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Abstract

I estimate a Gaussian two-factor affine term structure model of bond yields for three countries, the United States, the United Kingdom and Germany. I find a considerable time-varying component of excess returns in the data. They are positively correlated with the slope of the term structure and negatively with the short-term policy rate. In addition, the panel clearly indicates to co-movements in the same directions on an international level. When testing the estimated model for the expectations puzzle of the the term structure, at least at one end of the yield curve, this puzzle can be resolved when applying risk-adjusted yield changes.

Keywords: Term Structure, Term Premia, Kalman Filter, Maximum Likelihood.

JEL classification: G12, G15, E43.

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1 Introduction

In recent years, the concept of term premia has become a focus of attention for academics, policy makers as well as the investment community. This heightened interest was initially triggered by the puzzling behavior of long-term interest rates in the Unites States and in other industrialized countries (Greenspan, 2005). The interest-rate conundrum manifested itself in stable and even falling long-term bond yields despite a reversal in the short-term fed funds cycle. Over the period between June 2004 and February 2005, the FED decided to increase the target rate by over 120 basis points. Over the same time, the 10-year treasury rate lost temporarily over 100 basis points. Among the global saving glut, declining inflation expectations, reduced global macroeconomic and financial uncertainty that were cited as explanations attempts, shrinking bond term premia were the most promising fact to capture the conundrum within a coherent macroeconomic framework (Backus and Wright, 2007; Rudebusch et al., 2006; Kim and Wright, 2005).

The financial crisis, starting in the middle of 2007 and evoking distressing parallels to the Great Depression in the 1930s, serves as further indication for the increased sensitivity for overall risk attitudes. In particular, the explicit mentioning of risk and term premia has become commonplace in policy discussion, within central banks and its communication with the public.[1] In this respect, policy makers agree on an overall assessment of the causes of the crisis as being triggered by an heightened risk tolerance and appetite of market participants.[2] An environment of low interest rates promoted banks and other financial intermediaries to invest in more risky positions in 'search for yields' across a wider class of assets by making on the carry. In particular, the whole financial industry imitated the traditional banking model of 'maturity transformation' and expanded it to the 'originate and distribute' version of banking in which loans are pooled, tranched

[1] See, for instance, Kohn (2005); Bernanke (2006); Plosser (2007); Trichet (2008) among others for the United States and the euro area.

[2] For example, the Bundesbank identifies four main channels through which the financial crisis spread: (i) recklessness in securitization (ii) low risk perception (iii) slack lending standards and (iv) high credit expansion in the aftermath of 2003 (Zeitler, 2009).

and resold via securitization. The outcome could be documented in highly leveraged borrowing, in soaring asset prices and diminishing required risk premia on part of the market participants across the whole set of asset classes (Adrian and Shin, 2008).

The interest in term premia also coincides with developments in academia. By improving on the estimation and modeling of bond yields and corresponding term premia in asset markets, progress has been made in combining methods in financial and monetary economics in order to disentangle the structural macro sources of time-varying term premia. Since term premia have a great deal to do with private-sector's expectations of the future risk-neutral payoff of securities, identifying them relies on a model according to which agents can form these expectations. However, there is still no consensus how to measure or even define term premia since the latter heavily relies on model specification criteria and estimation techniques (Swanson, 2007). Still, measures of term premia provide many macroeconomic linkages. For instance, they seem to be counter-cyclical so that they can be used to predict changes in real economic activity (Hamilton and Kim, 2002; Favero et al., 2005; Ang et al., 2006).

In what follows, this paper concentrates on a term premium concept stemming solely from interest-rate risk. It originates from the uncertain path of future interest rates and, thus, risky payoffs provoke risk-averse investors to demand higher expected returns due to the danger of capital losses during the investment horizon. This sort of risk is the major concern when trading government securities. Liquidity risk and default risk are important extensions to interest-rate risk; they have showed up in particular in the aftermath of the financial crisis of 2007/08 and even in international government bonds markets. Still, I abstract from these issues and model overall interest-rate.

In order to review recent dynamics of international term premia, I build a two-factor affine term structure model that allows to estimate interest-rate risk premia and to forecast future interest rates along the yield curve. The focus is on three countries - the United States, United Kingdom and Germany covering the quarterly sample period 1970Q1:2008Q3. In contrast to most other research on term structure estimation, I use a model specification that holds across all three countries to make comparisons of the estimation results easier.

The decomposition of international yield curves confirms the presence of a large time-varying component of term premia. The latter are positively correlated with the slope of the term structure and negatively with the short-term policy instrument of the central bank (provided that the one-quarter rate is regarded as the policy rate). Moreover, they exhibit considerable co-movements in the same directions on an international level.

Tests of the expectations hypothesis of the term structure within the estimated models confirm the negative relationship between bond yield changes and the slope of the yield curve in the presence of large and time-varying term premia. If risk-adjusted yield changes are regressed onto the yield spread, the expectational puzzle of Campbell and Shiller (1991) can be to a large extend resolved.

The paper is structured as follows; Chapter 2 introduces the main idea of risk-neutral asset pricing that enables to separate term premia from observable bond yields. In a next, step I present the two-factor affine term structure model and how excess returns can be analytically derived from the model-implied yields. In Chapter 3, I estimate the model and provide results for the observed countries. Chapter 4 concludes.

2 Affine Term Structure Representations

2.1 General Set-up

The basic problem in term structure modeling is that market expectations about the path of interest rates are not observable. The future market might be seen as a good proxy but term premia might distort the information content of expectations. Separating expected interest rates from term premia is the main requirement for any yield curve model that tries to impose economically important no-arbitrage restrictions on the cross-movements of interest rates. Other yield curve specifications such as the Nelson-Siegel model are able to deliver a good fit as well as promising forecasting performances but they lack sound economic restrictions. One successful approach that stems from the finance literature and that has been growing very rapidly in recent years, are no-arbitrage term structure models. They rely on the general proposition that movements in the cross section of bonds yields are closely tied together.

The absence of arbitrage says that it is not possible to design a risk-free self-financing portfolio that yields more than the instantaneously return of the risk-free (short) rate within a time interval. Expected excess returns, then, are the result of explicit risk-taking. This means that arbitrage opportunities exists unless long-term bond yields are equal to risk-adjusted expectations of future short-term yields. The assumption of absence of arbitrage opportunities seems quite logical in bond markets in which arbitrage opportunities are traded away immediately and markets can be characterized as highly liquid. The so called affine dynamic term structure models (ATSM) are the most popular among the class of no-arbitrage term structure models. They are best tractable since they assume bond yields to be affine functions of a set of risk factors driving the whole yield curve. They enable to get closed-form solutions for interest rates and such models are maximally flexible to reproduce the moments of bond yields and excess returns. The pioneering work by Vasicek (1977) and Cox et al. (1985) consists of a particular simple form of an affine term structure model where the short-term interest rate is the single factor that drives the whole yield curve at one moment in time and where it describes comovements of bond yields of different maturities.

An equilibrium in such an affine framework requires that bond yields equal the path of expected risk-adjusted short rates. The most important implication of the absence of arbitrage is the existence of a positive stochastic process, with which all future payoffs are valued (Cochrane, 2001). The main task of modeling the term structure of interest rates is to find the evolution of the stochastic discount factor (SDF)- also called the pricing kernel- that allows to separate EH-consistent bond yields from term premia. The finance literature basically offers two ways how to specify the evolution of the SDF over time. They give the same functional form of the pricing kernel and for bond yields. If constructed appropriately, they should be equivalent to each other. The first way involves a direct specification of the evolution of the pricing kernel over time. Having pinned down the short-rate process and the market price of risk, it becomes possible to solve for the whole set zero-coupon bonds to construct the term structure of interest rates. The market price of risk describes the required excess return per unit of risk. It should be the same for all bonds and independent of time to maturity (Maes, 2004).

The second well established equivalent to the explicit specification of the pricing kernel in terms of its drift (short rate) and its volatility (market price of risk) is the concept of *risk-neutral pricing*. Central to this approach is that an asset's payoffs over its life are discounted by the uncertain future path of the riskless rate (the numéraire) where expectations are built as if agents are neutral towards financial risk. This implies that under this risk-neutral measure $\mathcal{Q}$, discounted bond price processes follow a martingale, i.e. $E_t^{\mathcal{Q}}[P_{n-T,t+T}P_{1,t+T}] = P_{n,t}P_{1,t}$ for all $T < n$ so that they are not predictable over time. The same is true for expected returns. What lies between expectations under the artificial, risk-neutral measure and the historical, data-generating measure[3] is again a specification for the market price of risk that captures agents' attitude towards risk. Deriving the pricing kernel with the help of the risk-neutral measure reveals that risk preferences of agents are implicit embedded in the pricing kernel as function of the state variables and in the change of the probability measure from the risk-neutral to the true measure (see for example Singleton, 2006, p.203).

The easiest and most intuitive way of thinking about risk-neutral evaluation is to recall the basic no-arbitrage asset pricing equation

$$1 + E_t^{\mathcal{P}}[R_{n,t+1}] = (1 + R_{f,t}) - \frac{cov_t(R_{n,t+1}, M_{t+1})}{E_t[M_{t+1}]}$$

[3] This measure is also referred to as the physical, true or actual measure.

which states that for any risky asset the expected return under the historical $\mathcal{P}$ measure equals the short rate plus a term that captures the covariance between the asset's return and the SDF. The risk-adjusted return of the bond with maturity n can be written as

$$
\begin{aligned}
1 + E_t^{\mathcal{P}}[R_{n,t+1}] + \frac{cov_t(R_{n,t+1}, M_{t+1})}{E_t[M_{t+1}]} &= (1 + R_{f,t}) \\
1 + E_t^{\mathcal{Q}}[R_{n,t+1}] &= (1 + R_{f,t}). \qquad (1)
\end{aligned}
$$

Taking the risk-neutral distribution for one-period returns basically means to shift the true distribution to the left. As a result, the risk-neutral pricing approach guarantees that all expected returns are equal to the risk-free return and agents price bonds as if they were risk-neutral due to zero expected excess returns. If so, risk-neutral pricing translates the distribution of the discounted asset price process to a martingale (random walk) by removing the predictable drift (mean).

