit rationing, let us presume for this section that banks apply solely credit rationing as disciplining device and do not additionally execute changes in the credit rate.⁴¹ For a constant credit rate $r_j(h) = r_j^\diamond \forall h$ offered to entrepreneurs of quality group j , banks break even in every financing in a competitive market environment only if entrepreneurs constantly choose the success probability $p_j^\diamond := p_B(r_j^\diamond)$. From entrepreneurs' best response function in Eq. (74) follows that an entrepreneur's optimal choice of the project's success probability is only constant for a given constant credit rate if the expected consequences of default in relation to the current credit volume are constant. Let us denote these expected relative consequences of default by $\sigma_j(h) := c_j(h)/\lambda_j(h)$. To implement entrepreneurs' probability choice p_j^\diamond for a given credit rate r_j^\diamond , the corresponding constant relative consequences of default σ_j^\diamond need, according to the equilibrium condition in Eq. (76), to satisfy

$$\sigma_j^\diamond = 1/p_j^\diamond - R_j + e'_j(p_j^\diamond). \quad (77)$$

Since disciplining consequences that do not occur in the very next period after default are assumed to be ineffective and entrepreneurs' preferences are modeled to be monotonic increasing in the credit volume, competing banks offer the maximum credit volume of one monetary unit to entrepreneurs after a successful project if they offer financing at all, that is, $\lambda_j(h^s) = 1 \forall h, j$; the same is generally true after non-financing, that is, after a history h^0 , but this

⁴¹Generally, I allow changes in the credit rate to happen as far as they do not contribute to changes in project utility. This is, however, not possible in my model unless the credit rate remains unchanged.

case is discussed specifically later in the text. An entrepreneur's project utility is therefore uniform after every history h^s and given by

$$U_j^s := U_j[(1, r_j^\diamond), p_j^\diamond] = p_j^\diamond R_j - 1 - e_j(p_j^\diamond). \quad (78)$$

Let us call U_j^s base utility of entrepreneurs of quality group j .

The credit volume after a history h^f , however, depends on the particular history h and not just on the fact that a default occurred after history h . More precisely, the credit volume $\lambda_j(h^f)$ depends on the number of previous consecutive defaults, which shall be seen in the following. With the hitherto results of this section, the expected consequences of default can be written as $c_j(h) = U_j^s - U_j(h^f) = [1 - \lambda_j(h^f)] U_j^s$. This shows the general relation between the expected consequences of default, $c_j(h)$, and the corresponding rationed credit volume after a default, $\lambda_j(h^f)$. By rearranging we obtain a recursive relation between the current credit volume, $\lambda_j(h)$, and the subsequent credit volume after default, $\lambda_j^\diamond(h^f)$, that corresponds to the constant expected relative consequences σ_j^\diamond that are required to implement p_j^\diamond for a given r_j^\diamond ,

$$\lambda_j^\diamond(h^f) = 1 - \frac{\sigma_j^\diamond \lambda_j(h)}{U_j^s} = 1 + \alpha_j \lambda_j(h), \quad (79)$$

where I define $\alpha_j := -\sigma_j^\diamond/U_j^s$. There obviously only is credit rationing after default if $\alpha_j < 0$ which is true for $U_j^s, \sigma_j^\diamond > 0$. Of course, disciplining entrepreneurs in repeated lending can naturally not work if their participation constraint is even violated after a history h^s and they always refuse to take out a loan to operate the project. And since the credit volume is simply scaling the base utility U_j^s , credit rationing can neither induce a disciplining punishment if entrepreneurs' participation constraint is satisfied with equality after a successful project operation, that is, if the base utility is zero.

If $\alpha_j = -1$, Eq. (79) cannot be used. In that case, the required expected relative consequences, σ_j^\diamond , exactly match entrepreneurs' base utility U_j^s . Thus,

the punishment that is required after default is total redlining, which means that the borrower should not get a loan in the period after a default. It follows that the history h^{f0} always needs to be the direct successor of a history h^f , that is, $h^{f+} = h^{f0} \forall h$. Thus, Eq. (79) is not defined for histories $h = h^f$ if $\alpha_j = -1$ since it is not possible to default in a project when not operating a project. The case $\alpha_j = -1$ is therefore the single exception to the previously stated rule that the credit volume after a default depends on the number of previous consecutive defaults. In this case we simply require non-financing after every default, and after success or redlining we are back to the base utility, that is, $\lambda_j(h^s) = \lambda_j(h^0) = 1 \forall h, j$ if financing is possible at all.

If $\alpha_j \neq -1$, the recursive relation in Eq. (79) is generally applicable after every history h to calculate the credit volume after a default that matches the expected relative consequences of default σ_j^\diamond . Starting with a history h^s which, as we already know, is characterized by $\lambda_j(h^s) = 1 \forall h, j$, we recursively obtain

$$\lambda_j^\diamond(h^{sf}) = 1 + \alpha_j \forall h, \quad (80)$$

$$\lambda_j^\diamond(h^{sff}) = 1 + \alpha_j + \alpha_j^2 \forall h, \quad (81)$$

$$\lambda_j^\diamond(h^{sf^n}) = 1 + \alpha_j + \alpha_j^2 + \alpha_j^3 + \dots + \alpha_j^n \forall h, \quad (82)$$

where h^{sf^n} denotes a series of $n \in \mathbb{N}^0$ consecutive defaults subsequent to a history h^s . The closed form formula for this geometric series is given by

$$\lambda_j^\diamond(h^{sf^n}) = \frac{1 - \alpha_j^{n+1}}{1 - \alpha_j}. \quad (83)$$

If entrepreneurs' beliefs about the credit volumes that banks offer after all possible histories h^{sf^n} match with the sequence $\lambda_j^\diamond(h^{sf^n})$, then entrepreneurs expect constant relative consequences σ_j^\diamond for a credit rate r_j^\diamond and disciplining by credit rationing can work. But entrepreneurs can reasonably expect these consequences only if they are, first, feasible and, second, robust to financing market competition.

Consequences are not credible after a history h if they require a punishment after some following history h^\rightarrow that is too large to be implemented. That is the case if the required punishing credit volume ever falls below zero, or technically if $\exists n \in \mathbb{N}^0 : \lambda_j^\diamond(h^{sf^n}) < 0$. The sequence $\lambda_j^\diamond(h^{sf^n})$ is Cauchy for $|\alpha_j| < 1$ (and $\alpha_j \neq 0$) and converges to the limit $\lim_{n \rightarrow \infty} \lambda_j^\diamond(h^{sf^n}) = 1/(1 - \alpha_j)$. Under the condition $-1 < \alpha_j < 0$, the elements of the convergent sequence fluctuate around the limit with decreasing distance for increasing n , which follows from $\partial \lambda_j^\diamond(h^f)/\partial \lambda_j(h) = \alpha_j$ and is illustrated in Fig. 14: the greater the current credit volume, the greater next period's credit rationing after default. The smallest value of the convergent sequence is therefore found for $n = 1$, that is, after a history h^{sf} , since the maximum credit volume of one monetary unit is provided after histories h^s . This smallest credit volume $\lambda_j^\diamond(h^{sf}) = 1 + \alpha_j$ is obviously strictly greater than zero if the condition for convergence holds. Thus, if the sequence $\lambda_j^\diamond(h^{sf^n})$ is Cauchy, it is $\lambda_j^\diamond(h^{sf^n}) > 0 \forall n \in \mathbb{N}^0$ and consequences of default that are in line with the sequence $\lambda_j^\diamond(h^{sf^n})$ are feasible for a strictly positive base utility. Proposition II.4 subsumes.