The construction of a dynamic term structure models relies on several functional relations which allow to adequately price all bonds along the yield curve. These primary ingredients are (i) the *risk-neutral* time-series process of the state variables or risk factors, (ii) the *historical* time-series process for the state variables and (iii) the mapping between these risk factors and the short-term interest rate (Singleton, 2006). Together with an affine functional relationship between bond prices for any maturity n and the state variables, maturity-dependent parameter restrictions guarantee the absence of arbitrage. The parameter restrictions comprehend the parameters governing the relations between the state variables under both the physical and risk-neutral measure as well as the short rate equation. If these restrictions can be chosen to fulfill the basic asset pricing equation (1), then the guess for the solution function of bond prices has been correct. In fact, the guess-and-verify strategy of the parameter restrictions for bond yields ist actually the same method used in modeling the rational expectations equilibrium for difference equations in monetary economics. The method of undetermined coefficients suggests to guess a function of the state variables and to solve it according to the minimum state variable solution (McCallum, 1983).

In order to produce time-varying term premia, the literature basically works with two strategies which result both in a heteroscedastic model of the pricing kernel. Without this requirement, i.e. in case of a constant variance of the pricing kernel, the term structure would only produce constant excess returns. The first strategy allows for heteroscedastic risk factors (stochastic volatility). The conditional variance of theses factors are then mostly characterized by a square-root process of the factor themselves and translated into the pricing kernel.

The second way of modeling prices of risk is to generate time-varying term premia through state-dependent risk price parameters which are not driven by the conditional variance of the risk factors but by the state of the economy. The general formulation of Duffee (2002) and its division in a set of subfamilies introduced by Dai and Singleton (2003) include both strategies. It turns out that from an empirical perspective, constant volatility factors and stochastic prices of risk parameters perform best in fitting historical yield curve dynamics (Dai and Singleton, 2002, 2003).

The basic question is how to convert the risk-neutral measure to the historical measure (et vice versa). The finance literature shows this with Girsanov's Theorem in continuous time.[4] Following the work of Ang and Piazzesi (2003) and Singleton (2006), I rather specify the change of measure and the corresponding pricing kernel in discrete time. The starting point is the fact that future payoffs follow a stochastic process. Investors must form expectations and assign probabilities to the set of all possible events. In general, the density functions of a random variable under the risk-neutral and historical measure are $f_t^{\mathcal{Q}}(Z_{t+1})$ and $f_t^{\mathcal{P}}(Z_{t+1})$ respectively. The *Radon-Nikodym derivative* ξ_{t+1} of the $\mathcal{Q}$ measure with respect to the $\mathcal{P}$ measure satisfies

$$\frac{d\mathcal{Q}}{d\mathcal{P}} = \xi_{t+1} = \frac{f_t^{\mathcal{Q}}(Z_{t+1})}{f_t^{\mathcal{P}}(Z_{t+1})}.$$

[4] See for instance Baxter and Rennie (1996); Bingham and Kiesel (2004).

Since the random variable Z_{t+1} is a stochastic process over time, so must be the Radon-Nikodym derivative. If one wants to know the risk-neutral expectations at time t of a random variable in $t+1$, then the amount of change of measure during this time interval is just ξ_{t+1}/ξ_t so that the transformation between $\mathcal{Q}$ and $\mathcal{P}$ can be written as

$$E_t^{\mathcal{Q}}(Z_{t+1}) = E_t^{\mathcal{P}}(\xi_{t+1} Z_{t+1})\xi_t^{-1}$$

and ξ_{t+1} follows a log-normal process

$$\xi_{t+1} = \xi_t \exp(-0.5\lambda_t^\top \lambda_t - \lambda_t^\top \varepsilon_{t+1}). \tag{2}$$

The *Novikov condition*[5], implying that the variation in λ_t is finite, makes the derivative a strictly positive exponential martingale, i.e. it behaves like a random walk (see Appendix D Duffie, 2003). $\mathcal{Q}$ is an equivalent martingale measure of $\mathcal{P}$ since ξ_{t+1} is a martingale and so is Z_{t+1} under $\mathcal{Q}$. The conversion form the physical to the risk-neutral measure guarantees that under the risk-neutral measure, all expected asset prices and returns are not predictable. This is the basic assertion if investors are risk-neutral.

With this expectations relation in mind, the fundamental asset pricing theory shows that any zero-coupon bond is the presented value, discounted by the expected path of the risk-free interest rate under the risk-neutral measure $\mathcal{Q}$. Recall the basic asset pricing equation for n-period bonds, especially for a one-period bond

$$P_{n,t} = E_t^{\mathcal{P}}[M_{t+1} P_{n-1,t+1}]$$
$$P_{1,t} = E_t^{\mathcal{P}}[M_{t+1}].$$

[5] This condition formally states that $(0.5 \prod_{t=1}^{T} \lambda_t^\top \lambda_t < \infty)$.

The pricing kernel is the essential variable for pricing the sequence of bonds along the yield curve. In particular, if investors are risk-neutral, the pricing kernel is simply the negative exponent of the continuously compounded risk-free interest rate

$$M_{t+1} = e^{-i_{1,t}}. \tag{3}$$

For risk-averse investors, however, the pricing kernel is modified according to

$$M_{t+1} = e^{-i_{1,t}} \frac{\xi_{t+1}}{\xi_t} \tag{4}$$

Substituting (2) in (4) gives

$$M_{t+1} = e^{-i_{1,t}} e^{-0.5\lambda_t^\top \lambda_t - \lambda_t^\top \varepsilon_{t+1}}. \tag{5}$$

The pricing kernel is the negative of the short rate and its variance is the negative of the market price of risk, i.e. $var_t(M) = -\lambda_t^\top$. If λ_t is constant over time, the pricing kernel is homoscedastic and expected excess returns of n-period bonds are constant. Instead, if λ_t varies over time, bonds may exhibit changing expected excess returns.

Since the right-hand side of (5) is log-normally distributed with $\varepsilon_{t+1} \sim N(0, I)$, the pricing kernel for $t+1$ always equals the short rate so that

$$\begin{aligned}\log E_t^{\mathcal{P}}[M_{t+1}] &= E_t^{\mathcal{P}}[\log M_{t+1}] + 0.5 var_t(\log M_{t+1}) \\ &= -i_{1,t} - 0.5\lambda_t^\top \lambda_t + 0.5\lambda_t^\top I \lambda_t.\end{aligned}$$

Hence,

$$E^{\mathcal{P}}[M_{t+1}] = e^{-i_{1,t}}$$

and there is no source of uncertainty (risk) in the pricing kernel. Again, the risk-free asset does not load any risk since it is perfectly correlated with the SDF and its covariance is zero. Typically, an instantaneously maturing bond carries such an interest since there are no uncertain (stochastic) payoffs in the immediate next time interval. However, note, that short-term yields are random variables from the vantage point of date t, and $i_{1,t+i} (i > 0)$ might be correlated with the payoff stream, an asset generates. Risk-averse agents want to get compensated via market prices of risk λ_t if there is covariation between the path of expected short rates and future prices of zero-coupon bonds.

Any term structure model under the historical measure can be then expressed as

$$P_{n,t} = E_t^{\mathcal{P}} \left[\frac{\xi_{t+1}}{\xi_t} \exp(-i_{1,t}) P_{n-1,t+1} \right]$$

and using the recursion argument as

$$P_{n,t} = E_t^{\mathcal{P}} \left[\frac{d\mathcal{Q}}{d\mathcal{P}} \exp \left(\sum_{i=0}^{n-1} -i_{1,t+i} \right) \right]. \tag{6}$$

Similarly, under the risk-neutral measure, bond prices follow

$$P_{n,t} = E_t^{\mathcal{Q}} \left[\exp(-i_{1,t}) P_{n-1,t+1} \right]$$

and

$$P_{n,t} = E_t^{\mathcal{Q}} \left[\exp \left(\sum_{i=0}^{n-1} -i_{1,t+i} \right) \right]. \tag{7}$$

This section introduces the class of discrete affine term structure models (ATSMs) which have become increasingly popular among monetary economists.[6] It typically starts with a parametrization of the factor dynamics under the historical probability measure.

[6] See for instance, Ang and Piazzesi (2003); Rudebusch et al. (2007); Bekaert et al. (2006); Hordahl et al. (2006) among others.

Thereby, N factors follow a VAR process with stochastic volatilities of innovations. State vector dynamics $[X_{1,t}, X_{2,t}, .., X_{n,t}]^\top$ can be expressed

$$X_t = \mu + \phi X_{t-1} + \Sigma S_t \varepsilon_t \tag{8}$$

where Σ is a $N \times N$ constant, S_t is a diagonal matrix ("volatility matrix") describing the conditional variance of the factors and $\varepsilon_t \sim (0, I)$ are sources of risk. This general formulation allows the state vector to be homoscedastic or heteroscedastic. Dai and Singleton (2000) uniquely categorize the family of N-factor ATSMs into $N+1$ subfamilies $A_M(N)$ with N factors and M number of factors that are present in the conditional factor variances.

Evaluating (5), the law of motion of the short rate and the market prices of risk specify the pricing kernel. The short-term interest rate is given by

$$i_{1,t} = \delta_0 + \delta_1^\top X_t \tag{9}$$

so that it simply depends on a constant term and it is linear in the state variables. The parameter vector $\delta_1^\top$ with size $N \times 1$ represents the loadings on these unobservable factors X_t.

Now, I highly reduce the complexity of the model and present an $A_0(N)$ that is mostly applied in monetary economics.[7] It has the convenient feature that it works with constant volatility of risk factor dynamics, i.e. $M = 0$ and $S_t = I_{N \times N}$, but introduces time-varying excess returns via changing market prices of risk. The market price of risk vector is given by

$$\lambda_t = \lambda_0 + \lambda_1 X_t. \tag{10}$$

[7] Ang and Piazzesi (2003); Ang et al. (2007); Dai and Philippon (2005); Rudebusch and Wu (2004).

Taking equation (5) as the nominal pricing kernel which prices all bonds in the economy, the total gross return of any bond n satisfies $E_t^{\mathcal{P}}[(1+R_{n,t+1})M_{t+1}] = 1$ so that bond prices can be derived recursively as

$$P_{n+1,t} = E_t^{\mathcal{P}}[M_{t+1}P_{n,t+1}]. \tag{11}$$

The state dynamics of (8) together with (5) form an essentially affine Gaussian term structure model where bond prices are given by

$$P_{n,t} = \exp\left(A_n + B_n^\top X_t\right) \tag{12}$$

and the bond specific factor loadings follow the recursions

$$\begin{aligned} A_n &= A_{n-1} + B_{n-1}^\top(\mu - \Sigma\lambda_0) + \frac{1}{2}B_{n-1}^\top \Sigma\Sigma^\top B_{n-1} - \delta_0 \\ B_n^\top &= B_{n-1}^\top(\phi - \Sigma\lambda_1) - \delta_1 \end{aligned} \tag{13}$$

with $A_1 = -\delta_0$ and $B_1 = -\delta_1$. These difference equations can be derived by induction as described in Appendix 1.

Since continuously-compounded interest rates are related to the logarithm of bond prices, $i_{n,t}$ is given by

$$\begin{aligned} i_{n,t} &= -n^{-1}\log(P_{n,t}) \\ &= n^{-1}(-A_n - B_n^\top X_t) \\ &= a_n + b_n^\top X_t \end{aligned} \tag{14}$$

with $a_n = -A_n/n$ and $b_n = -B_n/n$ (Ang and Piazzesi, 2003). Interest rates are also affine functions of the state vector where the loadings b_n describe how much variation in the state dynamics is translated into the term structure of interest rates.

Since interest rates take an affine form, expected returns are also affine in the state variables. Recall, that the holding-period return on an n-period zero-coupon bond for τ periods in excess of the return on a τ-period bond is given by

$$xr_{n,t+\tau} = p_{n-\tau,t+\tau} - p_{\tau,t} - \tau i_{1,t} \qquad = A_{n-\tau} + B_{n-\tau}^{\top} X_{t+\tau} - A_n - B_n^{\top} + A_\tau + B_\tau^{\top} X_t.$$

Conditional expected returns can be computed using

$$E_t^{\mathcal{P}}[xr_{n,t+\tau}] = A_n^{xr} + B_n^{xr\top} X_t$$

where $A_n^{xr} = A_{n-\tau} - A_n + A_\tau$ and $B_n^{xr\top} = B_{n-\tau}^{\top}\phi^{\tau} + B_\tau^{\top} - B_n^{\top}$. The slope coefficients on expected excess returns can be written as

$$B_n^{xr\top} = B_{n-\tau}^{\top}[\phi^{\tau} - (\phi - \Sigma\lambda_1)^{\top}]$$

so that the one-period expected excess return follows

$$\begin{aligned} E_t^{\mathcal{P}}[xr_{n,t+1}] &= A_n^{xr} + B_n^{xr\top} X_t \\ &= B_{n-1}^{\top}\Sigma\lambda_0 - 0.5 B_{n-1}^{\top}\Sigma\Sigma^{\top} B_{n-1} + B_{n-1}^{\top}\Sigma\lambda_1 X_t. \end{aligned} \tag{15}$$

Expected excess returns compromises three terms (i) a Jensen's inequality term $-0.5B_{n-1}^{\top}\Sigma\Sigma^{\top}B_{n-1}$, (ii) a constant term premium $B_{n-1}^{\top}\Sigma\lambda_0$ and (iii) a time-varying term premium $B_{n-1}^{\top}\Sigma\lambda_1 X_t$.[8] As previously discussed, the term premium is governed by the vector λ_t. A negative sign leads to positive expected excess returns. If a positive shock ε_{t+1} hits one of the state variables, according to equation (12), this lowers (expected) bond prices and triggers declining holding-period returns. When λ_t is negative, the shock drives

[8] The difference between the pure form of the expectations hypothesis (PEH, no constant part at all) and general risk-neutrality is obvious. If investors are risk-neutral, they still price Jensen's inequality as apposed to the PEH. To bo conrete, Jensen's inequality needs not to be interpreted as a 'risk premium' since it does not compensate for explicit risk-taking but its existence only allows the expectations hypothesis (EH, constant part included) to hold.

up the logarithm of the pricing kernel so that the realized negative correlation between returns and the SDF leads to a positiv pricing kernel. Such a dynamic is consistent with risk-averse investors. In order to afford a positively sloped yield curve, at least one parameter in the vector of market prices of risk must be significantly negative. In contrast, when λ_t is positive, this works like a hedge, since the shock drives down the pricing kernel so that bond returns are positively correlated with the pricing kernel. In sum, risk premia are negative.

The $A_0(N)$ model of the yield curve reveals that a non-zero λ_t vector affects the long-run mean of the term structure so that on average, it can be upward-sloping as confirmed by the stylized facts. The pure expectations hypothesis (PEH) is a contradiction to this observation as it would postulate an economic environment in which investors on average expect rising short-term interest rates. In fact, the PEH predicts a mean yield curve that is flat or even slightly falling due to Jensen's inequality. If $\lambda_0 \neq 0$ and $\lambda_1 = 0$, the expectations hypothesis (EH) holds. Time-varying market prices of risk $\lambda_1 \neq 0$ can be understood as a rejection of the EH so that yield spreads do not necessarily predict future changes in interest rates but rather time-varying dynamics of term premia.

2.2 A Two-Factor Affine Term Structure Model

The first question in estimating an ATSM surrounds the choice on the general model - whether it is Gaussian with constant variance of the state variables or whether it can be described as a stochastic volatility model in a CIR-style format. This basically means to ask wether observed excess returns and the failure of the EH is produced by stochastic market prices of risk or by stochastic interest rate volatility. Dai and Singleton (2002) clearly argue for the former force. Furthermore, the number of risk factors (state variables) driving the underlying interest rates must be determined. According to the level, slope and curvature evidence of Litterman and Scheinkmann (1991) it is reasonable to work with two or three factors in order to adequately fit bond yields and so, most research follows this line. It must also be evaluated whether these factors are treated as latent or observable factors. Initially, ATSMs have been estimated with pure latent variables but the attempt to combine macroeconomic and finance topics in coherent models breed the inclusion of observable macro variables (Ang and Piazzesi, 2003; Dai and Philippon, 2005, among many other work). This strand of literature helps in exploring the linkages between interest rate behavior and the business cycle.

The choice of the number and form of risk factors is not the only preselection for the estimation. Any sensible parametrization of ATSMs must be theoretically admissible and econometrically identified. The theoretical model is *admissible* if it rules out negative conditional variances and negative short-term interest rates. From a statistical point of view, factor models need to be specified in a sense that one cannot find a 'rotation' of the risk factors that leaves bond yields unchanged. An ATSM is said to be in its canonical form if finding restrictions on the state-space system makes it possible to get it econometrically identified (Dai and Singleton, 2000).[9]

A second estimation question mainly deals with the set of bond prices or yields with which one can estimate an ATSM. The frequency of interest rates typically ranges from weekly up to quarterly observations depending on the aim what the yield curve model tries to capture. It may be reasonable to use CIR-models on a weekly basis since stochastic volatility emerges mainly at short frequencies. Quarterly data may be applied in a Gaussian setting. This is more likely the case in the macro-finance literature in which the macroeconomic model anyway works with constant variances of its state variables. The estimation can become quite difficult if the sample on interest rate data is too short to accurately provide reliable information about the data-generating dynamics of the risk factors. Due to highly persistent interest rates, the sample may suffer from a sufficient

[9] The problem is that specific numerical values of the underlying parameters give rise to the same term structure. In this respect, invariant transformations of the original ATSM by restricting and normalizing specific parameter constellations before the estimation is a procedure to guarantee identification of the model and to present it in its canonical form. Dai and Singleton (2000) show that if one restricts a specific set of parameters, this allows to treat the more 'interesting' parameters to the econometrician as free parameters.

number of mean-reversion observations so that estimates on long-term expectations of the short rate might get distorted. To overcome this problem, some studies include survey information on the expected path of the short rate that support to pin down the estimated parameters of the data-generating drift of the state variables (Kim and Orphanides, 2005).

The following model to be estimated applies the sketch of Section (2.1) with the $A_0(N)$ workhorse among the no-arbitrage models as mostly adopted by monetary economists. The motivation of estimating the Gaussian model specification results from previous studies that document the overwhelmingly success in fitting historical behavior of bond yields and basic diagnostics on the observed empirical puzzles when testing the expectations hypothesis within single-equations regressions. Many studies on these countries differ in model specification, data selection and the use of survey data which make a comparison of cross-country results awkward. Moreover, some studies simplify the model so as to allow only for constant risk premia if estimated jointly with a DSGE model whose solution only produces homoscedatic pricing kernels.[10] Therefore, I restrict the estimations of the ATSMs for the US, UK and Germany to the two-factor Gaussian term structure model $A_0(2)$ for each country.