Proposition II.4 (Feasibility of Disciplining by Credit Rationing). *Disciplining by credit rationing is feasible if $\alpha_j \in [-1, 0)$ or, equivalently, $0 < \sigma_j^\diamond \leq U_j^s$.*

Like in the stage game and the first best solution, disciplining by credit rationing cannot lead to financing of entrepreneurs of generic group G_\emptyset since disciplining by credit rationing is evidently never feasible in that case because of $U_j^s < 0$. For entrepreneurs of generic group G_0 , however, Lemma II.3 proofs that disciplining by credit rationing can be feasible and allow financing, while financing always fails for these entrepreneurs without disciplining, as shown in the stage game solution.

Lemma II.3. *Entrepreneurs of generic group G_0 exist for whom disciplining by credit rationing is feasible, or stated formally, $\exists j \in G_0 : \alpha_j \in [-1, 0)$.*

Proof. We have $j \in G_0$ if $e'_j(p) > R_j - 1/p \forall p \in \omega_j^p$. This is equivalent to the statement $e'_j(p_j^\tau) > R_j - 1/p_j^\tau$ since the reaction functions are closest to each

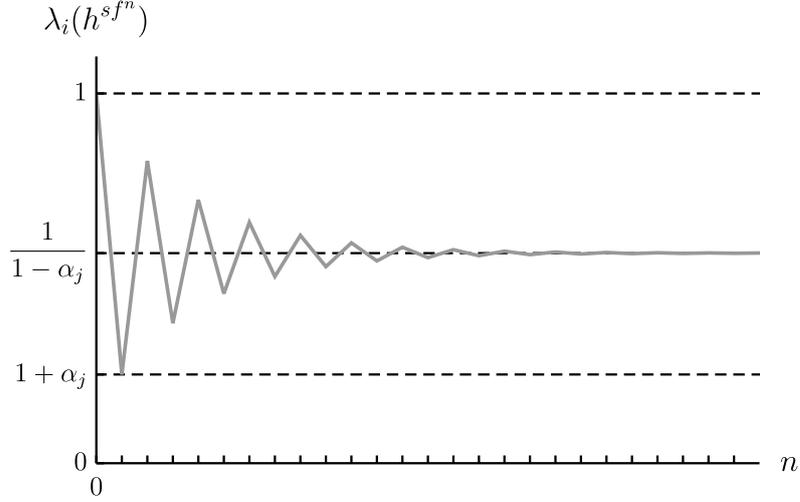


Figure 14: Illustration of the sequence $\lambda_j^\diamond(h^{sf^n})$ for $\alpha_j \in (-1, 0)$.

other at p_j^τ for $j \in G_0$. Solving for R_j gives $R_j < R_j^0 := e'_j(p_j^\tau) + 1/p_j^\tau$. The feasibility constraint in Proposition II.4 reduces to $\sigma_j^\diamond \leq U_j^s$ since $0 < \sigma_j^\diamond$ is clearly always true for $j \in G_0$. Rewriting and solving for R_j yields

$$R_j \geq R_j^\diamond := \frac{1 + p_j^\diamond + p_j^\diamond [e_j(p_j^\diamond) + e'_j(p_j^\diamond)]}{p_j^\diamond + (p_j^\diamond)^2}. \quad (84)$$

For the same p , $R_j^0 > R_j^\diamond$ reduces to $e'_j(p) > e_j(p)/p$ which has been shown to be always true under the model's assumptions in the proof of Lemma II.2. Further we obviously have $R_j^0 > 1$. Hence, groups j of entrepreneurs with qualities $\pi_j = (R_j, e_j(p), \bar{p}_j)$ exist that satisfy $R_j^\diamond \leq R_j < R_j^0$ for some disciplining contract θ_j^\diamond and they therefore simultaneously belong to generic group G_0 and meet the feasibility constraint of disciplining by credit rationing. ■

Whenever we see financing in the stage game equilibrium (that is, for entrepreneurs of generic groups G_1 and G_2), disciplining by credit rationing is feasible. This can be seen from the following argument. If we have a financing equilibrium (θ_j^*, p_j^*) in the stage game, the base utility U_j^s for $\sigma_j^\diamond = 0$ is equal to the stage game equilibrium utility $U_j(\theta_j^*, p_j^*)$ which is strictly positive according to Lemma II.2. Thus, there are clearly at least some arbitrarily small though strictly positive consequences σ_j^\diamond for which disciplining by credit rationing

provides a strictly positive base utility $U_j^s \geq \sigma_j^\diamond$ and therefore satisfies the feasibility condition in Proposition II.4.

Feasibility of disciplining by credit rationing, however, is a necessary but not a sufficient condition that entrepreneurs' believes about future credit volumes match with the sequence $\lambda_j^\diamond(h^{sf^n})$. The punishment also needs to be robust to strategic interaction in a competitive financing market, that is, banks should not be expected to deviate. That means that pursuing the consequences in the face of a default needs to be an equilibrium choice of banks although fully comprehensive credit registries equip banks with a fully comprehensive assessment of entrepreneurs' qualities that cannot be further refined by additional default information. To address this issue let us consider an entrepreneur of quality group j who is offered the financing contract $\theta_j(h^{sf^n}) = (\lambda_j^\diamond(h^{sf^n}), r_j^\diamond)$ to operate the project after a credit history h^{sf^n} that is characterized by n previous consecutive defaults. The bank that offers this contract requires the entrepreneur to apply effort $e_j(p_j^\diamond)$ to the project, which the entrepreneur only does if she expects corresponding relative consequences of σ_j^\diamond in case of a default. But, if the entrepreneur's project fails after history h^{sf^n} , why should a bank in the following period offer the credit volume $\lambda_j^\diamond(h^{sf^{n+1}})$ that corresponds to the expected consequences σ_j^\diamond instead of offering a greater or even the maximum credit volume of 1 which provides greater utility to the entrepreneur and could win her over to the bank?⁴² If a bank offers a credit volume greater than $\lambda_j^\diamond(h^{sf^{n+1}})$ after a history $h^{sf^{n+1}}$, the entrepreneur recognizes the deviation from her expected relative consequences σ_j^\diamond and makes a downward revision of her believes about future consequences of default according to this new information. But expecting less future consequences because of less currently experienced consequences shifts the entrepreneur's best response function and she applies less effort than $e_j(p_j^\diamond)$ to the current project for the same credit rate r_j^\diamond . This results in expected losses for the bank. Hence, it is in the own interest of a bank

⁴²Smaller credit volumes are inferior for entrepreneurs and therefore not a relevant strategic choice of banks in the face of competition.

to pursue consequences of σ_j^\diamond after default for a given feasible credit rate r_j^\diamond .