For estimation, the theoretical model is cast into state-space form where the transition equation and the measurement equation are built in accordance with the theoretical model. I repeat the basic equations of the ATSM as discussed in Section (2.1) and link them to the state-space from. The transition equation consists of two latent factors which follow a simple VAR(1) process with $X_t = [X_{1,t}, X_{2,t}]^\top$ and

$$X_t = \phi X_{t-1} + \Sigma \varepsilon_t \tag{16}$$

[10] Most work on affine term structure models rely on US data. See for instance, Adrian and Wu (2009); Bolder (2006); D'Amico et al. (2008); Dai and Singleton (2002); Duffee (2002); Dai and Philippon (2005) as well as Kim and Orphanides (2005); Kim and Wright (2005); Lemke (2006); Pericoli and Taboga (2008); Rudebusch and Wu (2007). For Germany, studies have been carried out by Cassola and Luís (2003); Fendel and Frenkel (2005) and for the UK Bianchi et al. (2009).

where ϕ is upper-triangular, Σ is a 2×2 constant with diagonal elements and the distribution of the factor innovations is $\varepsilon_t \sim (0, I_2)$. Following Dai and Philippon (2005), the means of the risk factors are normalized to zero ($\mu = [0, 0]^\top$) and the short rate is given by

$$i_{1,t} = \delta_0 + \delta_1^\top X_t \tag{17}$$

where δ_1 takes on the value $[1, 1]^\top$. To price all bonds, the stochastic discount factor takes the form:

$$M_{t+1} = \exp\left(-i_{1,t} - 0.5\lambda_t^\top \lambda_t - \lambda_t^\top \varepsilon_{t+1}\right) \tag{18}$$

and the vector of market prices is

$$\lambda_t = \lambda_0 + \lambda_1 X_t. \tag{19}$$

Bond yields satisfy

$$i_{n,t} = a_n + b_n^\top X_t \tag{20}$$

with initial conditions $a_0 = b_0 = 0$ and $a_1 = \delta_0$ and $b_1 = \delta_1$. Since Equation (20) represents the measurement equation, we may add a measurement error so that we get

$$\tilde{i}_{n,t} = a_n + b_n^\top X_t + u_t \tag{21}$$

with the simple specification that

$$u_t \sim (0, h^2 I_n).$$

This means that the difference between the theoretical and observed yields has the same variance for all maturities.[11]

The empirical analysis uses quarterly data for the US, UK and Germany (see Table 1 and the specification of the observation vector). Since interest rates are annualized but the model defines the length of a period as unit of time, the measurement equation has to be multiplied by 400. It implies that the model parameters are those corresponding to quarterly continuously-compounded yields and so are the time-series dynamics of the state variables but the measurement error is in annualized terms. As pointed by Lemke (2006), this strategy circumvents possible numerical difficulties for the estimation of the measurement error that would be otherwise very low when working with quarterly and not with annualized yields.

Table 1: Data for Estimation $A_0(2)$-model

	USA	GER	UK
Yields	Nelson-Siegel	Nelson-Siegel	Spline
Short Rate	LIBOR	FIBOR	Spline
Source	FED	BuBa	BoE
Sample	1972Q1:2008Q3	1973Q1:2008Q3	1970Q1:2008Q3
Measurement equation			
$n^\top$	$(1,4,12,20,28,40)^\top$	$(1,4,8,20,28,40)^\top$	$(1,4,8,12,20,28,40)^\top$

Before estimating the $A_0(2)$-model with maximum likelihood and the Kalman filter, some last technical comments are in order.[12] The parameters to be estimated are stacked into the vector $\theta = \left\{\text{vec}(\phi), \text{vec}(\Sigma), \lambda_0^\top, \text{vec}(\lambda_1), h^2\right\}^\top$. To assure that some parameters fulfil admissability conditions in numerical optimization, the likelihood function is

[11] I normalize the model by imposing the following restriction: (i) ϕ is upper-triangular, (ii) Σ is diagonal, (iii) the mean μ of the latent factors are zero and (iv) the loadings δ_1 on the short rate are each set to one. I further set δ_0 to the long-run mean of the short rate before estimation in order to reduce the number of parameters to be estimated.

[12] See Appendix 2 for details on the estimation.

reparameterized with the help of auxiliary parameters.[13] To begin the maximization procedure, the initial vector of starting values is based on a VAR(1) estimation with the 40-quarter yield and the difference between the 1-quarter yield and the 40-quarter yield taken as state variables. The VAR allows to load the initial matrices of the transition equation. Maximization of the likelihood is performed with Matlab in a two-stage maximization routine: first, the numerical Simplex routine *fminsearch* with a maximum of 3000 iterations is carried out; after this round, taking the parameters of the Simplex as initial vector, the derivative-based optimizer *fminunc* refines the parameters estimates.[14] Standard errors are computed in line with the quasi-maximum likelihood variance-covariance matrix of the estimated parameters as described in Hamilton (1994, section 5.8).[15]

3 Evidence on International Term Premia

Table (2) contains the maximum likelihood estimates of the parameters and associated t-statistics for the three countries subject to the cross-equations restrictions implied by the no-arbitrage assumption. An inspection of the state equation dynamics reveals that the first factor is highly persistent for all countries. In particular, in the UK, it nearly hits the boundary condition of 1 which might indicate to a non-stationary time-series property. The persistence of the second risk factor is smaller, though the low mean-reverting feature is apparent, too. This observation is basically the main result of all factor-based yield curve models, e.g. models based on principal component analysis or the dynamic Nelson-Siegel model (Diebold and Li, 2006). To this end, the variance structure confirms that the second factor is more volatile than the first one so that we can speak about the first variable as describing level movements of bond yields; whereas the second factor causes

[13] Especially, the diagonal elements of ϕ need to be smaller than 1 for stationarity. This is guaranteed by introducing $\phi_{ii}^{aux} = -\log(.999/\phi_{ii} - 1)$. In addition, the covariance matrix Σ needs to be strictly positive which is derived by setting $\Sigma_{ii}^{aux} = \log(\Sigma_{ii})$. Converting the two auxiliary matrices back in its original form reveals that the true values always fulfil the admissibility restrictions.

[14] A similar but much more intensive hands-on procedure is proposed by Duffee (2009). The written Matlab code is available on request.

[15] The Matlab function to calculate the standard errors is partly provided by Piazzesi and Schneider (2007) and Eric Jondeau on their websites.

the term structure to flip. The stationary assumption of the risk factors imposed in the underlying model may exhibit one decisive weakness. For a sufficiently long horizon, expected short-term interest rates must inevitably converge to its long-run mean, i.e. the constant part of the short rate equation δ_0. Thus, movements of forward rates with long maturity are always the reflex of time-varying forward premia. Some ATSMs modify the short rate process with the help of shifting endpoints so that long-run forecasts do not necessarily coincide with the constant term δ_0 (see Kozicki and Tinsley, 2001). This is mainly justified by varying inflation perceptions on part of market participants. The estimated models for Germany, US and UK reveal that the short rate does not asymptote at the considering maturities so that this potential drawback does not hold.[16]

Since affine yield curve models are able to extract term premia from observed bond prices, a special focus lies on the derived market prices of risk. As expected, the constant term λ_0 is negative at least for one entry in the USA, GER and UK so that the yield curve is on average upward sloping. In contrast to the time-varying components of the state prices, estimated parameters of the constant price term are rather small. This suggests that risk premia are driven by an important time-varying component as displayed by the λ_1 values in absolute terms. However, estimated standard errors and corresponding t-statistics indicate to econometrically insignificance for some individual parameters so that inference based on individual estimates should be exercised wit caution.[17] It is for that reason to abstract from an economic interpretation of single risk prices parameters. However, as pointed out by many other studies, it is hard to pin down the single parameters of market prices of risk so that lack of significance does not necessarily indicate to a poor model fit; they are rather essential to fit the data (Ang and Piazzesi, 2003; Hordahl et al., 2006; Moench, 2008).

[16] I also modified the log-likelihood function in line with Chernov and Mueller (2008) by adding a term premium component to the log-likelihood function. It introduces an additional burden and uses term premia as a last resort in fitting yields. It basically means that the model first tries to fit yields via the expectations hypothesis. If this does not work, it let term premia to do it. It turned out that the penalty did not alter the results at all.

[17] One line of technical defense is that standard errors calculated with Matlab tend to be higher than with other software programs such as Fortran (Duffee, 2009).

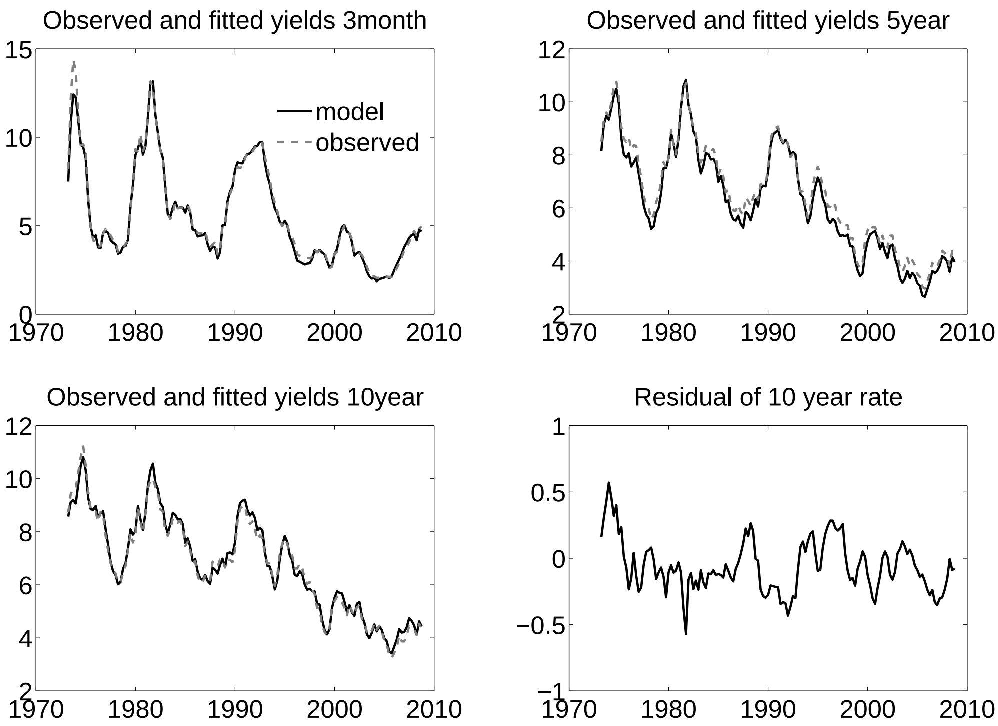
Table 2: Maximum Likelihood Parameter Estimates for $A_0(2)$

	USA		GER		UK	
ϕ_{1n}	0.960	0.070	0.967	0.038	0.998	-0.012
	(151.74)	(2.97)	(50.60)	(1.80)	(497.0)	(-0.92)
ϕ_{2n}	0	0.874	0	0.920	0	0.903
	(-)	(31.60)	(-)	(31.30)	(-)	(63.31)
$\Sigma_{1n} \times 10^2$	0.16	0	0.10	0	0.14	0
	(10.39)	(-)	(10.72)	(-)	(13.72)	(-)
$\Sigma_{2n} \times 10^2$	0	0.15	0	0.15	0	0.20
	(-)	(6.49)	(-)	(5.92)	(-)	(9.53)
δ_0	0.015		0.014		0.021	
	(-)		(-)		(-)	
λ_{01}	-0.125		-0.144		-0.079	
	(-2.73)		(-4.47)		(-3.86)	
λ_{02}	-0.260		0.014		-0.014	
	(-2.16)		(0.10)		(-0.14)	
$\lambda_{1(1n)}$	-22.821	73.282	-28.661	75.997	-6.271	-0.460
	(-6.88)	(10.96)	(-1.50)	(2.94)	(-6.44)	(-2.522)
$\lambda_{1(2n)}$	-26.129	25.269	-20.418	65.072	2.530	27.673
	(-6.48)	(3.25)	(-2.85)	(11.73)	(1.99)	(256.6)
h^2	0.267		0.260		0.231	
	(16.98)		(8.06)		(20.30)	
LR-Test:	36.63		2.158		2.003	
p-value:	0.00		0.14		0.16	

Notes: Parameter estimates of the $A_0(2)$ model based on ML and Kalman filter with t-statistics in parentheses. The parameters for which the t-statistic is not reported have been either restricted for admissability or estimated in advance to reduce the number of parameters to be estimated.

To see this, Figure (1) exemplary provides a plot of selected model-implied yields and its observed counterparts for Germany. The model fits the data well which should not come as a surprise due to its flexibel specification. The measurement error is quite small and quantified with 26 basis points in annual terms for Germany. Similar results are obtained for the US and UK.

Figure 1: Fitted and Observed Yields for Germany



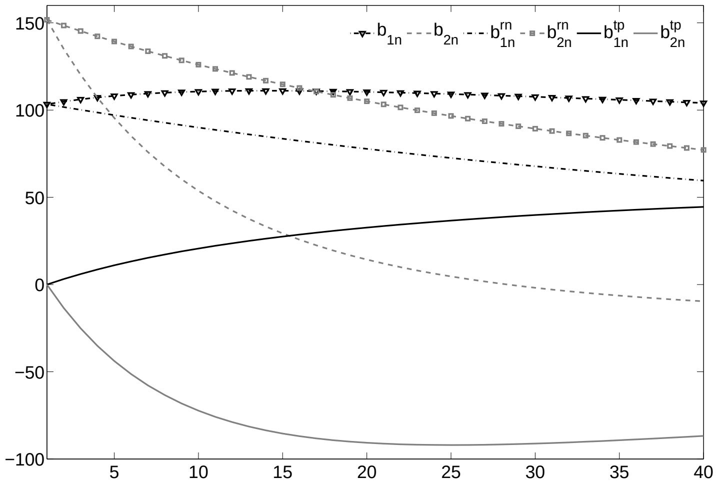
Notes: Model-implied yields are calculated with the individual yield loadings of the measurement equation.

To further reveal the deep characteristics of the estimated model, fitted yields can be decomposed to ask at any point in time, how much of the bond yield and forward rate corresponds to expected future interest rates and how much to yield- and forward term premia, respectively? Appendix 1 describes how to extract the different term premia concepts, risk-neutral yields and forward rates as well as model-implied expected one-period short-term interest rates. Figure (2) pictures the loadings of the German yield curve to one-standard deviation of shocks to the factors in basis points. The first factor is the level factor that induces an equal change in yields of all maturities (b_{1n}); whereas the second factor induces the curve to rotate (b_{2n}). When decomposing these responses into the two components of interest rate determination, i.e. the average expected short rate and the yield term premium, interest rates at the short end are mainly driven by the expectational part b_{1n}^{rn} of the first factor. At longer maturities, yield changes are mainly dominated by the growing term premium component of the first factor (b_{1n}^{tp}). As regards changes in the second risk factor, rotating yield curve dynamics are mainly caused by the increasing term premium component and not by the expectational part. This means that the presence of a normally sloped term structure indicates only partly to increasing short-term interest rate expectations. The main effect comes from the high b_{2n}^{tp} component at the long end relatively to the short end. Note that, in times of a normal spread, the second risk factor takes on negative values which triggers bond yields to rise due to the term premium loading.

The decomposition can also be displayed from a time-series perspective. The upper left panel of Figure (3) displays the 10-year yield and the decomposition into its risk-neutral level and the yield term premium. As can be seen, there is ample evidence for the time-varying component of the yield premium, although the latter only partly contributes to the run-up in bond yields through the early 1980s or 1990s for Germany. In the US, we rather see a mixture picture, where indeed both risk premia and risk-neutral yields account for the high interest rate levels.[18] What is eye-catching for all three countries is that after any one yield peak, short-rate expectations as imbedded in risk-neutral yields fell much faster than observed long-term yields indicating to an important premium component after the peaks. The lower left part of Figure (3) presents a decomposition for the 5-year forward rate. There is a sharp divergence between the time series of forward rates and expected

[18] The US yield curve decomposition is not reported here but a similar result can be found in Rudebusch et al. (2007) for comparison.

Figure 2: Instantaneous Yield Curve Response for Germany



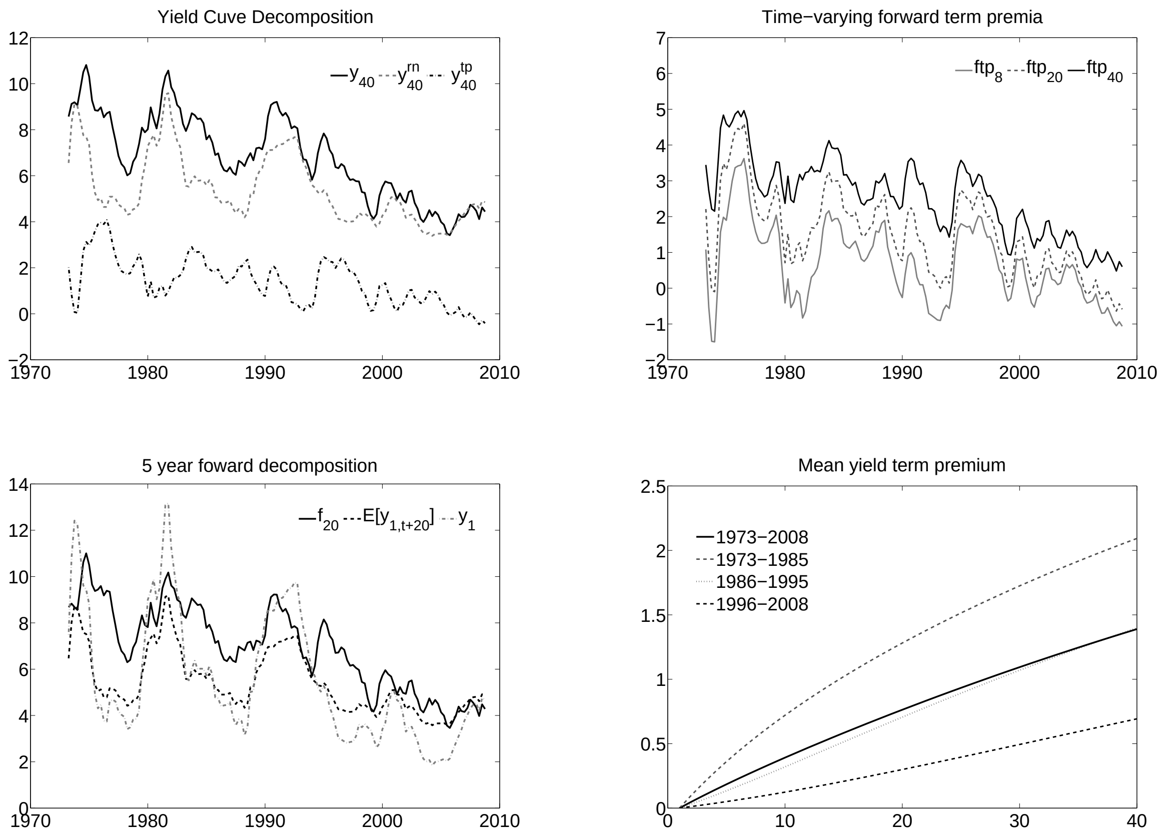
Notes: Factor loadings on yields $b_{1,2n}$, expected average short rates $b^{rn}_{1,2n}$ and yield term premia $b^{tp}_{1,2n}$ to one standard deviation shock for the German $A_0(2)$ model. Bond yields are decomposed as $i_{n,t} = n^{-1}\sum_{j=0}^{n-1} E_t[i_{t+j}] + \phi_{n,t}$.

future interest rates. The expected one-period rate in 5 years tracks the current short rate through the four policy-induced rate declines much more better in terms of dynamics and time of reversals. It confirms the finding that the (expected) short rate process follows a 'random-walk' pattern with the best estimate closely linked to the current level of the short rate, in particular for shorter maturities (Mankiw and Miron, 1986). Moreover, Cochrane and Piazzesi (2008) find for the US that expected future rates decline even faster than current rates.[19] Market participants know that in times of rate cutes, these will likely to continue to fall. With forward rates remaining unchanged, extra returns can be earned on buying the forward contracts. The opposite holds in an increasing expected short-rate environment where forward premia are rapidly eliminated. The term structure models basically recommend to 'get out' after the bottom of the short rate is reached.