Feasible and robust disciplining by credit rationing is only found in equilibrium if entrepreneurs prefer it over the repeated stage game solution. Since feasibility requires a positive base utility, entrepreneurs always prefer feasible disciplining by credit rationing over non-financing. Hence, disciplining by credit rationing is an equilibrium choice of entrepreneurs of generic group G_0 whenever it is feasible. For generic groups G_1 and G_2 the analysis is less straightforward. As shown before, feasible and robust disciplining by credit rationing provides a utility of U_j^s to entrepreneurs after every successful project, and after failure they receive the utility $U_j^s \lambda_j^\diamond(h^{sf^n})$, depending on the number of previous consecutive defaults n . Because of this path-dependency, the binomial tree of the project payoffs can show different payoff levels at the same nodes of the tree, contingent on the particular path (that is, on the particular credit history). The probability to get to node m of the binomial tree after t trials with n previous consecutive defaults is—neglecting the probability of continuation—given by

$$\text{prob}(t, m, n) = \binom{t - (n + 1)}{m - n} (p_j^\diamond)^{t-m} (1 - p_j^\diamond)^m, \quad (85)$$

where $0 \leq n \leq m \leq t$. The probability that n previous consecutive defaults occur after t periods is then, for $n < t$, given by

$$\sum_{m=n}^{t-1} \text{prob}(t, m, n) = (1 - p_j^\diamond)^n p_j^\diamond. \quad (86)$$

After a history h^s , the utility that entrepreneurs of quality group j can expect to receive in the t -th period after history h^s can therefore be stated as⁴³

$$\delta^t (1 - p_j^\diamond)^t U_j^s \lambda_j^\diamond(h^{sf^n}) + \delta^t \sum_{n=0}^{t-1} U_j^s \lambda_j^\diamond(h^{sf^n}) (1 - p_j^\diamond)^n p_j^\diamond. \quad (87)$$

⁴³While the assumption that delayed consequences of default are ineffective affects entrepreneurs' effort choice, entrepreneurs' contract choice is not directly affected.

The total expected utility for disciplining by credit rationing is then given by

$$\begin{aligned} TU_j^\diamond &= \lim_{T \rightarrow \infty} \sum_{t=1}^T \left[\delta^t (1 - p_j^\diamond)^t U_j^s \lambda_j^\diamond(h^{sf^t}) + \delta^t \sum_{n=0}^{t-1} U_j^s \lambda_j^\diamond(h^{sf^n}) (1 - p_j^\diamond)^n p_j^\diamond \right] \\ &= \frac{U_j^s}{(1 - \delta)[1 - (1 - p_j^\diamond)\delta\alpha_j]}. \end{aligned} \quad (88)$$

Disciplining by credit rationing is preferred by entrepreneurs of quality group j over the repeated stage game solution if $TU_j^* < TU_j^\diamond$ which simplifies to the condition

$$U_j(\theta_j^*, p_j^*) < \frac{U_j^s}{1 - (1 - p_j^\diamond)\alpha_j}. \quad (89)$$

Lemma II.4 proofs that there is always disciplining by credit rationing that is superior to the repeated stage game equilibrium for entrepreneurs of generic group G_1 . And there also is superior disciplining in a wide parameter space, albeit not always, for entrepreneurs of a group $j \in G_2$, as proven in Lemma II.4 and illustrated by Fig. 15 in which curly braces mark the p_j^* space where disciplining by credit rationing is clearly preferred. Figure 16 finally shows the new equilibria that can emerge with disciplining by credit rationing for different generic groups in comparison to the stage game equilibria.

Lemma II.4. $TU_j^* < TU_j^\diamond$ is always true for $j \in G_1$. For $j \in G_2$, $TU_j^* < TU_j^\diamond$ is true if $e_j''(p_j^*) < 1/[(p_j^*)^2 - (p_j^*)^3]$.⁴⁴

Proof. The stage game financing equilibrium, (r_j^*, p_j^*) , is characterized by no disciplining, that is, zero consequence of default and we therefore have $\lim_{(r_j^\diamond, p_j^\diamond) \rightarrow (r_j^*, p_j^*)} TU_j^\diamond = TU_j^*$.⁴⁵ Thus, if the total utility of disciplining by credit

⁴⁴The condition for generic groups G_2 captures most and the most reasonable, albeit not all, of the cases where $TU_j^* < TU_j^\diamond$ is true. There are some $p_j^\diamond > p_j^b$ for which disciplining by credit rationing is also superior to the repeated stage game equilibrium for $p_j^* \in [p_j^a, p_j^b]$.

⁴⁵Expected consequences of default shift an entrepreneur's best response function upwards. Thus, there are two intersection points of the entrepreneur's shifted best response function and banks' offer curve. With the same expected consequences of default, the upper intersection point (that is, the one at greater p) provides greater utility than the lower one since moving along banks' offer curve towards the lower intersection point (which is always smaller than the tangency point p_j^c) results in a decreasing base utility.

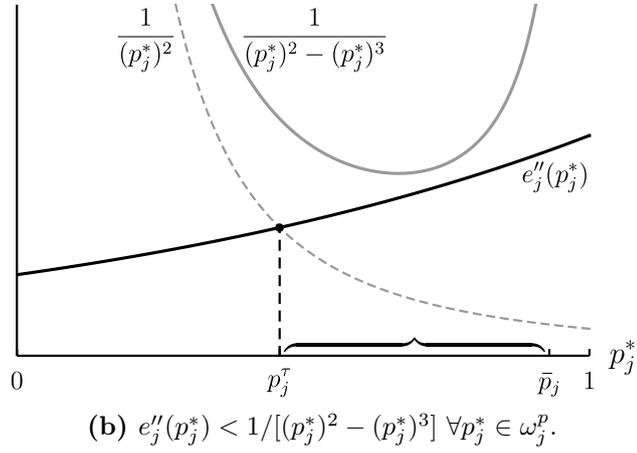
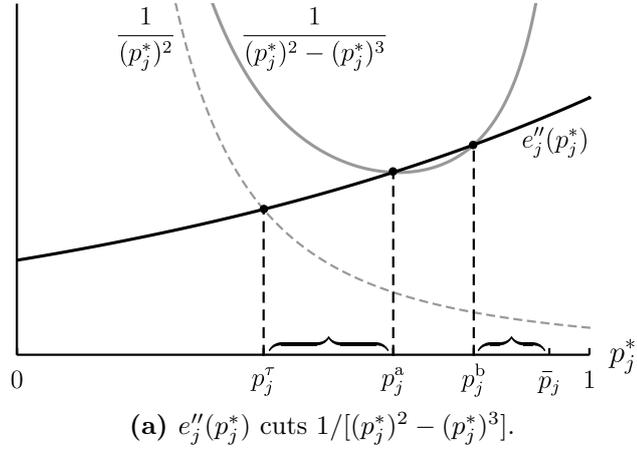


Figure 15: Illustration of Lemma II.4: p_j^* space (curly braces) where $TU_j^* < TU_j^\diamond$ for generic groups G_1 and G_2 .

rationing (that is, the right-hand side of Eq. (89)) has a positive slope in p at the point (r_j^*, p_j^*) , $TU_j^* < TU_j^\diamond$ is clearly true. Calculating the first partial derivative of $U_j^s/[1 - (1 - p_j^\diamond)\alpha_j]$ at p_j^* and considering that $\sigma_j(p_j^*) = 0$ gives the following condition for a positive slope of TU_j^\diamond at p_j^* :

$$R_j - e'_j(p_j^*) > (1 - p_j^*) [e''_j(p_j^*) - 1/(p_j^*)^2]. \quad (90)$$

For the stage game equilibrium applies $R_j - e'_j(p_j^*) = 1/p_j^*$. Equation (90) hence becomes

$$e''_j(p_j^*) < \frac{1}{(p_j^*)^2 - (p_j^*)^3}. \quad (91)$$

Since $1/[(p_j^*)^2 - (p_j^*)^3] > 1/(p_j^*)^2$ and $1/(p_j^\tau)^2 = e_j''(p_j^\tau)$, the condition is always satisfied for $p_j^* = p_j^\tau$, that is, for entrepreneurs of generic group G_1 . For $j \in G_2$, it is $p_j^* > p_j^\tau$ and we need further analysis. $e_j''(p)$ intersects $1/(p^2 - p^3)$ twice, tangentially once, or never in the domain $p \in (0, 1)$. Denoting these intersections p_j^a and p_j^b , where $p_j^a \leq p_j^b$, we can state that Eq. (91) holds if $p_j^* \in [p_j^\tau, p_j^a)$ or $p_j^* \in (p_j^b, \bar{p}_j)$. If there is no intersection, Eq. (91) holds for every $p_j^* \geq p_j^\tau$. Figure 15 illustrates. ■

By comparing the disciplining equilibrium with the repeated stage game equilibrium we can assess the efficiency of disciplining by credit rationing. For entrepreneurs of generic group G_0 , disciplining can Pareto improve the market efficiency by overcoming financing market failure and partially healing underinvestment in projects that occur without disciplining. Overinvestment in projects due to disciplining by credit rationing does not happen since disciplining is never feasible if financing is not possible in the first best solution. This is true since the feasibility constraint of disciplining by credit rationing is strictly stricter than the condition for financing in the first best world.

In cases in which financing is possible without disciplining (in generic groups G_1 and G_2), disciplining by credit rationing can Pareto improve the market outcome and heal underinvestment in effort since effort levels are always higher in comparison to the (repeated) stage game equilibrium. Depending on the parameter set of the market segment, entrepreneurs' effort can be increased closer to the first best level, even reach the first best level, or sporadically overshoot it. This results in lower default probabilities due to disciplining by credit rationing. Proposition II.5 subsumes.

Proposition II.5 (Efficiency of Disciplining by Credit Rationing). *Disciplining by credit rationing can Pareto improve the efficiency of the financing market and reduce default probabilities in presence of comprehensive credit information sharing. It can, first, overcome market failure and thereby mitigate underinvestment in projects (for $j \in G_0$) and, second, mitigate underinvestment in effort (for $j \in G_1 \cup G_2$).*

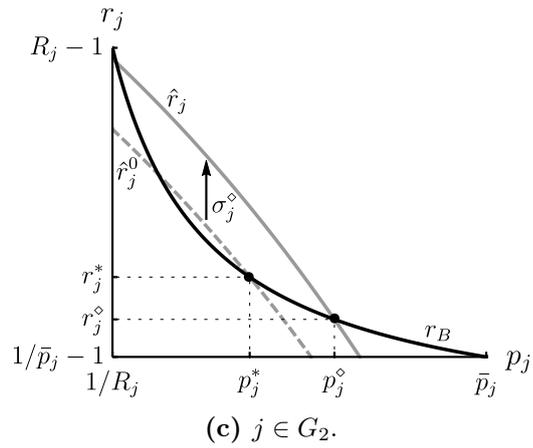
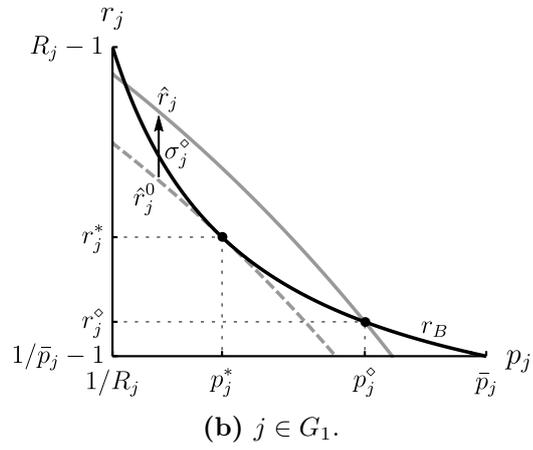
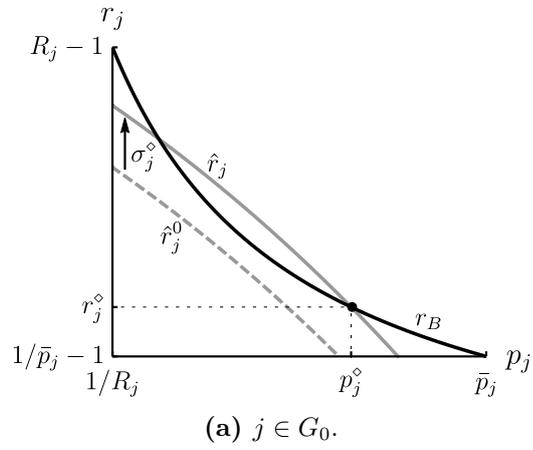


Figure 16: Best response functions and financing equilibria with and without disciplining by credit rationing for different generic groups G .

The positive effects of disciplining by credit rationing are achieved at the price of equilibrium credit rationing in the sense of Stiglitz and Weiss (1981): whenever entrepreneurs are punished with lower credit volumes after default, they are willing to borrow more at the same and even at a higher credit rate, and banks actually do lend more to quality-wise identical entrepreneurs,⁴⁶ but rationing is yet stable in equilibrium. However, aggregate credit rationing in the market (or its segments G) is, in the context of my model, shown to be connected to improvements of the market's efficiency. Credit rationing is therefore, as in de Mesa and Webb (1992), not a case for government intervention, contrary to a substantial previous literature on credit rationing (Jaffee and Russell, 1976; Keeton, 1979; Stiglitz and Weiss, 1981, 1983; Williamson, 1986).