[19] The authors build a modified affine term structure model based on monthly data. Their findings can be hardly captured in the quarterly frequency of the estimated model in this section.

To summarize, both forward premia and yield premia derived by the affine model show considerable dynamics with a secular decline starting at the beginning of the mid 1990s (upper and lower right panel of Figure 3). The analysis clearly shows that interest-rate risk premia are positively correlated with the term spread and negatively with short-term interest rates. In an environment of rising (expected) short rates, there are two effects in opposite directions: first, with fixed risk premia, long-term bond prices may fall; and second, a falling risk premium induces bond prices to rise and long-term yields to decline. The evidence shows that the second effect tends to dominate the first. For that reason, the slope of the yield curve falls as interest rates are rising, leading to a positive correlation between the spread and expected excess returns. This is exactly the story what the expectational puzzle in its reduced regression form of Campbell and Shiller (1991) tells about.

Figure 3: Decomposing the German Yield Curve



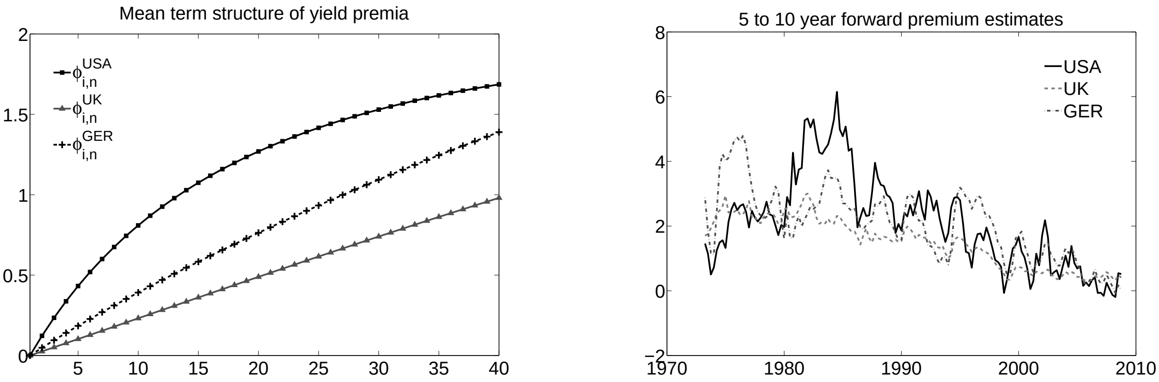
Further characteristics of international risk premia are sketched out in the upper left graph of Figure (4). The mean of the term structure of yield premia is rising with time to maturity for all three countries, with the USA averaging at 1.7 per cent for a 10-year bond, followed by Germany with 1.4 an UK with 1.0 per cent over the sample period 1973Q1:2008Q3. As already observed for Germany, these have generally tended to trend downwards over time. The average forward term premia for 5 up to 10 year forward contracts for all three countries have been in a range between -1 to $+2$ percentage points in the last ten years of the sample. In addition, the panel clearly indicates to co-movements in the same directions on an international level.[20] Still, for the UK, the time-varying component of term premia are much more smaller than for the US and Germany.

What can also be read from the data is the considerable fall of UK term premia in 1997 that were accompanied by falling long-term bond yields. This fact coincides with two major decisions in UK monetary policy, i.e. the given operational independence of the Bank of England in May 1997 and the Monetary Policy Committee's announcement of a symmetric inflation target of 2.5 percentage points for annual RPIX inflation in June 1997. These events have altered the shape of the yield curve considerably; they allow for a straightforward economic interpretation: inflationary risk seems to drive risk premia at the long end of the yield curve. Piazzesi and Schneider (2006) find that the UK real yield curve is on average downward sloped, while the nominal yield curve slopes up. In May and June 1997, the two curves behaved differently, with the real curve remaining unchanged and the nominal curve declining and flattening. This picture presumably reflects the effect of falling inflation expectations *and* lower inflation risk premia as estimated by the $A_0(2)$ for the UK.

According to the expectations hypothesis (EH), the yield on a n-period bond should increase one-to-one when the term spread widens. Evidence, however, reports the opposite

[20] These findings are supported by Diebold et al. (2008). The authors identify a significant 'global' yield curve factor that accounts for much of the variation in international yield curve dynamics.

Figure 4: International Risk Premia

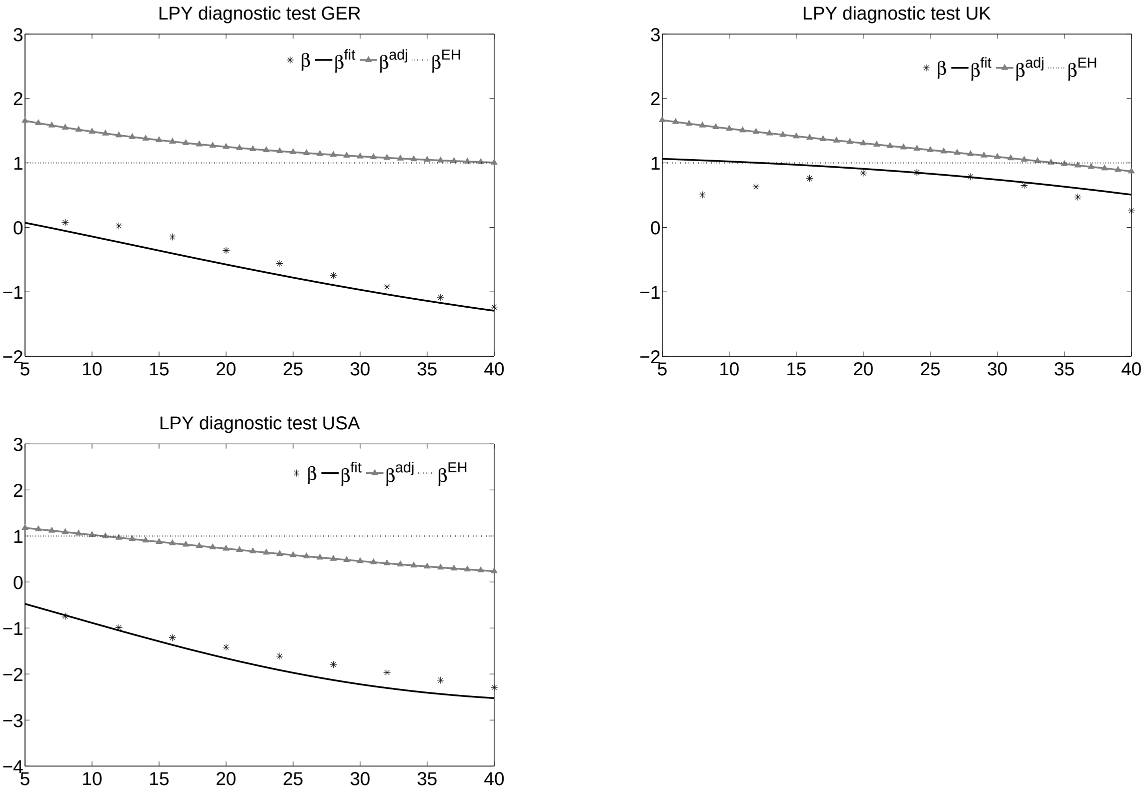


with a negative relationship between bond yield changes and the slope of the yield curve in the presence of large and time-varying term premia.[21] On this regard, Dai and Singleton (2002) have proposed to run two diagnostics for estimated ATSMs to test the ability of the model at its maximized parameter values to reproduce the stylized facts on the expectational puzzle. The first test asks whether the model shares the property that the pattern of the sample coefficient of fitted yields from an regression of yield changes onto the scaled yield spread matches the CS-regression of actual yield data - the authors call it the LPY(i) test. Matching LPY(i) means that the ATSM describes the historical behavior of yields und der the $\mathcal{P}$-measure. In addition, the authors suggest to run a second kind of test -LPY(ii)- to concentrate on realized risk premia. If the model captures the risk-neutral dynamics well, a CS-regression with risk-adjusted yields changes onto the scaled yield spread should give a coefficient of unity.

In Figure (5), I plot the results for the three countries. As in Campbell (1995), the graphs confirm that the historical coefficients are significantly lower than one and decreasing with time to maturity. The results for the UK build an exception with a small hump at a maturity of 20 quarters. Moreover, for the US and Germany, the coefficients of

[21] See the extensive literature on the failure of the expectations hypothesis: Mankiw and Miron (1986), Campbell and Shiller (1984, 1991), Hardouvelis (1994), Rudebusch (1995), Tzavalis and Wickens (1997), Rudebusch and Wu (2007), Bekaert et al. (1997), Bekaert and Hodrick (2001), Bekaert et al. (2002), Cuthbertson and Nitzsche (2006).

Figure 5: Yield Curve Fitting Diagnostics



Notes: Regression coefficients of (adjusted) yield changes on the scaled yield spread. Model implied expected excess returns are calculated as $e_{n-h,t+h} = E_t^{\mathcal{P}}[p_{n-h,t+h} - p_{n,t} + p_{h,t}]$ with regression diagnostics LPY(i): $i_{n-h,t+h} - i_{n,t} = \alpha_n + \beta^{fit} h(i_{n,t} - i_{1,t})/(n-h)$ and LPY(ii): $i_{n-h,t+h} - i_{n,t} + 1/(n-h)e_{n-h,t+h} = \alpha_n + \beta^{adj} h(i_{n,t} - i_{1,t})/(n-h)$. The diagnostic tests are carried out for a holding period of one year ($h = 4$ in the quarterly model). The stars (β) are the coefficients of the original data, the solid line (β^{fit}) are the coefficients for model-implied yields, β^{adj} represent the results for risk-adjusted yield changes and β^{EH} is the EH-benchmark.

the model-implied yields follow closely the coefficients of the original data set with downward sloping features. The results for the UK are somehow mixed where model-implied coefficients coincide with β only from 5-year bond yields onward to longer maturities.