4 Limitations of Disciplining by Credit Rate Tightening

Instead of rationing the credit volume, banks could also tighten the credit rate for the next loan as a disciplining consequence after an entrepreneur's default. Higher credit rates actually are the only possible unfavorable consequences of default in Vercammen (1995) and Padilla and Pagano (2000).

Let us regard the credit volume to remain unchanged to isolate the effect of credit rate tightening. Due to competition, the maximum credit volume of 1 monetary unit will therefore be offered whenever financing is offered at all, that is, $\lambda_j(h) = 1 \forall h, j$. For a credit contract $(1, r_j^\triangleright(h))$ offered to entrepreneurs of quality group j after history h , banks break even if entrepreneurs choose the corresponding success probability $p_j^\triangleright(h) := p_B[r_j^\triangleright(h)]$. But entrepreneurs only choose the required effort level $e_j[p_j^\triangleright(h)]$ if it satisfies their best response function in Eq. (74), that is, if they believe about future credit rates manifest in consequences of default of $c_j^\triangleright(h) = 1 + r_j^\triangleright(h) - R_j + e'_j[p_j^\triangleright(h)]$. From Eq. (76),

⁴⁶Entrepreneurs of the same quality obtain greater credit volumes than others if they either did not default in the previous project, or they are characterized by a different (not necessarily greater) number of previous consecutive defaults.

an equilibrium requires that

$$c_j^\triangleright(h) = 1/p_j^\triangleright(h) - R_j + e'_j[p_j^\triangleright(h)] \quad \forall h. \quad (92)$$

According to the equilibrium condition, equilibrium pairs $(r_j^\triangleright(h), p_j^\triangleright(h))$ always lie on banks' best response function $r_B(p_j)$. Since $r_B(p_j)$ is monotonic decreasing in p_j , a tighter credit rate results in a lower effort level and therefore leads to a higher probability of default. This constitutes a conceptual problem of disciplining by credit rate tightening. Increasing the credit rate as disciplining punishment reduces the incentives to perform of defaulting entrepreneurs in comparison to succeeding ones of the same quality. Entrepreneurs therefore become more likely to default again if they have defaulted once.

Another limitation of disciplining by credit rate tightening is that it fails to be feasible in many cases where disciplining by credit rationing is feasible. This is illustrated in the following. Positive expected consequences of default shift entrepreneurs' best response functions upwards. Let us define the newly arising intersection points of entrepreneurs' and banks' best response functions to be at $r_j^{\text{lo}}(h)$ and $r_j^{\text{hi}}(h)$, where $r_j^{\text{lo}}(h) \leq r_j^{\text{hi}}(h)$. Both of these credit rates can be achieved for the same expected consequences but after a history h^s the smaller credit rate $r_j^{\text{lo}}(h)$ (that corresponds to a higher success probability) provides a greater base utility to entrepreneurs and therefore dominates the larger credit rate $r_j^{\text{hi}}(h^s)$. Thus, in equilibrium we have $r_j^\triangleright(h^s) = r_j^{\text{lo}}(h^s)$ for the corresponding $c_j^\triangleright(h^s)$.

Due to the assumption that disciplining consequences are only effective if they occur in the very next period after a default, in a competitive financing market the same loan contract is offered to and chosen by every entrepreneur of the same quality group j after a successful project. The project utility of entrepreneurs of the same quality group j is therefore uniform after every

history h^s and given by

$$\bar{U}_j^\triangleright := U_j[(1, r_j^\triangleright(h^s)), p_j^\triangleright(h^s)] = p_j^\triangleright(h^s) R_j - 1 - e_j[p_j^\triangleright(h^s)] = \text{const. } \forall h. \quad (93)$$

Let us call \bar{U}_j^\triangleright base utility of entrepreneurs of quality group j in case of disciplining by credit rate tightening. If a default occurs after a history h^s , the expected consequences of default $c_j^\triangleright(h^s)$ are realized in the next period if banks offer a tightened credit rate $r_j^\triangleright(h^{sf})$ such that the project utility after default satisfies $U_j[(1, r_j^\triangleright(h^{sf})), p_j^\triangleright(h^{sf})] = \bar{U}_j^\triangleright - c_j^\triangleright(h^s)$. For $n \in \mathbb{N}^0$ consecutive defaults subsequent to a history h^s , we can write the general relation between the consequences $c_j^\triangleright(h^{sf^n})$ that are expected to take effect in the next period in case of defaulting in the current project and the credit rate $r_j^\triangleright(h^{sf^{n+1}})$ that actually brings these expected consequences into effect after a default as

$$c_j^\triangleright(h^{sf^n}) = \bar{U}_j^\triangleright - U_j[(1, r_j^\triangleright(h^{sf^{n+1}})), p_j^\triangleright(h^{sf^{n+1}})]. \quad (94)$$

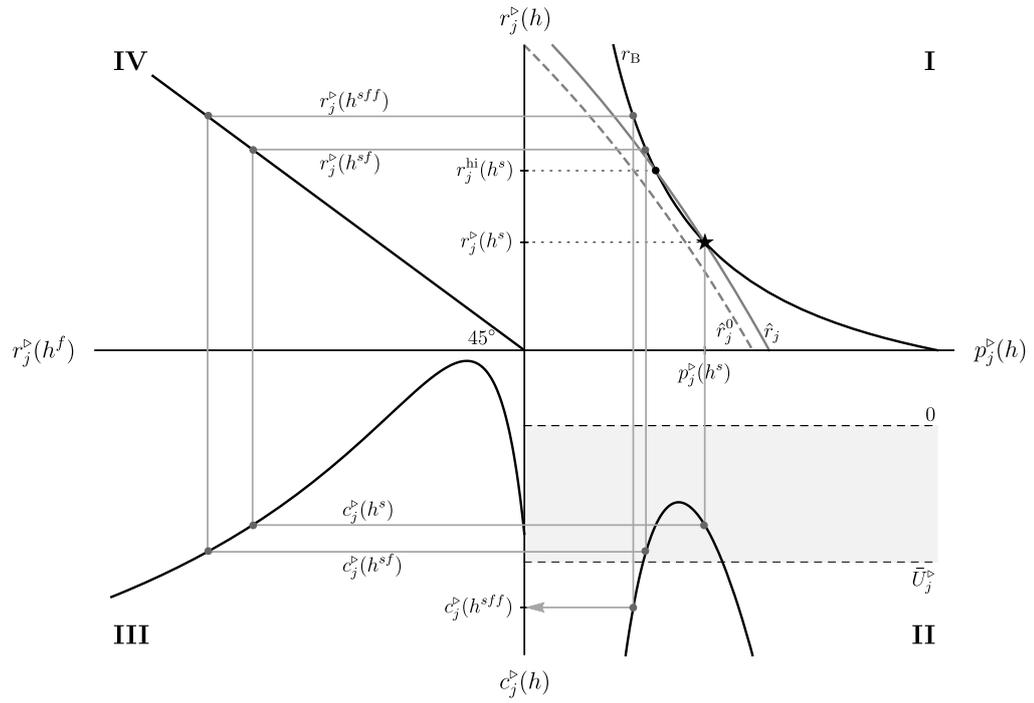
To be feasible, disciplining by credit rate tightening obviously requires that $0 < c_j^\triangleright(h^{sf^n}) \leq \bar{U}_j^\triangleright \forall n$.

For the same credit contract terms $(1, r_j)$ after a history h^s , it is $p_j^\triangleright(h^s) = p_j^\diamond$ and the required consequences of default are the same for disciplining by credit rate tightening and disciplining by credit rationing: it is $c_j^\triangleright(h^s) = \sigma_j^\diamond$ and also $\bar{U}_j^\triangleright = U_j^s$. If $0 < c_j^\triangleright(h^s) = \sigma_j^\diamond \leq \bar{U}_j^\triangleright = U_j^s$, disciplining by credit rationing is feasible, as I have shown previously, because the punishment after the first default is the most severe. This is, however, not generally true for disciplining by credit rate tightening. If the punishing credit rate $r_j^\triangleright(h^{sf})$ that implements the expected consequences $c_j^\triangleright(h^s)$ after a history h^{sf} is characterized by $r_j^\triangleright(h^{sf}) > r_j^{\text{hi}}(h^s)$, the expected consequences after the second consecutive default need to be greater than those after the first, that is, $c_j^\triangleright(h^{sf}) > c_j^\triangleright(h^s)$. This follows from the fact that the previous consequences result in an intersection point of the best response functions at $r_j^{\text{hi}}(h^s)$ and increased expected consequences are

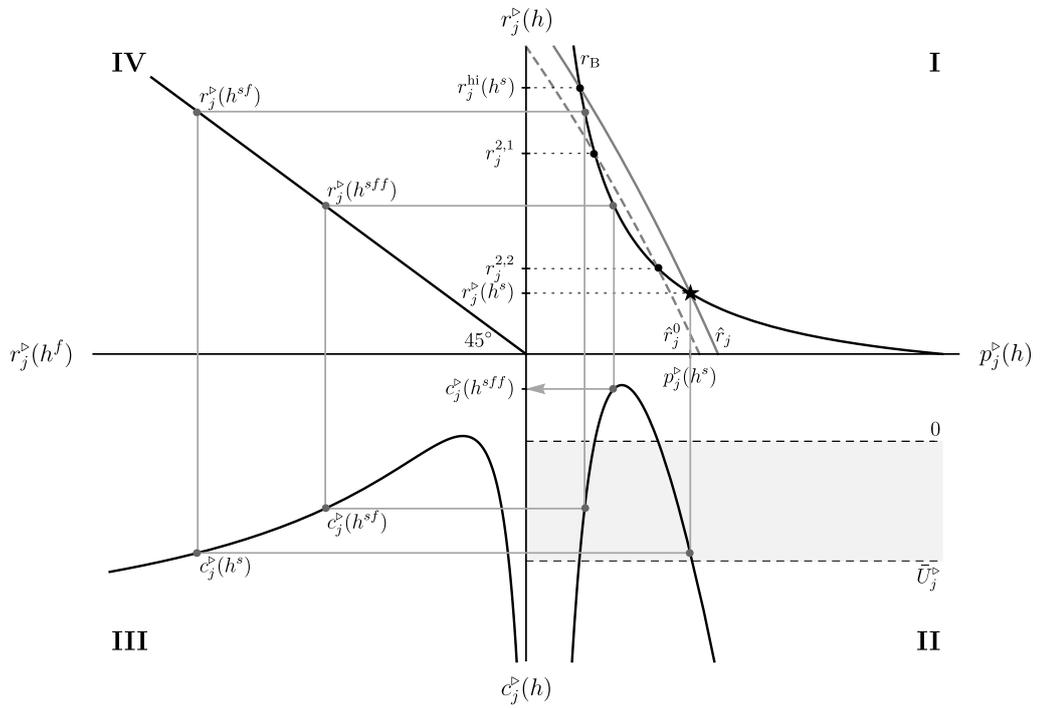
required to shift entrepreneurs' best response functions further upwards to give an intersection at a larger credit rate, as required in equilibrium. This process goes on: greater consequences of default result in an even tighter credit rate which in turn requires even greater consequences after a further default, and so forth. The consequences of default do in general not converge in the number of consecutive defaults for reasonable effort functions. This leads to a punishment excess for consecutive defaults that lets disciplining by credit rate tightening break down while disciplining by credit rationing is feasible.

Figure 17a illustrates this punishment excess in a four-quadrant visualization. Quadrant I shows the local equilibrium condition after a history h by means of the best response functions of the bank (solid black) and the entrepreneurs without (dashed gray) and with disciplining by credit rate tightening (solid gray). For a credit rate $r_j^\triangleright(h)$ after a history h , the required success probability $p_j^\triangleright(h)$ is obtained. Quadrant II gives the corresponding expected consequences $c_j^\triangleright(h)$ according to the equilibrium condition in Eq. (92). The lightgray filling marks the space where the feasibility constraint, $0 < c_j^\triangleright(h) \leq \bar{U}_j^\triangleright$, is locally⁴⁷ satisfied. The punishing credit rate, $r_j^\triangleright(h^f)$, that realizes the expected consequences after a default is given by quadrant III following Eq. (94). Quadrant IV transfers the tightened credit rate back to quadrant I where the process begins anew. Starting at the black star in quadrant I for a given credit contract $(1, r_j^\triangleright(h^s))$ after a history h^s , the solid lightgray lines guide the way through the quadrants. It can be seen how implementing consequences of default requires even greater consequences of further defaults at a growing rate. This punishment excess finally results in required consequences that are too large to be implemented as they leave the lightgray feasibility space in quadrant II. Hence, disciplining by credit rate tightening fails to be feasible for the given contract. Disciplining by credit rationing, however, works since $\sigma_j^\diamond = c_j^\triangleright(h^s)$ apparently satisfies the feasibility constraint.

⁴⁷I call it 'locally' because the graph shows the feasibility constraint only after one history, but feasible disciplining by credit rate tightening requires satisfaction after every history.



(a) Punishment excess.



(b) Punishment undershooting.

Figure 17: Limitations of disciplining by credit rate tightening in four quadrants.

The limitations of disciplining by credit rate tightening are particularly severe for entrepreneurs of generic group G_2 . Since negative punishment after default is neither reasonable nor possible, an additional constraint for the punishing credit rate is given by $r_j^\triangleright(h^{sf^n}) \notin [r_j^{2,1}, r_j^{2,2}] \forall n$. That means that the tightened credit rate has to satisfy $r_j^\triangleright(h^s) > r^{2,1}$ after a history h^{sf} ,⁴⁸ which requires immense punishment after default while, as shown before, disciplining by credit rationing is always feasible for little consequences in case of $j \in G_2$.