Whether the model matches the LPY(ii) test mainly relies on the ability to generate highly persistent market risk premia. If this is so, risk-adjusted yield changes regressed on the scaled slope should result in a horizontal line with parameter values of 1. The grey lines β^{adj} display the model-implied coefficients. What we can observe is that the risk premium goes always in the right direction towards the EH-consistent line. Especially for Germany and UK, the test of LPY(ii) is positive and significant for longer maturities; meanwhile the opposite holds for the US where the risk-adjusted line converges to β^{EH} at shorter maturities.

To comprehend the findings, some further considerations are in order. The relatively high success of LPY(i) and LPY(ii) depends on the modeling assumptions, in particular on the flexible prices of risk that vary with the level of the sources of risk. If not modeled in this way, the test diagnostic would be rather disappointing.[22] Moreover, the assumption of a zero factor correlation (ϕ =diag) performs poor in matching LPY(i) and LPY(ii) although a conventional likelihood ratio test would not reject the restriction of zero-factor correlation for Germany and the UK.[23] It is therefore the combination of market prices of risk and factor correlation that allows to make LPY positive (see also Dai and Singleton, 2002, for this result).

To this end, the fact that only one end of the risk-adjusted coefficients along the yield curve successfully lies within the EH-range, may be due to the inability of the $A_0(2)$ to reproduce all dynamics within the yield curve. For instance, money markets are regularly exposed to non-continuous distortions stemming from 'flight to quality' or regulatory issues. This lack can be rectified by the inclusion of a third or even a fourth risk factor as outlined by Dai and Singleton (2002) among others. It can be assumed that this modification might support the risk-adjusted coefficient line to fall within the EH-theoretical line across all maturities.

4 Final Remarks

Affine term structure models are a powerful tool to extract bond premia from observable bond prices. I have estimated a two factor Gaussian term structure model for three

[22] If I estimate the model with the assumption that λ_1 has zero entries in its off-diagonal elements or if it is even empty, the risk-adjusted coefficients would not lie near β^{EH}.

[23] The restricted model implies a diagonal matrix of ϕ. The LR statistic is χ^2 with 1 degree of freedom. The 5 and 1 percent critical values are 3.84 and 6.64.

countries, the Unites Stated, the United Kingdom and Germany. Despite the well-known burden of estimating such models, I obtain compelling results: there is a clear downward trend of international excess returns earned on holding long-term government bonds. The empirical analysis reveals that the Gaussian subclass of ATSMs with correlated factors and flexible risk parameters can to a large extent help resolving the expectational puzzles surrounding the expectations hypothesis at least at one end of the yield curve. The decomposition of international yield curve confirms the presence of a large time-varying component of term premia. The latter are positively correlated with the slope of the term structure and negatively with the short-term policy instrument of the central banks (provided that the 1 quarter rate is regarded as the policy rate).

Although the model does not capture any macroeconomic content in terms of the conduct of monetary policy or the impact of inflation and output on yield curve dynamics, the UK case in 1997 reveals that to a large extend inflation expectations as well as inflationary risk drive the long end of the yield curve. In this respect, Gürkaynak et al. (2006) point out that 'inflation targeters' such as the Bank of England with a credible inflation target successfully help to anchor market-participants views regarding the distribution of long-run inflation outcomes. The opposite can be found in the US where despite a clear downward trend of risk premia, the time-varying component is still much higher compared to the UK.

The estimated model with only latent factors is incapable to explain the interactions between monetary policy and the yield. Fortunately, a bulk of papers have began to explore the links between the long end of the yield curve and the macroeconomy in an integrated macro-finance framework.[24] What these paper do is to derive standard macro models and explore their term structure implications. However, they typically do not model feedback effects running from long-term yields to the macroeconomy. They lack an important transmission mechanism which needs to explored, in particular against the

[24] See Rudebusch et al. (2006); Ang et al. (2007); Rudebusch and Swanson (2008); Kato and Hisata (2008); Ang et al. (2008).

background of rising long-term yields in an environment of very low short-term interest rates which is a challenge for effectiveness of monetary policy.

Apppendix 1

The derivation of the difference equations follow the guess-and-verify strategy similar to the method of undetermined coefficients supposed by McCallum (1983). For convenience, the relevant starting equations are

$$
\begin{aligned}
X_t &= \mu + \phi X_{t-1} + \Sigma\varepsilon_t \\
P_{n,t} &= E_t^{\mathcal{P}}[M_{t+1}P_{n-1,t+1}] \\
M_{t+1} &= \exp(-i_{1,t} - 0.5\lambda_t^\top\lambda_t - \lambda_t^\top\varepsilon_{t+1}) \\
i_{i,t} &= \delta_0 + \delta_1 X_t
\end{aligned}
$$

Duffie and Kan (1996) guess a solution for bond prices as

$$P_{n,t} = \exp\left(A_n + B_n X_t\right).$$

For a one-period bond, it can be easily shown that

$$P_{1,t} = \exp(-i_{1,t}) = \exp(-\delta_0 - \delta_1 X_t). \tag{22}$$

Matching coefficients yields $A_1 = -\delta_0$ and $B_1 = -\delta_1$. Recursive solving and matching coefficients can also be applied to a n-period bond:

$$
\begin{aligned}
P_{n,t} &= E_t^{\mathcal{P}}[M_{t+1}P_{n-1,t+1}] \\
&= E_t^{\mathcal{P}}\left[\exp(-i_{1,t} - 0.5\lambda_t^\top\lambda_t - \lambda_t^\top\varepsilon_{t+1})\exp(A_{n-1} + B_{n-1}^\top X_{t+1})\right] \\
&= E_t^{\mathcal{P}}\left[\exp(-\delta_0 - \delta_1 X_t - 0.5\lambda_t^\top\lambda_t - \lambda_t^\top\varepsilon_{t+1})\exp(A_{n-1} + B_{n-1}^\top(\mu + \phi X_t + \Sigma\varepsilon_{t+1}))\right] \\
&= \exp(-\delta_0 - \delta_1 X_t - 0.5\lambda_t^\top\lambda_t + A_{n-1} + B_{n-1}^\top\mu + B_{n-1}^\top\phi X_t)E_t^{\mathcal{P}}\left[\exp((B_{n-1}^\top\Sigma - \lambda_t^\top)\varepsilon_{t+1})\right].
\end{aligned}
$$

The sources of uncertainty ε_{t+1} are assumed to be log-normal distributed with mean zero and variance $I_{N\times N}$ so that $E(e^{b\varepsilon}) = \exp(0.5bIb^\top)$. This result makes it possible to modify the expectations term of the recursive solution:

$$\begin{aligned}
P_{n,t} = &\exp(-\delta_0 - \delta_1 X_t - 0.5\lambda_t^\top \lambda_t + A_{n-1} + B_{n-1}^\top \mu + B_{n-1}^\top \phi X_t)\dots \\
&\exp((B_{n-1}^\top \Sigma - \lambda_t^\top) var_t(\varepsilon)(B_{n-1}^\top \Sigma - \lambda_t^\top)^\top \\
= &\exp(-\delta_0 - \delta_1 X_t - 0.5\lambda_t^\top \lambda_t + A_{n-1} + B_{n-1}^\top \mu + B_{n-1}^\top \phi X_t)\dots \\
&\exp(0.5B_{n-1}^\top \Sigma^\top \Sigma B_{n-1} - B_{n-1}^\top \Sigma \lambda_t + 0.5\lambda_t^\top \lambda_t).
\end{aligned}$$

Substituting $\lambda_t = \lambda_0 + \lambda_1 X_t$ and netting out gives

$$\begin{aligned}
= &\exp(-\delta_0 - \delta_1 X_t + A_{n-1} + B_{n-1}^\top \mu + B_{n-1}^\top \phi X_t)\dots \\
&0.5B_{n-1}^\top \Sigma^\top \Sigma B_{n-1} - B_{n-1}^\top \Sigma(\lambda_0 + \lambda_1 X_t)
\end{aligned}$$

To this end, matching coefficients yields

$$\begin{aligned}
\exp(A_n + B_n X_t) = &\exp(A_{n-1} + B_{n-1}^\top(\mu - \Sigma\lambda_0) + \frac{1}{2}B_{n-1}^\top \Sigma\Sigma^\top B_{n-1} - \delta_0 \\
&+ B_{n-1}^\top(\phi - \Sigma\lambda_1)X_t - \delta_1 X_t)
\end{aligned}$$

so that scalar A_n and vectors B_n with size $N \times 1$ follow complex difference equations

$$\begin{aligned}
A_n &= A_{n-1} + B_{n-1}^\top(\mu - \Sigma\lambda_0) + \frac{1}{2}B_{n-1}^\top \Sigma\Sigma^\top B_{n-1} - \delta_0 \\
B_n^\top &= B_{n-1}^\top(\phi - \Sigma\lambda_1) - \delta_1.
\end{aligned}$$

Continuously-compounded interest rates then follow

$$
\begin{aligned}
i_{n,t} &= -n^{-1}\log(P_{n,t}) \\
&= n^{-1}(-A_n - B_n^\top X_t) \\
&= a_n + b_n^\top X_t
\end{aligned}
$$

with $a_n = -A_n/n$ and $b_n = -B_n/n$.