But even if this constraint is met for $j \in G_2$, disciplining by credit rate tightening is still very prone to failing. Whenever punishment excess is avoided by $r_j^\triangleright(h^{sf}) < r_j^{\text{lo}}(h^s)$, with the same argument as before we can state that now the expected consequences after the second default need to decrease to induce an intersection of the best response functions. Hence, the required punishing credit rate, $r_j^\triangleright(h^{sf^n})$, can (and in many cases does) run between $r^{2,1}$ and $r^{2,2}$ for some n and thereby undershoots the possible punishment which induces failure. This is illustrated by the four-quadrant visualization in Fig. 17b which also shows the discussed requirement of immense initial consequences. The graph is to be understood exactly like Fig. 17a. It can be seen how, for a given contract (marked by the black star), punishment undershooting leads to a failure of disciplining by credit rate tightening, while disciplining by credit rationing is apparently feasible.

5 Policy Implications for Credit Reporting Systems

Important parameters of credit reporting systems are, first, the type and the level of detail of the information included in the registry, second, the participants of the credit reporting system that provide and access the information and, third, the transparency of the first two parameters.

⁴⁸A rate $r_j^\triangleright(h^s) < r^{2,2}$ is not feasible since best response functions intersect at $r^{2,2}$ for zero consequences, and $\bar{U}_j^s - c_j^\triangleright(h^s)$ decreases in $p_j^\triangleright(h^s)$. Hence, from Eq. (94), increasing expected consequences of default result in increasing $r_j^\triangleright(h^s)$.

To allow for disciplining as presented in my model, credit reporting apparently needs to include default information as well as contract-specific data like credit volumes or credit rates. Actual credit registries usually contain contract-specific information but there is few previous literature that can explain why lenders share these data instead of only sharing information concerning borrower quality (like payment and default information). An exception is Bennardo et al. (2015) who show that the possibility of multi-bank lending can produce incentives to overborrow which can be mitigated by sharing contract-specific information about past debts. My work provides a different and very direct reason to share such information since setting up and maintaining disciplining has been shown to require specifics about previous contracts.

My model demonstrates that disciplining is not compromised if the registry contains more information than required and it even works for comprehensive registries. Overly informative credit registries are therefore noncritical for disciplining. The previous theoretical literature about borrower discipline due to credit information sharing focuses on reputation effects and finds that too much information can lead to inferior outcomes since reputation effects diminish with an increasing informativeness of credit registries (Vercammen, 1995; Padilla and Pagano, 2000).⁴⁹ It is concluded that the amount of quality-relevant information that is shared is a critical element to the design of credit registries. Hence, to save reputation effects from diminishing and finally disappearing when the credit registries mature and become more comprehensive, the literature suggests to reduce the informativeness of credit registries by partially preventing access to credit histories (Vercammen, 1995) or by randomizing credit information sharing (Padilla and Pagano, 2000). However, artificially confining the informativeness of credit registries in the suggested ways comes with problems. First, other positive effects of credit information sharing are hurt, namely the mitigation of adverse selection (Pagano and Jappelli, 1993) and hold-up problems (Padilla

⁴⁹The negative effect of too much information has also been obtained by Diamond (1991) and Crémer (1995) in different frameworks.

and Pagano, 1997). Second, partial information sharing can be used by banks as a strategic tool to reduce banking competition since new entrants into the banking market are particularly affected by confined credit registries and therefore prevented from competitive bidding for some borrowers (Bouckaert and Degryse, 2006).

Participants of credit reporting systems need not only be banks. The credit registry can also be opened to non-lenders. A broader access to registries is of economic value if the participants of the credit reporting system mutually contribute to the registry to quickly obtain extensive, more detailed, and differentiated information. This leads to efficiency-enhancing mitigation of adverse selection and hold-up problems in different sectors, but also to quickly diminishing reputation effects. A broad access to credit registries can be problematic if the registry contains only scarce information. In that case, the reputation effect can be very strong and easily become too strong. If numerous participants of different sectors evaluate a borrower's reputation on the basis of her loan repayment, this can lead to inefficient underinvestment and even prevent financing entirely. To control the magnitude of the total reputation effects seems non-trivial.

If, however, a broadly accessed credit registry is informationally confined in order to preserve reputation effects, problems related to those in Bouckaert and Degryse (2006) can arise, as outlined in the following. Borrowers' incentives to perform can be increased either by reputation effects or by disciplining. While reputation effects have the huge problem to diminish with more information, disciplining is robust to information but is more expensive than "passive" reputation effects because it requires to implement consequences of default. If these consequences are implemented with deteriorating credit terms, they reduce banks' profits and business volumes after defaults.⁵⁰ Consequently, it would be in the interest of banks to outsource costly disciplining, which would

⁵⁰Although disciplining by adjusting next period's credit terms can increase market efficiency, as shown in my model, it is not perfectly efficient in increasing the efficiency.

require consequences that are exogenous to the credit business and therefore not lead to a deterioration of lending terms after default.⁵¹ One can think of two basic ways to achieve this. First, by building on non-monetary social costs. However, these can usually only occur in very special cases, for example in joint-liability microlending (Stiglitz, 1990; Varian, 1990; Besley and Coate, 1995). Second, by broadening access to credit registries and have others, in particular non-lenders, implement consequences of loan defaults.

To execute the second point, banks need to artificially confine the broadly shared information such that the resulting reputation effects constitute consequences of default. This can be satisfied by a confinement that preserves reputation effects and makes them take effect after the undesirable event of a loan default, which is in line with “black-listing” practices to share only negative information and to strictly limit the memory of the registry to current entries. To subsume, banks can have incentives to use confined information sharing as a strategic tool to cheaply create exogenous consequences of default. In this way, banks outsource consequences of default to others by using credit reporting as a system of finger-pointing on defaulters.

The discussed problems can be prevented by policies that do not confine credit reporting artificially but, instead, promote positive and negative credit reporting to obtain a more complete picture of borrowers, and that foster a level playing field for all participants of credit reporting systems.⁵² A level playing field requires, for instance, absence of discrimination towards particular participants of the credit reporting system, that is, equal access for all participants to read information and also to supplement data. Then, more differentiated and accurate information about the data subjects arise in greater pace. This can

⁵¹Another shortcoming of disciplining is solved by exogenous consequences: consequences of default affect borrowers asymmetrically. That is, tighter credit rates are not paid if the borrower defaults again, and scaled-down project returns due to rationed credit are anyway not earned if the project fails again. Thus, if a defaulting borrower defaults again, she is generally not at all or much less affected by the consequences of the previous default than in case of a success, which partially opposes the desired effect of disciplining.

⁵²See also World Bank (2011) for a discussion.

both mitigate the possibility to exploit credit registries for strategic reasons and reduce market inefficiencies due to asymmetric information.

My work also implicates that consequences of default to discipline borrowers can only be effective if they are expected by borrowers, and they are efficient if the expectations are sufficiently accurate. Transparency about the extend of information sharing and the access to registries is therefore essential to enable borrowers to form accurate believes about the total consequences of default.

6 Conclusion

The existing literature shows that credit information sharing between lenders disciplines borrowers and increases their incentives to perform because defaulting with one lender ruins the reputation with every other lender. This reputation effect, however, is fragile since the existence and the extent of the effect heavily depend on and vary with the particular information set that lenders possess about particular borrowers. A new piece of data affects lenders' assessment of a borrower's quality and therefore her reputation less, the more comprehensive lenders' information about the borrower is.

The purpose of this work is to show that credit information sharing can induce borrower discipline beyond "passive" reputation effects by actively using the information shared. I therefore provide a different approach to discipline borrowers on the basis of credit information sharing: classical disciplining.

In a multi-period model of repeated lending, I show that disciplining is feasible and can reduce problems of moral hazard by increasing borrowers' incentives to perform. Disciplining can Pareto improve the efficiency of the financing market and reduce defaults by overcoming market failure and mitigating underinvestment in projects and in effort. But unlike reputation effects, disciplining is robust to the level of informativeness of credit registries (as long as the required information is included) and even works for comprehensive and unrestricted

credit registries. Hence, to complement and compensate reputation effects with disciplining seems to be a better and more efficient approach than, as suggested in the literature, preventing reputation effects from diminishing by constraining or randomizing credit information sharing which provokes inefficiencies on other levels.

The model also yields results beyond its main purpose. First, disciplining requires contract-specific data of past debts and can therefore provide a direct explanation why this information is actually shared in addition to quality-relevant information (Bennardo et al., 2015). Second, disciplining borrowers by *pro rata* rationing their next credit after a default shows to be more promising than tightening the credit rate since the latter reduces borrowers incentives to perform after a default and can lead to punishment excesses. This result is a possible explanation for the observation that banks rather change credit availability than credit rates when their costs of lending shift (Petersen and Rajan, 1994). Third, disciplining by credit rationing constitutes aggregate equilibrium credit rationing in the sense of Stiglitz and Weiss (1981). But since it enhances the efficiency of the market, my model provides a rare case of efficient equilibrium credit rationing that is not a case for government intervention.

Future research could compare different concept of disciplining in more detail and in different frameworks. On the one hand, the problems of credit rate tightening could be studied in more depth. On the other hand, new disciplinary devices like tightening collateral requirements could be analyzed. Future empirical work could, first, try to distinguish different effects of credit information sharing and, second, endeavor to filter reputation effects in order to distinguish them from, for instance, disciplining. That could also enable to discriminate between efficient equilibrium credit rationing, as found in my model, and inefficient rationing.

Epilogue

My thesis studied contemporary issues in relationship banking and emphasized the importance of information for the lending business. The first part was motivated by the tremendous problems that lax and uninformed lending to households caused in the recent past, as unveiled by the financial crisis of 2007/2008. I argued that household lending can be improved if banks engage in saving relationships with these households prior to lending since the information that can be gathered in saving relationships is of high relevance for the lending business. Individuals who manage to save on a regular basis should likely be able to regularly repay debt. Although my work concentrates on saving relationships, related arguments can be brought forward for transaction accounts. Information about spending behavior provide insight into households' capabilities of managing and planing their finances, which should significantly help to assess their borrower qualities.

The model I developed in Part I showed that savings-linked relationship lending can improve the efficiency of the financing market and mitigate adverse selection problems. I found this in particular to be the case for housing finance which was a major source of problems in the recent past. I identified *Bausparen*, a product of housing finance that performed well through the crisis, to contain the concept of savings-linked relationship lending. Hence, *Bausparen* shares in this respect the efficiency that I found for savings-linked relationship lending in the model.

The current low-interest period has led *Bausparkassen*—Germany's special institutes for *Bauspar* contracts—into an existential crisis. That, however, does not show that savings-linked relationship lending has failed. The problems have rather arisen from other specifics of *Bausparen*. First, the interest rate hedging (which was the main focus of the previous literature about *Bausparen*) that *Bauspar* contracts provide have become very costly for old contracts with

comparably high interest on savings. At the same time the demand for *Bauspar* loans has dried-out since fixed credit rates from old contracts have not been capable of competing with low market rates. Second, the investment universe of German *Bausparkassen* is, besides making *Bauspar* loans, legally limited to investment-grade securities which have earned hardly any return over the past years. Third, trying to juridically tackle the aforementioned problems by canceling expensive *Bauspar* contracts and by fighting for regulatory easing have led to a severe reputational damage for *Bausparkassen*.

All the signs are that *Bausparen* and *Bausparkassen* are forced to change permanently. My work emphasizes that savings-linked relationship lending is an economically meaningful element in the concept of *Bausparen* which is worth preserving in this change. Even more, learning about borrowers in prior relationships could be executed in a more rigorous way using state-of-the-art data warehousing and information technology.

Bausparkassen are not exclusively faced with forces to adapt. The whole banking industry is currently undergoing a tremendous change. Besides policy issues like extended banking regulation, the change is mainly driven by a severe damage of trust in traditional banks,⁵³ a new social awareness of the banking business, changing customer needs, and new technology. These factors mitigate market entry barriers and thereby allow for increased competition in the financial services industry. Numerous new players in the banking sector focus on technology to improve banking services and customer experiences, and to capture market segments that traditional banks refuse to serve. Technology is intensively used to establish information-based lending. That is, credit-relevant information about borrowers is produced and analyzed using new methods and technologies to obtain improved assessments of borrower qualities.

Extrapolating this trend, credit-relevant information about loan-demanding

⁵³This is due to numerous unveiled cases of opportunistic, excessive, unethical, and even fraudulent and corrupt behavior of national and international banks.

agents will be produced in growing quantity and with increasing sophistication. Advanced credit reporting systems will collect, connect and process this information with increasing pace and reliability. Due to this development, credit registries can potentially become more and more comprehensive and informative, which reduces the efficiency-enhancing reputation effects of credit information sharing that can induce incentives to perform. In times of vast technological and informational progress, it seems hardly possible to restrict the informativeness of credit registries to preserve reputation effects, as recommended in the literature. Contrary to these suggestions, Part II of my thesis provided a different and probably more viable approach to increase borrower discipline on the basis of credit information sharing: classical disciplining. I showed that disciplining can reduce problems of moral hazard and increase borrowers' incentives to perform in spite of highly comprehensive and unrestricted credit information sharing. From my work follows that unconfined credit reporting, a level playing field, and a high level of transparency are in support of disciplining and also provoke efficiency of credit information sharing while preventing exploitation. These properties of credit information sharing also foster ethical responsibility for which a world of increased information production and storage will call.

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