It is straightforward to calculate model-implied forward rates. Since forward rates are defined as

$$f_{n,t} = p_{n+1,t} - p_{n,t}$$

they can be easily computed as

$$f_{n,t} = (A_{n+1} - A_n) + (B_{n+1}^\top - B_n^\top)X_t.$$

Risk-neutral yields and forward rates can be defined as those that would prevail if investors did not price risk (λ_t) and all other parameters remain unchanged. The simple recursions for deriving risk-neutral rates can be defined as

$$
\begin{aligned}
A_n^{rn} &= A_{n-1}^{rn} + B_{n-1}^{rn,\top}\mu + \frac{1}{2}B_{n-1}^{rn,\top}\Sigma\Sigma^\top B_{n-1}^{rn} - \delta_0 \\
B_n^{rn,\top} &= B_{n-1}^{rn,\top}\phi - \delta_1.
\end{aligned}
$$

Bond risk premia are therefore computed as the difference between the model-implied yields and forwards and its artificial counterparts derived as if investors were risk-neutral.

$$
\begin{aligned}
\phi_{i_n,t} &= i_{n,t} - i_{n,t}^{rn} \\
\phi_{f_n,t} &= f_{n,t} - f_{n,t}^{rn}.
\end{aligned}
$$

If we want to impose the pure form of the expectations hypothesis (PEH), not only are investors insensitive to risk, but interest rates need to be deterministic. This is achieved by setting the variance-covariance matrix of the state variables equal to zero. Expected interest rates then follow

$$E_t[i_{1,t+n}] = f_{n,t}^{rnc}$$

where $f_{n,t}^{rnc}$ is the risk-neutral forward rate minus the convexity effect due to Jensen's inequality.

Appendix 2

This appendix introduces the statistical state-space model. It describes the tools that are employed for the estimation of various term structure models (see for subsequent work Harvey, 1990; Hamilton, 1994; Gourieroux and Monfort, 1997; Lemke, 2006).

Structure of the State Space Model

A state-space model is a representation of the joint dynamics of an observable random vector y_t with size $N \times 1$ that can be generally described by an unobservable state vector α_t with size $(r \times 1)$. It consists of a measurement equation and a transition equation. The former governs the evolution of the state vector, the latter specifies the empirical link between the set of observable variables and the state. Such a model is said to be Gaussian if the innovation to the state space are normally distributed. The representation can be written as

$$\alpha_t = c + T\alpha_{t-1} + D\eta_t \tag{23}$$

where c is a $N \times 1$ vector T is a $N \times N$ matrix and D is $r \times g$. The measurement equation is given by

$$y_t = b + M\alpha_t + \varepsilon_t \tag{24}$$

where M is a $n \times r$ matrix and b is an $N \times 1$ vector. The state innovations ($g \times 1$) and measurement errors ($N \times 1$) are normally distributed with the first two moments given by

$$\begin{pmatrix} \eta_t \\ \varepsilon_t \end{pmatrix} = \mathcal{N} \sim \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} Q & 0 \\ 0 & H \end{pmatrix} \right)$$

so that the disturbances are uncorrelated and independent to each other. The initial conditions are $\alpha_0 \sim (a_0, P_0)$ and $E(\eta_t \alpha_0), E(\varepsilon_t \alpha_0) = 0$.

Kalman Filter

The Kalman filter is an algorithm to estimate and extract the evolution of the unobservable state variables given the sequence of observable variables y_t via a feedback-control rule. It calculates linear least square forecasts of the state vector on the basis of data observed through time $t-1$. Optimality is achieved by minimizing the mean squared error (MSE) of the state variables. Thereby, the best *a priori* estimate of the state variable vector is $E_t(a_t|\mathcal{I}_{t-1})$ and its variance-covariance matrix is $E_t(\Sigma_t|\mathcal{I}_{t-1})$, conditionally on all information $\mathcal{I}$ available at time $t-1$ where Σ_t is the MSE. Since y_t has not been yet observed, the prediction of the observable variables takes the form of $E_t(\hat{y}_t|\mathcal{I}_{t-1})$ and the error of this forecast is $y_t - \hat{y}_{t|t-1}$ with variance-covariance matrix F_t after the measure has been observed. Continuing from here, the Kalman filter recursively updates the *a priori* estimates of the conditional means and (co)-variances to yield the *a posteriori* estimates to get $E_t(a_t|\mathcal{I}_t)$ and $E_t(\Sigma_t|\mathcal{I}_t)$, respectively.

The Kalman filter algorithm typically starts in **Step 1** with an initialization of the first two moments of the measurement equation

$$a_{0|0} = \bar{a}_0 \quad \Sigma_{0|0} = \bar{\Sigma}_0$$

The system of prediction equations can be summarized in **Step 2** as

$$\begin{aligned}
a_{t|t-1} &= c + T a_{t-1|t-1} \\
\Sigma_{t|t-1} &= T\Sigma_{t-1|t-1}T^{\top} + DQD^{\top} \\
\hat{y}_{t|t-1} &= b + M a_{t|t-1} \\
v_t &= \hat{y}_{t|t-1} - y_t \\
F_t &= M\Sigma_{t|t-1}M^{\top} + H.
\end{aligned} \tag{25}$$

After Step 2 y_t is now observed and the current value of a_t can be updated. To this, a coefficient matrix K_t (Kalman gain) is introduced with which the difference between the *a posteriori* and the *a priori* estimate of the variance-covariance matrix of the state variables can be minimized. It is a weighting matrix that defines to what extend the difference between the *a priori* estimate and the observed measure is weighted in the *a posteriori* estimate. It is proportional to the mean squared error of the forecast for the state vector and inversely related to the mean squared error of the observable vector. The higher K_t the greater is the weight of the observable measure on the *a posteriori* estimate of the state equation. The system of updating equation can be documented in **Step 3** as

$$\begin{aligned}
K_t &= \Sigma_{t|t-1}M^{\top}F_t^{-1} \\
a_{t|t} &= a_{t|t-1} + K_t(y_t - \hat{y}_{t|t-1}) \\
\Sigma_{t|t} &= \Sigma_{t|t-1} - K_t M \Sigma_{t|t-1}.
\end{aligned}$$

In **Step 4**, this procedure is repeated by setting $t = t + 1$ if $t < T$ and one goes back to Step 2.

The Kalman filter provides the sequence of conditional means and covariances for the relevant conditional distributions. In estimation, the initial mean and covariance of the state variables are calculated as their unconditional equivalents (provided that the transition equation is stationary).[25] Thus, $a_{0|0}$ is chosen as

$$a_{0|0} = (I - T)^{-1}c$$

and the covariance-variance matrix is given in a column vector as

$$\Sigma_{0|0} = \text{vec}[I - (T \otimes T)]^{-1}\text{vec}(Q).$$

Maximum Likelihood Estimation

If the parameters describing the state space are unknown, they can be estimated with maximum likelihood (ML). For a given distributional assumption of the innovations, the ML estimate of an unknown parameter set is the value that maximizes the probability density. For a linear Gaussian model (normality assumption of innovations) with a set of unknown elements stacked in the vector ϑ, the conditional density function of a simple VAR(1) process with $y_t \sim N(\mu, \Omega)$ and dimension N can be written in general as

$$f(y_t|\mathcal{I}_{t-1};\vartheta) = (2\pi)^{-N/2}|\Omega|^{-1/2}\exp\left[-\frac{1}{2}(y_t - \mu)^\top\Omega^{-1}(y_t - \mu)\right].$$

The joint density function from observation t through T satisfies

$$f(y_T|\mathcal{I}_{T-1};\vartheta) = \prod_{t=1}^{T} f(y_t|\mathcal{I}_{t-1};\vartheta)$$

[25] See Hamilton (1994, p. 378).

and the log-likelihood function is given as

$$\ln \mathcal{L}(\vartheta) = -\frac{NT}{2}\log(2\pi) - \frac{T}{2}\log|\Omega| - \frac{1}{2}\sum_{t=1}^{T}\left[(y_t-\mu)^\top \Omega^{-1}(y_t-\mu)\right].$$

The Kalman filter can be used to calculate the sequence of conditional means and (co)-variances so that for above state-space specification, the distribution of y_t conditional on $\mathcal{I}_{t-1}$ is given by (25)

$$y_t|\mathcal{I}_{t-1} \sim N(\hat{y}_{t|t-1}, F_t)$$

with $\hat{y}_{t|t-1} = b + Ma_{t|t-1}$. Accordingly, the log-likelihood function becomes

$$\ln \mathcal{L}(\vartheta) = -\frac{NT}{2}\log(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\left[\log|F_t| + v_t^\top F_t^{-1} v_t\right].$$

This function can be maximized with numerical optimization techniques (see Lemke, 2006, p.79): (i) choose an initial value for $\vartheta = \vartheta_0$, (ii) run the Kalman filter and store the sequences ϑ_t and F_t, (iii) use them to compute the log-likelihood and (iv) use an optimization procedure that repeats steps (i)-(iii) until a maximizer $\hat{\vartheta}$ has been found.

To this end, if the sample size is sufficiently large, the distribution of the maximum likelihood estimate $\hat{\vartheta}$ can be approximated as

$$\hat{\vartheta} = N(\vartheta_0, T^{-1}I^{-1})$$

where I is the information matrix. It can be estimated in two ways. The first way is to calculate the Hessian to get

$$I_H = -\frac{1}{T}\left[\frac{\partial^2 \ln \mathcal{L}(\vartheta)}{\partial\vartheta\partial\vartheta^\top}\right].$$

The second way is based on the outer-product estimate

$$I_{OP} = \frac{1}{T}\sum_{t=1}^{T}[h(\hat{\vartheta}, \mathcal{I})][h(\hat{\vartheta}, \mathcal{I})]^{\top}$$

where $h(.)$ denotes the the vector of derivatives evaluated at $\hat{\vartheta}$.

According to Hamilton (1994, section 5.8), the variance-covariance matrix for $\hat{\vartheta}$ can be then given as

$$Cov(\hat{\vartheta}) = \frac{1}{T}[I_H I_{OP}^{-1} I_H]^{-1}.$$

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